BOUNDleness of TOEPLITZ OPERATORS
GENERATED BY THE CAMPANATO-TYPE
FUNCTIONS AND RIESZ TRANSFORMS
ASSOCIATED WITH SCHÖDINGER OPERATORS

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Abstract: In the paper, we study the boundedness of the Toeplitz operators generated by the
Campanato-type functions and Riesz transforms associated with the Schrödinger operators. Using
the sharp maximal function estimate, we establish the boundedness of the Toeplitz operator \( \Theta^b \)
on the Lebesgue space, which extend the previous results about the commutators.

Keywords: Commutator; Campanato-type functions; Riesz transform; Schrödinger operator

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1 Introduction

Let \( \mathcal{L} = -\Delta + V \) be a Schrödinger operator on \( \mathbb{R}^n (n > 3) \), where \( \Delta \) is the Laplacian
on \( \mathbb{R}^n \) and \( V \neq 0 \) is a nonnegative locally integrable function. The problems about the
Schrödinger operators \( \mathcal{L} \) were well studied (see [1–3] for example). Especially, Fefferman [1],

The commutators generated by the Riesz transforms associated with the Schrödinger
operators and BMO functions or Lipschitz functions also attracted much attention (see [4–6]
for example). Chu [7], consider the boundedness of commutators generalized by the BMO_{x}
functions and the Riesz transform \( \nabla(-\Delta + V)^{-1/2} \) on Lebesgue spaces. Mo et al. [8]
established the boundedness of commutators generated by the Campanato-type functions
and the Riesz transforms associated with Schrödinger operators.

First, let us introduce some notations. A nonnegative locally \( L^q(\mathbb{R}^n) \) integrable function
\( V \) is said to belong to \( B_q (1 < q < \infty) \) if there exists \( C = C(q,V) > 0 \) such that the reverse
Hölder’s inequality

\[
\left( \frac{1}{|B|} \int_B V(x)^q \, dx \right)^{1/q} \leq C \left( \frac{1}{|B|} \int_B V(x) \, dx \right)
\]  

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holds for every ball $B$ in $\mathbb{R}^n$.

Let $T_{j,1} = \nabla(-\triangle + V)^{-1/2}$ or $T_{j,1} = \pm I$ and $T_{j,2}$ be a linear operator which is bounded on $L^p(\mathbb{R}^n)$ space, for $j = 1, 2, \cdots, m$. Let $M_b f = bf$, where $b$ is a locally integrable function on $\mathbb{R}^n$, then the Toeplitz operator is defined by

$$\Theta^b = \sum_{j=1}^{m} T_{j,1} M_b T_{j,2}.$$ 

About the Toeplitz operator, there are some results. Mo et al. [9] established the boundedness of the Toeplitz operator generalized by the singular integral operator with nonsmooth kernel and the generalized fractional. Liu et al. [11] investigated the boundedness of the Toeplitz operator related to the generalized fractional integral operator.

The commutator $[b, T](f) = b T(f) - T(bf)$ is a particular case of the Toeplitz operators. Inspired by [7–10], we will consider the boundedness of the Toeplitz operators generated by the Campanato-type functions and Riesz transforms associated with Schödinger operators.

**Definition 1.1** Let $f \in L_{\text{loc}}(\mathbb{R}^n)$, then the sharp maximal function associated with $\mathcal{L} = -\Delta + V$ is defined by

$$M^p_{\mathcal{L}}(f)(x) = \begin{cases} \sup_{x \in B} \frac{1}{|B|} \int_{B(x,s)} |f(y) - f_{B(x,s)}|dy, & \text{when } s < \rho(x), \\ \sup_{x \in B} \frac{1}{|B(x,s)|} \int_{B(x,s)} |f(y)|dy, & \text{when } s \geq \rho(x), \end{cases}$$

where $\rho$ is define by

$$\rho(x) = \sup \left\{ r > 0 : \frac{1}{m-2} \int_{B(x,r)} V \leq 1 \right\}.$$ 

**Definition 1.2** [12, 13] Let $\mathcal{L} = -\triangle + V$, $p \in (0, \infty)$ and $\beta \in \mathbb{R}$. A function $f \in L^p_{\text{loc}}(\mathbb{R}^n)$ is said to be in $\Lambda^\beta,p_{\mathcal{L}}(\mathbb{R}^n)$, if there exists a nonnegative constant $C$ such that for all $x \in \mathbb{R}^n$ and $0 < s < \rho(x) \leq r$,

$$\left\{ \frac{1}{|B(x,s)|^{1+p\beta}} \int_{B(x,s)} |f(y) - f_{B(x,s)}|^p dy \right\}^{1/p} + \left\{ \frac{1}{|B(x,r)|^{1+p\beta}} \int_{B(x,r)} |f(y)|^p dy \right\}^{1/p} \leq C,$$

where $f_B = \frac{1}{|B|} \int_{B} f(y) dy$ for any ball $B$. Moreover, the minimal constant $C$ as above is defined for the norm of $f$ in the space $\Lambda^\beta,p_{\mathcal{L}}(\mathbb{R}^n)$ and denote by $\|f\|_{\Lambda^\beta,p_{\mathcal{L}}(\mathbb{R}^n)}$.

