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ORLICZ MIXED INTERSECTION BODIES

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Abstract: In this paper, we investigate the Orlicz mixed intersection bodies and its properties. By using geometric analysis, we introduce the concept of Orlicz mixed intersection bodies and obtain the continuity and affine invariant property of the Orlicz mixed intersection bodies operator. By applying the integral methods and Steiner symmetrization, the affine isoperimetric inequality for Orlicz mixed intersection bodies is established.

Keywords: convex bodies; Orlicz mixed intersection bodies; Steiner symmetrization; affine isoperimetric inequality

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1 Introduction

Let $\mathcal{K}^n$ denote the set of convex bodies (compact, convex subsets with nonempty interiors) in Euclidean n-space $\mathbb{R}^n$; Let $\mathcal{K}_o^n$ denote the set of convex bodies containing the origin in their interiors in $\mathbb{R}^n$; $\mathcal{S}_o^n$ denotes the set of star bodies (about the origin) in $\mathbb{R}^n$. We use $|K|$ to denote the n-dimensional volume of a body K. Besides, write S^{n-1} for the unit sphere in $\mathbb{R}^n$.

For a compact set $K \subset \mathbb{R}^n$ which is star-shaped with respect to the origin, we will use $\rho_K = \rho(K, \cdot) : \mathbb{R}^n \backslash \{o\} \to [0, \infty)$ to denote its radial function. That is,

$$\rho_K(x) = \rho(K, x) = \max\{\lambda \geq 0 : \lambda x \in K\} \text{ for all } x \in \mathbb{R}^n \backslash \{o\}.$$

If ρ_K is continuous and positive, then K is called star body. In [1], Lutwak introduced the intersection body IK of each $K \in \mathcal{S}_o^n$, which is the star body with radial function

$$\rho(IK, u) = v(K \cap u^{\perp}), \quad u \in S^{n-1},$$

where $v(\cdot)$ denotes $(n-1)$-dimensional volume and $u^{\perp}$ is the hyperplane orthogonal to u.

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For $0 < p < 1$, the L_p-intersection body I_pK of $K \in \mathcal{S}_o^n$ is defined by (see, e.g., Cardner and Giannopoulos [2], Yuan [3])

$$\rho(I_pK, u)^p = \int_K |x \cdot u|^{-p} \mathrm{d}x, \quad u \in S^{n-1}, \tag{1.1}$$

where $x \cdot u$ is the usual inner product of $x, u \in \mathbb{R}^n$, and integration is with respect to Lebesgue measure.

Haberl and Ludwig [4] also gave the following definition of L_p-intersection body for convex polytopes and investigated its characterization. There is a different constant between the L_p-intersection body of (1.1) and the following L_p-intersection body in [5]. For $u \in S^{n-1}$ and $0 < p < 1$, the L_p-intersection body of $K \in \mathcal{S}_o^n$ is defined by

$$\rho(I_pK, u)^p = \frac{1}{\Gamma(1-p)} \int_K |x \cdot u|^{-p} \mathrm{d}x,$$

where Γ denotes the Gamma function.

Intersection body [1] played an important role for the solution of the celebrated Busemann-Petty problem, and found applications in geometric tomography [6], affine isoperimetric inequalities [7, 8] and the geometry of Banach spaces [9]. For more details about intersection body we refer the reader to [10–14].

Progress towards an Orlicz Brunn-Minkowski theory was made by Lutwak, Yang and Zhang [15, 16]. This theory plays such a crucial role that it is undeniably applied to a number of areas of geometry. The more development of the Orlicz Brunn-Minkowski theory can be found in [17–31]. Recently, Ma et al. [32] considered convex function $\varphi : \mathbb{R}\backslash\{o\} \to (0, \infty)$ such that $\lim_{t\to\infty} \varphi(t) = 0$, $\lim_{t\to 0} \varphi(t) = \infty$ and $\varphi(0) = \infty$. This means that φ must be increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. They assumed that φ is either strictly increasing on $(-\infty, 0)$ or strictly decreasing on $(0, \infty)$, and denoted by Φ the class of such φ. Further, for $K \in \mathcal{S}_o^n$, they gave the concept of Orlicz intersection body $I_\varphi K$ as the star body whose radial function is given by

$$\rho_{I_\varphi K}^{-1}(x) = \sup\left\{\lambda > 0 : \frac{1}{|K|} \int_K \varphi\left(\frac{|x \cdot y|}{\lambda}\right) \mathrm{d}y \le 1\right\}, \ x \in \mathbb{R}^n, \tag{1.2}$$

when $\varphi(t) = |t|^{-p}$, $0 < p < 1$, then $I_\varphi K = \frac{1}{|K|} I_pK$.

In this paper, we also consider the above convex function φ and assume that K, L are two star bodies (about the origin) in $\mathbb{R}^n$ with volume $|K|$ and $|L|$. If $\varphi \in \Phi$, then the Orlicz mixed intersection bodies $I_\varphi(K, L)$ of K, L as the star body whose radial function at $x \in \mathbb{R}^n$ is given by

$$\rho_{I_\varphi(K,L)}^{-1}(x) = \sup\left\{\lambda > 0 : \frac{1}{|K||L|} \int_K \int_L \varphi(\frac{|x \cdot (y-z)|}{\lambda}) \mathrm{d}y\mathrm{d}z \le 1\right\}, \tag{1.3}$$

when $\varphi(t) = |t|^{-p}$ with $0 < p < 1$, and $K = L$ in (1.3), then $I_\varphi(K, K) = I_p(K, K)$, where the radial function of $I_p(K, K)$ is given by

$$\rho(I_p(K, K), x)^p = \frac{1}{|K|^2} \int_K \int_K |x \cdot (y - z)|^{-p} \mathrm{d}y\mathrm{d}z.$$

Our main result deals with the affine isoperimetric inequality for Orlicz mixed intersection bodies. Here and in the following, for $K \in \mathcal{K}^n$, we denote by B_K the n-ball with the same volume as K centered at the origin.

Theorem 1.1 If $K, L \in \mathcal{K}_o^n$ and $\varphi \in \Phi$, then

$$|I_\varphi^*(K,L)| \geq |I_\varphi^*(B_K, B_L)|$$

with equality if K and L are dilates of each other and have the same midpoints.

2 Notations and Orlicz Mixed Intersection Bodies

For a body $K \in \mathcal{K}_o^n$, its support function, $h_K = h(K, \cdot) : S^{n-1} \to \mathbb{R}$ is defined by

$$h_K(u) = \max\{u \cdot x : x \in K\}, \quad u \in S^{n-1}.$$

For $c > 0$, the support function of the convex body $cK = \{cy : y \in K\}$ is

$$h_{cK}(x) = ch_K(x), \quad x \in \mathbb{R}^n.$$

Let $\mathrm{GL}(n)$ denote the group of linear transformations. For $T \in \mathrm{GL}(n)$, write T^t for the transpose of T, T^{-1} for the inverse of T and T^{-t} for the inverse of the transpose of T. The support function of the image $TK = \{Ty : y \in K\}$ is given by

$$h_{TK}(x) = h_K(T^t x), \quad x \in \mathbb{R}^n.$$

According to the definition of radial function ρ_K, for $K \in \mathcal{S}_o^n$ and $a > 0$, we easily get

$$\rho_K(ax) = a^{-1}\rho_K(x) \quad \text{and} \quad \rho_{aK}(x) = a\rho_K(x). \tag{2.1}$$

For $T \in \mathrm{GL}(n)$, the radial function of the image $TK = \{Ty : y \in K\}$ of $K \in \mathcal{S}_o^n$ is given by

$$\rho(TK, x) = \rho(K, T^{-1}x) \text{ for all } x \in \mathbb{R}^n. \tag{2.2}$$

The radial distance between $K, L \in \mathcal{S}_o^n$ is defined by

$$\tilde{\delta}(K, L) = \sup_{u \in S^{n-1}} |\rho(K,u) - \rho(L,u)|.$$

For $K \in \mathcal{S}_o^n$, define the real number R_K and r_K by

$$R_K = \max_{x \in K} \rho_K(x), \quad r_K = \min_{x \in K} \rho_K(x). \tag{2.3}$$

For $K \in \mathcal{K}_o^n$, then the polar body K^* of K is defined by

$$K^* = \{x \in \mathbb{R}^n : x \cdot y \leq 1, \forall y \in K\}.$$

It is easily proved that $(K^*)^* = K$ if $K \in \mathcal{K}_o^n$.

From the definitions of support and radial function, it follows obviously that for each $K \in \mathcal{K}_o^n$, we easily get

$$h_{K^*}(u) = \frac{1}{\rho_K(u)} \quad \text{and} \quad \rho_{K^*}(u) = \frac{1}{h_K(u)} \quad \text{for all } u \in S^{n-1}.$$

In the following write x, y for vectors in $\mathbb{R}^n$ and x', y' for vectors in $\mathbb{R}^{n-1}$. We will use (x', s), (y', s) for vectors in $\mathbb{R}^n = \mathbb{R}^{n-1} \times \mathbb{R}$. For a convex body K and a direction $u \in S^{n-1}$, let K_u denote the image of the orthogonal projection of K onto $u^\perp$, the subspace of $\mathbb{R}^n$ orthogonal to u. The undergraph and overgraph functions, $\underline{l}_u(K, \cdot) : K_u \to \mathbb{R}$ and $\bar{l}_u(K, \cdot) : K_u \to \mathbb{R}$, of K in the direction u are given by

$$K = \{y' + tu : -\underline{l}_u(K, y') \le t \le \bar{l}_u(K, y') \text{ for } y' \in K_u\}.$$

Therefore the Steiner symmetral $S_u K$ of $K \in \mathcal{K}_o^n$ in the direction u is defined by the body whose orthogonal projection onto $u^\perp$ is identical to that of K and whose undergraph and overgraph functions are

$$\underline{l}_u(S_u K, y') = \bar{l}_u(S_u K, y') = \frac{1}{2}\underline{l}_u(K, y') + \frac{1}{2}\bar{l}_u(K, y'). \tag{2.4}$$

For more details about the Steiner symmetrization we can refer to [33].

The following lemma will be required.

Lemma 2.1 (see [16], Lemma 1.2) Suppose $K \in \mathcal{K}_o^n$ and $u \in S^{n-1}$. For $y' \in \operatorname{relint} K_u$, the overgraph and undergraph functions of K in direction u are given by

$$\bar{l}_u(K, y') = \min_{x' \in u^\perp} \{h_K(x', 1) - x' \cdot y'\}$$

and

$$\underline{l}_u(K, y') = \min_{x' \in u^\perp} \{h_K(x', -1) - x' \cdot y'\}.$$

From the definition of Orlicz mixed intersection bodies (1.3) and φ is strictly increasing on $(-\infty, 0)$ or strictly decreasing on $(0, \infty)$, we have the function

$$\lambda \mapsto \int_K \int_L \varphi\left(\frac{|x \cdot (y - z)|}{\lambda}\right) \mathrm{d}y \mathrm{d}z$$

is strictly increasing on $(0, \infty)$ and it is also continuous. Thus we have

Lemma 2.2 If $K, L \in \mathcal{S}_o^n$, then for $u_0 \in S^{n-1}$,

$$\frac{1}{|K||L|} \int_K \int_L \varphi\left(\frac{|u_0 \cdot (y - z)|}{\lambda_0}\right) \mathrm{d}y \mathrm{d}z = 1$$

if and only if $\rho^{-1}_{I_\varphi(K,L)}(u_0) = \lambda_0$.

Lemma 2.3 (see [34]) Let $x = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$ and $y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n$. If $0 < m_1 \le x_k \le M_1$, $0 < m_2 \le y_k \le M_2$, $k = 1, \cdots, n$, then

$$\left(\sum_{k=1}^n x_k^2\right)\left(\sum_{k=1}^n y_k^2\right) \le \left(\frac{\sqrt{\frac{M_1 M_2}{m_1 m_2}} + \sqrt{\frac{m_1 m_2}{M_1 M_2}}}{2}\right)^2 \left(\sum_{k=1}^n x_k y_k\right)^2.$$

Lemma 2.3 implies that if $x, y, z \in \mathbb{R}^n$, then there exists a constant $c_0 \in (0,1)$ such that

$$|x \cdot (y-z)| \geq c_0 \|x\| \|y-z\|. \tag{2.5}$$

If K and L are bodies in $\mathbb{R}^n$, their multiplicative $d_{ab}(K,L)$ is defined by (see [35]) $d_{ab}(K,L) = \inf\{ab : a, b > 0, K \subseteq bL, L \subseteq aK\}$. Whenever we write

$$\frac{1}{a}\|x\| \leq \|x\|_K \leq b\|x\|, \quad \frac{1}{b}\|y\| \leq \|y\|_L \leq a\|y\|. \tag{2.6}$$

Denote by $c_\varphi > 0$ the constant to meet $c_\varphi = \min\{c > 0 : \max\{\varphi(c), \varphi(-c)\} \leq 1\}$ for each $\varphi \in \Phi$. And denote distant $d(K,L)$ of convex bodies K, L by

$$d(K,L) := \max\{|x-y| : x \in K, y \in L\}. \tag{2.7}$$

Lemma 2.4 If $K, L \in \mathcal{S}_o^n$ and $\varphi \in \Phi$, then for all $u \in S^{n-1}$, there exists a positive constant b and $c_0 \in (0,1)$ such that

$$\frac{c_0}{c_\varphi}\left|\frac{1}{bR_K} - \frac{b}{r_L}\right| \leq \rho^{-1}_{I_\varphi(K,L)}(u) \leq \frac{d(K,L)}{c_\varphi}.$$

Proof Let $x_0 \in S^{n-1}$ and $\rho^{-1}_{I_\varphi(K,L)}(x_0) = \lambda_0$. Then

$$\frac{1}{|K||L|}\int_K\int_L \varphi\left(\frac{|x_0 \cdot (y-z)|}{\lambda_0}\right)\mathrm{d}y\mathrm{d}z = 1. \tag{2.8}$$

We first obtain the upper estimate. From the definition of c_φ, either $\varphi(c_\varphi) = 1$ or $\varphi(-c_\varphi) = 1$. If $\varphi(c_\varphi) = 1$, by (2.7), we have

$$\begin{aligned}
\varphi(c_\varphi) = 1 &= \frac{1}{|K||L|}\int_K\int_L \varphi\left(\frac{|x_0 \cdot (y-z)|}{\lambda_0}\right)\mathrm{d}y\mathrm{d}z \\
&\geq \frac{1}{|K||L|}\int_K\int_L \varphi\left(\frac{d(K,L)}{\lambda_0}\right)\mathrm{d}y\mathrm{d}z \\
&= \varphi\left(\frac{d(K,L)}{\lambda_0}\right).
\end{aligned}$$

With the definition of φ, φ is strictly increasing on $(-\infty, 0)$ or strictly decreasing on $(0, \infty)$. So we can immediately obtain $c_\varphi \leq \frac{d(K,L)}{\lambda_0}$, i.e., $\lambda_0 \leq \frac{d(K,L)}{c_\varphi}$.

On the other hand, we obtain the lower estimate. Note that $\varphi(c_\varphi) = 1$, $x_0 \in S^{n-1}$, from

(2.5), (2.6) and (2.3), it follows that

$$\begin{aligned}\varphi(c_\varphi)=1&=\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|x_0\cdot(y-z)|}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z\\&\le\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|c_0\|y-z\||}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z\\&\le\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{c_0|\|y\|-\|z\||}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z\\&\le\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{c_0|\frac{1}{b}\|y\|_K-b\|z\|_L|}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z\\&=\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{c_0}{\lambda_0}\Big|\frac{1}{b\rho_K(y)}-\frac{b}{\rho_L(z)}\Big|\Big)\mathrm{d}y\mathrm{d}z\\&\le\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{c_0}{\lambda_0}\Big|\frac{1}{bR_K}-\frac{b}{r_L}\Big|\Big)\mathrm{d}y\mathrm{d}z\\&=\varphi\Big(\frac{c_0}{\lambda_0}\Big|\frac{1}{bR_K}-\frac{b}{r_L}\Big|\Big),\end{aligned}$$

where $b>0$ and $c_0\in(0,1)$. Since φ is strictly decreasing on $(0,\infty)$, we immediately get $c_\varphi\ge\frac{c_0}{\lambda_0}\big|\frac{1}{bR_K}-\frac{b}{r_L}\big|$, i.e., $\lambda_0\ge\frac{c_0}{c_\varphi}\big|\frac{1}{bR_K}-\frac{b}{r_L}\big|$. We complete the proof.

The following shows that the Orlicz mixed intersection bodies operator $I_\varphi:\mathcal{S}_o^n\to\mathcal{S}_o^n$ is continuous.

Lemma 2.5 Suppose $\varphi\in\Phi$. If $K_i,L_i\in\mathcal{S}_o^n$ and $K_i\to K\in\mathcal{S}_o^n$, $L_i\to L\in\mathcal{S}_o^n$, then $I_\varphi(K_i,L_i)\to I_\varphi(K,L)$.

Proof For $\varphi\in\Phi$ and φ is convex, continuous, and either strictly increasing on $(-\infty,0)$ or strictly decreasing on $(0,\infty)$, then for $u_0\in S^{n-1}$, we will show that

$$\rho^{-1}_{I_\varphi(K_i,L_i)}(u_0)\to\rho^{-1}_{I_\varphi(K,L)}(u_0).$$

Suppose $\rho^{-1}_{I_\varphi(K_i,L_i)}(u_0)=\lambda_i$, by Lemma 2.2, we have

$$\frac{1}{|K_i||L_i|}\int_{K_i}\int_{L_i}\varphi\Big(\frac{|u_0\cdot(y-z)|}{\lambda_i}\Big)\mathrm{d}y\mathrm{d}z=1.\tag{2.9}$$

From Lemma 2.4, there exists a positive constant b and $c_0\in(0,1)$ such that

$$\frac{c_0}{c_\varphi}\Big|\frac{1}{bR_{K_i}}-\frac{b}{r_{L_i}}\Big|\le\lambda_i\le\frac{d(K_i,L_i)}{c_\varphi}.$$

Since $K_i\to K\in\mathcal{S}_o^n$, $L_i\to L\in\mathcal{S}_o^n$, we have $d(K_i,L_i)\to d(K,L)>0$, $R_{K_i}\to R_K>0$, $r_{L_i}\to r_L>0$ and there exist α, β such that $0<\alpha\le\lambda_i\le\beta<\infty$ for all i. Let $\{\lambda_*\}$ be a convergent subsequence of $\{\lambda_i\}$, and suppose that $\lambda_*\to\lambda_0$, together with the continuous of φ and (2.9), we have

$$\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|u_0\cdot(y-z)|}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z=1,$$

which by Lemma 2.2 yields $\rho^{-1}_{I_\varphi(K,L)}(u_0) = \lambda_0$. This shows that

$$\rho^{-1}_{I_\varphi(K_i,L_i)}(u_0) \to \rho^{-1}_{I_\varphi(K,L)}(u_0).$$

But for radial function on S^{n-1} pointwise and uniform convergence are equivalent (see Schneider [8], p.54). Thus the pointwise convergence $\rho^{-1}_{I_\varphi(K_i,L_i)} \to \rho^{-1}_{I_\varphi(K,L)}$ on S^{n-1} completes the proof.

We next show that the Orlicz mixed intersection bodies operator is also continuous in φ.

Lemma 2.6 For $K, L \in \mathcal{S}^n_o$ and $\varphi_i \to \varphi \in \Phi$, then $I_{\varphi_i}(K,L) \to I_\varphi(K,L)$.

Proof Let $K, L \in \mathcal{S}^n_o$ and $u_0 \in S^{n-1}$. We will show that $\rho^{-1}_{I_{\varphi_i}(K,L)} \to \rho^{-1}_{I_\varphi(K,L)}$. For $\varphi \in \Phi$ with φ is convex, continuous, and either strictly increasing on $(-\infty, 0)$ or strictly decreasing on $(0,\infty)$, and let $\rho^{-1}_{I_{\varphi_i}(K,L)}(u_0) = \lambda_i$, i.e.,

$$\frac{1}{|K||L|}\int_K\int_L \varphi_i\bigg(\frac{|u_0\cdot(y-z)|}{\lambda_i}\bigg)\mathrm{d}y\mathrm{d}z = 1, \tag{2.10}$$

then together with Lemma 2.4 we have that there exists a positive constant b and $c_0 \in (0,1)$ such that

$$\frac{c_0}{c_{\varphi_i}}\bigg|\frac{1}{bR_K} - \frac{b}{r_L}\bigg| \le \lambda_i \le \frac{d(K,L)}{c_{\varphi_i}}.$$

Since $\varphi_i \to \varphi \in \Phi$, we have $c_{\varphi_i} \to c_\varphi > 0$ and thus there exist α, β such that $0 < \alpha \le \lambda_i \le \beta < \infty$ for all i. Denote by $\{\lambda_*\}$ the arbitrary convergent subsequence of $\{\lambda_i\}$, and suppose that $\lambda_* \to \lambda_0$, together with (2.10), we immediately have

$$\frac{1}{|K||L|}\int_K\int_L \varphi\bigg(\frac{|u_0\cdot(y-z)|}{\lambda_0}\bigg)\mathrm{d}y\mathrm{d}z = 1,$$

which by Lemma 2.2 yields $\rho^{-1}_{I_\varphi(K,L)}(u_0) = \lambda_0$. This shows that $\rho^{-1}_{I_{\varphi_i}(K,L)}(u_0) \to \rho^{-1}_{I_\varphi(K,L)}(u_0)$. Since the radial function $\rho_{I_{\varphi_i}(K,L)} \to \rho_{I_\varphi(K,L)}(u_0)$ pointwise on S^{n-1} they converge uniformly and hence $I_{\varphi_i}(K,L) \to I_\varphi(K,L)$.

Lemma 2.7 Suppose $\varphi \in \Phi$. For $K, L \in \mathcal{S}^n_o$ and a linear transformation $T \in \mathrm{GL}(n)$, then $I_\varphi(TK,TL) = T^{-t}(I_\varphi(K,L))$.

Proof Suppose $x_0 \in \mathbb{R}^n$ and

$$\rho^{-1}(I_\varphi(TK,TL), x_0) = \lambda_0. \tag{2.11}$$

Let $s = Ty$, $t = Tz$, then $|TK| = |\mathrm{det}T||K|$, $|TL| = |\mathrm{det}T||L|$, $\mathrm{d}s = |\mathrm{det}T|\mathrm{d}y$, $\mathrm{d}t = |\mathrm{det}T|\mathrm{d}z$, where $|\mathrm{det}T|$ is the absolute value of the determinant of T. From Lemma 2.2, we have

$$\begin{aligned}1&=\frac{1}{|TK||TL|}\int_{TK}\int_{TL}\varphi\Big(\frac{|x_0\cdot(s-t)|}{\lambda_0}\Big)\mathrm{d}s\mathrm{d}t\\&=\frac{1}{|\mathrm{det}T|^2|K||L|}\int_K\int_L\varphi\Big(\frac{|x_0\cdot(Ty-Tz)|}{\lambda_0}\Big)|\mathrm{det}T|^2\mathrm{d}y\mathrm{d}z\\&=\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|(x_0\cdot Ty)-(x_0\cdot Tz)|}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z\\&=\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|(T^tx_0\cdot y)-(T^tx_0\cdot z)|}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z\\&=\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|T^tx_0\cdot(y-z)|}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z,\end{aligned}$$

i.e., $\rho^{-1}_{I_\varphi(K,L)}(T^tx_0)=\lambda_0$. By (2.2), we have $\lambda_0=\rho^{-1}_{I_\varphi(K,L)}(T^tx_0)=\rho^{-1}_{T^{-t}(I_\varphi(K,L))}(x_0)$. Combining with equality (2.11), we immediately obtain

$$\rho^{-1}(I_\varphi(TK,TL),x_0)=\rho^{-1}(T^{-t}(I_\varphi(K,L)),x_0).$$

That is, $I_\varphi(TK,TL)=T^{-t}(I_\varphi(K,L))$.

3 Proofs of Main Results

Lemma 3.1 Suppose $\varphi\in\Phi$ is strictly convex and $K,L\in\mathcal{K}^n_o$. If $u\in S^{n-1}$ and $x_1',x_2'\in u^\perp$, then

$$\rho^{-1}_{I_\varphi(S_uK,S_uL)}(\frac{x_1'+x_2'}{2},1)\le\frac{1}{2}\rho^{-1}_{I_\varphi(K,L)}(x_1',1)+\frac{1}{2}\rho^{-1}_{I_\varphi(K,L)}(x_2',-1)\tag{3.1}$$

and

$$\rho^{-1}_{I_\varphi(S_uK,S_uL)}(\frac{x_1'+x_2'}{2},-1)\le\frac{1}{2}\rho^{-1}_{I_\varphi(K,L)}(x_1',1)+\frac{1}{2}\rho^{-1}_{I_\varphi(K,L)}(x_2',-1).\tag{3.2}$$

Equality in either inequality holds if $\rho_{I_\varphi(K,L)}(x_1',1)=\rho_{I_\varphi(K,L)}(x_2',-1)$ with K and L are dilates having the same midpoints.

Proof We only prove (3.1). Inequality (3.2) can be established in the same way.

For each $z'\in K_u$, $y'\in L_u$, let $\sigma_{z'}=\sigma_{z'}(u)=|K\cap(z'+\mathbb{R}u)|$ and $\sigma_{y'}=\sigma_{y'}(u)=|L\cap(y'+\mathbb{R}u)|$ be the lengths of the chords $K\cap(z'+\mathbb{R}u)$, $L\cap(y'+\mathbb{R}u)$.

Define $m_{z'}=m_{z'}(u)$ by $m_{z'}(u)=\frac{1}{2}\underline{l}_u(K,z')+\frac{1}{2}\overline{l}_u(K,z')$ such that $z'+m_{z'}u$ is the midpoint of the chord $K\cap(z'+\mathbb{R}u)$. And define $m_{y'}=m_{y'}(u)$ by $m_{y'}(u)=\frac{1}{2}\underline{l}_u(L,y')+\frac{1}{2}\overline{l}_u(L,y')$ such that $y'+m_{y'}u$ is the midpoint of the chord $L\cap(y'+\mathbb{R}u)$. Note that the midpoints of the chords of K in the direction u lie in a subspace if and only if there exists a $\mu\in K_u$ such that $\mu\cdot z'=m_{z'}$ for all $z'\in K_u$. Similarly, the midpoints of the chords of L in the direction u lie in a subspace if and only if there exists a $\mu\in L_u$ such that

$\mu \cdot y' = m_{y'}$ for all $y' \in L_u$. If $\lambda_1 > 0$, then we have

$$\begin{aligned}
&\int_K \int_L \varphi\bigg(\frac{|(x_1',1)\cdot(y-z)|}{\lambda_1}\bigg)\mathrm{d}y\mathrm{d}z \\
&= \int_K \int_L \varphi\bigg(\frac{|(x_1',1)\cdot((y',s)-(z',t))|}{\lambda_1}\bigg)\mathrm{d}(y',s)\mathrm{d}(z',t) \\
&= \int_{K_u}\int_{m_{z'}-\frac{\sigma_{z'}}{2}}^{m_{z'}+\frac{\sigma_{z'}}{2}} \int_{L_u}\int_{m_{y'}-\frac{\sigma_{y'}}{2}}^{m_{y'}+\frac{\sigma_{y'}}{2}} \varphi\bigg(\frac{|x_1'\cdot(y'-z')+(s-t)|}{\lambda_1}\bigg)\mathrm{d}y'\mathrm{d}s\mathrm{d}z'\mathrm{d}t \\
&= \int_{K_u}\int_{-\frac{\sigma_{z'}}{2}}^{\frac{\sigma_{z'}}{2}} \int_{L_u}\int_{-\frac{\sigma_{y'}}{2}}^{\frac{\sigma_{y'}}{2}} \varphi\bigg(\frac{|x_1'\cdot(y'-z')+m_{y'}+s_1-m_{z'}-t_1|}{\lambda_1}\bigg)\mathrm{d}y'\mathrm{d}s_1\mathrm{d}z'\mathrm{d}t_1 \\
&= \int_{S_uK}\int_{S_uL} \varphi\bigg(\frac{|x_1'\cdot(y'-z')+(s_1-t_1)+(m_{y'}-m_{z'})|}{\lambda_1}\bigg)\mathrm{d}(y',s_1)\mathrm{d}(z',t_1) \qquad (3.3)
\end{aligned}$$

by making the change of variables $s = m_{y'} + s_1$ and $t = m_{z'} + t_1$.

On the other hand, for $\lambda_2 > 0$, we have

$$\begin{aligned}
&\int_K \int_L \varphi\bigg(\frac{(|x_2',-1)\cdot(y-z)|}{\lambda_2}\bigg)\mathrm{d}y\mathrm{d}z \\
&= \int_K \int_L \varphi\bigg(\frac{(|x_2',-1)\cdot((y',s)-(z',t))|}{\lambda_2}\bigg)\mathrm{d}(y',s)\mathrm{d}(z',t) \\
&= \int_{K_u}\int_{m_{z'}-\frac{\sigma_{z'}}{2}}^{m_{z'}+\frac{\sigma_{z'}}{2}} \int_{L_u}\int_{m_{y'}-\frac{\sigma_{y'}}{2}}^{m_{y'}+\frac{\sigma_{y'}}{2}} \varphi\bigg(\frac{|x_2'\cdot(y'-z')-(s-t)|}{\lambda_2}\bigg)\mathrm{d}y'\mathrm{d}s\mathrm{d}z'\mathrm{d}t \\
&= \int_{K_u}\int_{-\frac{\sigma_{z'}}{2}}^{\frac{\sigma_{z'}}{2}} \int_{L_u}\int_{-\frac{\sigma_{y'}}{2}}^{\frac{\sigma_{y'}}{2}} \varphi\bigg(\frac{|x_2'\cdot(y'-z')-m_{y'}+s_1+m_{z'}-t_1|}{\lambda_2}\bigg)\mathrm{d}y'\mathrm{d}s_1\mathrm{d}z'\mathrm{d}t_1 \\
&= \int_{S_uK}\int_{S_uL} \varphi\bigg(\frac{|x_2'\cdot(y'-z')+(s_1-t_1)-(m_{y'}-m_{z'})|}{\lambda_2}\bigg)\mathrm{d}(y',s_1)\mathrm{d}(z',t_1) \qquad (3.4)
\end{aligned}$$

by making the change of variables $s = m_{y'} - s_1$ and $t = m_{z'} - t_1$.

Let

$$x_0' = \frac{1}{2}x_1' + \frac{1}{2}x_2' \quad \text{and} \quad \lambda_0 = \frac{1}{2}\lambda_1 + \frac{1}{2}\lambda_2.$$

From the convexity of φ, it follows that

$$\begin{aligned}
2\varphi\bigg(\frac{|x_0'\cdot(y'-z')+(s_1-t_1)|}{\lambda_0}\bigg) \le &\frac{\lambda_1}{\lambda_0}\varphi\bigg(\frac{|x_1'\cdot(y'-z')+(s_1-t_1)+(m_{y'}-m_{z'})|}{\lambda_1}\bigg) \\
&+ \frac{\lambda_2}{\lambda_0}\varphi\bigg(\frac{|x_2'\cdot(y'-z')+(s_1-t_1)-(m_{y'}-m_{z'})|}{\lambda_2}\bigg).
\end{aligned} \qquad (3.5)$$

By (3.3), (3.4) and (3.5), it yields that

$$
\begin{aligned}
&\frac{\lambda_1}{\lambda_0}\int_K\int_L\varphi\Big(\frac{|(x_1',1)\cdot(y-z)|}{\lambda_1}\Big)\mathrm{d}y\mathrm{d}z+\frac{\lambda_2}{\lambda_0}\int_K\int_L\varphi\Big(\frac{|(x_2',-1)\cdot(y-z)|}{\lambda_2}\Big)\mathrm{d}y\mathrm{d}z\\
=&\frac{\lambda_1}{\lambda_0}\int_{S_uK}\int_{S_uL}\varphi\Big(\frac{|x_1'\cdot(y'-z')+(s_1-t_1)+(m_{y'}-m_{z'})|}{\lambda_1}\Big)\mathrm{d}(y',s_1)\mathrm{d}(z',t_1)\\
&+\frac{\lambda_2}{\lambda_0}\int_{S_uK}\int_{S_uL}\varphi\Big(\frac{|x_2'\cdot(y'-z')+(s_1-t_1)-(m_{y'}-m_{z'})|}{\lambda_2}\Big)\mathrm{d}(y',s_1)\mathrm{d}(z',t_1)\\
\geq&2\int_{S_uK}\int_{S_uL}\varphi\Big(\frac{|x_0'\cdot(y'-z')+(s_1-t_1)|}{\lambda_0}\Big)\mathrm{d}(y',s_1)\mathrm{d}(z',t_1)\\
=&2\int_{S_uK}\int_{S_uL}\varphi\Big(\frac{|(x_0',1)\cdot(y-z)|}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z. \qquad (3.6)
\end{aligned}
$$

Let

$$\rho^{-1}_{I_\varphi(K,L)}(x_1',1)=\lambda_1 \quad \text{and} \quad \rho^{-1}_{I_\varphi(K,L)}(x_2',-1)=\lambda_2.$$

From Lemma 2.2, we get

$$\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|(x_1',1)\cdot(y-z)|}{\lambda_1}\Big)\mathrm{d}y\mathrm{d}z=1$$

and

$$\frac{1}{|K||L|}\int_K\int_L\varphi\Big(\frac{|(x_2',-1)\cdot(y-z)|}{\lambda_2}\Big)\mathrm{d}y\mathrm{d}z=1.$$

Combining with the fact that $|K|=|S_uK|$, $|L|=|S_uL|$ and (3.6), we get

$$\frac{1}{|S_uK||S_uL|}\int_{S_uK}\int_{S_uL}\varphi\Big(\frac{(x_0',1)\cdot(y-z)}{\lambda_0}\Big)\mathrm{d}y\mathrm{d}z\leq 1.$$

In light of the continuity of φ and (1.3) yields

$$\rho^{-1}_{I_\varphi}(S_uK,S_uL)(\frac{x_1'+x_2'}{2},1)\leq\frac{1}{2}\lambda_1+\frac{1}{2}\lambda_2$$

with the equality requiring equality in (3.5) for all $z'\in K_u$, $y'\in L_u$, and $s_1\in[-\frac{\sigma_{y'}}{2},\frac{\sigma_{y'}}{2}]$, $t_1\in[-\frac{\sigma_{z'}}{2},\frac{\sigma_{z'}}{2}]$.

Since φ is strictly convex, this means that we must have φ can not be linear in a neighborhood of the origin given by

$$\frac{x_1'\cdot(y'-z')+(s_1-t_1)+(m_{y'}-m_{z'})}{\lambda_1}=\frac{x_2'\cdot(y'-z')+(s_1-t_1)-(m_{y'}-m_{z'})}{\lambda_2} \qquad (3.7)$$

for all $s_1\in(-\frac{\sigma_{y'}}{2},\frac{\sigma_{y'}}{2})$, $t_1\in(-\frac{\sigma_{z'}}{2},\frac{\sigma_{z'}}{2})$. Choosing $\lambda_1=\rho^{-1}_{I_\varphi(K,L)}(x_1',1)$ and $\lambda_2=\rho^{-1}_{I_\varphi(K,L)}(x_2',-1)$, equation (3.7) immediately yields

$$\rho^{-1}_{I_\varphi(K,L)}(x_1',1)=\lambda_1=\lambda_2=\rho^{-1}_{I_\varphi(K,L)}(x_2',-1)$$

and

$$(x_2' - x_1') \cdot y' = 2m_{y'}, \quad (x_2' - x_1') \cdot z' = 2m_{z'}$$

for all $y' \in L_u$ and $z' \in K_u$.

But this means that the midpoints $\{(y', m_{y'}) : y' \in L_u\}$ and $\{(z', m_{z'}) : z' \in K_u\}$ of the chords of L, K parallel to u lie in the subspaces

$$\{(y', \frac{x_2' - x_1'}{2} \cdot y') : y' \in L_u\} \quad \text{and} \quad \{(z', \frac{x_2' - x_1'}{2} \cdot z') : z' \in K_u\}$$

of $\mathbb{R}^n$, respectively. As we can observe the equality holds if $\rho_{I_\varphi(K,L)}(x_1', 1) = \rho_{I_\varphi(K,L)}(x_2', -1)$ with K and L are dilates having the same midpoints.

Lemma 3.2 Suppose $K, L \in \mathcal{K}_o^n$, $\varphi \in \Phi$. If $u \in S^{n-1}$, then

$$I_\varphi^*(S_u K, S_u L) \subseteq S_u(I_\varphi^*(K, L)).$$

Proof Let $y' \in \operatorname{relint}(I_\varphi^*(K, L)_u)$. According to Lemma 2.1, there exist $x_1' = x_1'(y')$ and $x_2' = x_2'(y')$ in $u^\perp$ such that

$$\overline{l}_u(I_\varphi^*(K, L), y') = h_{I_\varphi^*(K,L)}(x_1', 1) - x_1' \cdot y' \tag{3.8}$$

and

$$\underline{l}_u(I_\varphi^*(K, L), y') = h_{I_\varphi^*(K,L)}(x_2', -1) - x_2' \cdot y'. \tag{3.9}$$

From (2.4), (3.8), (3.9) and Lemma 3.1, it follows that

$$\begin{aligned}
\overline{l}_u(S_u I_\varphi^*(K, L), y') &= \frac{1}{2}\overline{l}_u(I_\varphi^*(K, L), y') + \frac{1}{2}\underline{l}_u(I_\varphi^*(K, L), y') \\
&= \frac{1}{2}(h_{I_\varphi^*(K,L)}(x_1', 1) - x_1' \cdot y') + \frac{1}{2}(h_{I_\varphi^*(K,L)}(x_2', -1) - x_2' \cdot y') \\
&= \frac{1}{2}h_{I_\varphi^*(K,L)}(x_1', 1) + \frac{1}{2}h_{I_\varphi^*(K,L)}(x_2', -1) - \big(\frac{x_1' + x_2'}{2}\big) \cdot y' \\
&\geq h_{I_\varphi^*(S_u K, S_u L)}(\frac{x_1' + x_2'}{2}, 1) - \big(\frac{x_1' + x_2'}{2}\big) \cdot y' \\
&\geq \min_{x' \in u^\perp} \{h_{I_\varphi^*(S_u K, S_u L)}(x', 1) - x' \cdot y'\} \\
&= \overline{l}_u(I_\varphi^*(S_u K, S_u L), y')
\end{aligned} \tag{3.10}$$

and

$$\begin{aligned}
\underline{l}_u(S_u I_\varphi^*(K, L), y') &= \frac{1}{2}\overline{l}_u(I_\varphi^*(K, L), y') + \frac{1}{2}\underline{l}_u(I_\varphi^*(K, L), y') \\
&= \frac{1}{2}(h_{I_\varphi^*(K,L)}(x_1', 1) - x_1' \cdot y') + \frac{1}{2}(h_{I_\varphi^*(K,L)}(x_2', -1) - x_2' \cdot y') \\
&= \frac{1}{2}h_{I_\varphi^*(K,L)}(x_1', 1) + \frac{1}{2}h_{I_\varphi^*(K,L)}(x_2', -1) - \big(\frac{x_1' + x_2')}{2}\big) \cdot y' \\
&\geq h_{I_\varphi^*(S_u K, S_u L)}(\frac{x_1' + x_2'}{2}, -1) - \big(\frac{x_1' + x_2'}{2}\big) \cdot y' \\
&\geq \min_{x' \in u^\perp} \{h_{I_\varphi^*(S_u K, S_u L)}(x', -1) - x' \cdot y'\} \\
&= \underline{l}_u(I_\varphi^*(S_u K, S_u L), y'),
\end{aligned} \tag{3.11}$$

(3.10) and (3.11) give the inclusion.

Proof of Theorem 1.1 Combining with the Steiner symmetrization argument, there is a sequence of direction $\{u_i\}$, such that the sequences $\{K_i\}$ and $\{L_i\}$ converge to B_K and B_L, respectively, where the sequences $\{K_i\}$ and $\{L_i\}$ are defined by

$$K_i = S_{u_i} \cdots S_{u_1} K \quad \text{and} \quad L_i = S_{u_i} \cdots S_{u_1} L$$

with $|K| = |K_i|$ and $|L| = |L_i|$. Thus $|K| = |B_K|$ and $|L| = |B_L|$.

Since the Steiner symmetrization keeps the volume, by Lemma 3.2, we have

$$|I^*_\varphi(K_i, L_i)| = |I^*_\varphi(S_{u_i}K_{i-1}, S_{u_i}L_{i-1})| \le |I^*_\varphi(K_{i-1}, L_{i-1})| \le \cdots \le |I^*_\varphi(K, L)|,$$

when $i \to \infty$, we have $|I^*_\varphi(B_K, B_L)| \le |I^*_\varphi(K, L)|$.

According to the equality condition of Lemma 3.1, above equality holds if K and L are dilates of each other and have the same midpoints.

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Orlicz 混合相交体

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摘要: 本文研究了Orlicz 混合相交体及其性质. 利用几何分析方法提出了Orlicz 混合相交体的概念, 获得了Orlicz 混合相交体算子的连续性和仿射不变性. 通过积分方法和Steiner 对称, 建立了Orlicz 混合相交体的仿射等周不等式.

关键词: 凸体; Orlicz 混合相交体; Steiner 对称; 仿射等周不等式

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