**Remak 1.1** [12] When $p \in [1, \infty)$, $\Lambda^{0,p}_{\mathcal{L}}(\mathbb{R}^n) = \text{BMO}_{\mathcal{L}}(\mathbb{R}^n)$. And, when $0 \leq \beta < \infty$ and $p_1, p_2 \in [1, \infty)$, $\Lambda^{\beta,p_1}_{\mathcal{L}}(\mathbb{R}^n) = \Lambda^{\beta,p_2}_{\mathcal{L}}(\mathbb{R}^n)$ and $\|f\|_{\Lambda^{\beta,p_1}_{\mathcal{L}}(\mathbb{R}^n)} \sim \|f\|_{\Lambda^{\beta,p_2}_{\mathcal{L}}(\mathbb{R}^n)}$. For simplicity, we denote $\Lambda^{\beta,p}_{\mathcal{L}}(\mathbb{R}^n)$ by $\Lambda^{\beta}_p(\mathbb{R}^n)$.

**Lemma 1.1** (see [2, 7]) Suppose that $V \in B_q(n/2 \leq q < n)$ satisfies the condition

$$\int_{B(x,R)} \frac{V(y)}{|x-y|^{n-1}}dy \leq \frac{1}{R^{n-1}} \int_{B(x,R)} V(y)dy,$$
then the kernel $K(x, y)$ of operator $\nabla(-\Delta + V)^{-1/2}$ satisfies the following estimates: there exists a constant $\delta > 0$ such that for any nonnegative integrate $i$,

$$|K(x, y)| \leq \frac{C_i}{(1 + m(x, V)|x - y|)^{\tau}|x - y|^n},$$

$$|K(x + h, y) - K(x, y)| \leq C\frac{|h|^{\delta}}{|x - y|^{n+\delta}}$$

for $0 < |h| < \frac{|x - y|}{2}$.

Hence, $\nabla(-\Delta + V)^{-1/2}$ is bounded on $L^p(\mathbb{R}^n)$ space for $1 < p \leq p_0$, where $1/p_0 = 1/q - 1/n$.

Throughout this paper, the letter $C$ always remains to denote a positive constant that may vary at each occurrence but is independent of the essential variable.

### 2 Theorems and Lemmas

**Theorems 2.1** Let $V \in B_q$ satisfy (1.2) for $n/2 \leq q < n$. Let $0 \leq \beta < 1$, $b \in \Lambda_{2}^{\beta}(\mathbb{R}^n)$, $1 < \tau < \infty$ and $1 < s < p_0$, where $1/p_0 = 1/q - 1/n$. Suppose that $\Theta^1f = 0$ for any $f \in L^r(\mathbb{R}^n)$ ($1 < r < \infty$), then there exists a constant $C > 0$ such that

$$M_{\beta}^{\delta}(\Theta^b f)(x) \leq C \sum_{j=1}^{m} \|b\|_{\Lambda_{2}^{\beta}} \|M_{s, n\beta}(T_{j}2f)(x).$$

**Theorems 2.2** Let $V \in B_q$ satisfy (1.2) for $n/2 \leq q < n$ and $1 < p_0 < \infty$ satisfy $1/p_0 = 1/q - 1/n$. Suppose that $\Theta^1f = 0$ for any $f \in L^r(\mathbb{R}^n)$ ($1 < \tau < \infty$, $0 < \beta < 1$, and $b \in \Lambda_{2}^{\beta}(\mathbb{R}^n)$. Then for $1 < r < \min\{1/\beta, p_0\}$ and $1/p = 1/r - \beta$, there exists a constant $C > 0$, such that

$$\|\Theta^{b}f\|_{L^p} \leq C \sum_{j=1}^{m} \|b\|_{\Lambda_{2}^{\beta}} \|f\|_{L^r}.$$

**Theorems 2.3** Let $V \in B_q$ satisfy (1.2) for $n/2 \leq q < n$ and $1 < p_0 < \infty$ satisfy $1/p_0 = 1/q - 1/n$. Suppose that $\Theta^1f = 0$ for any $f \in L^r(\mathbb{R}^n)$ ($1 < \tau < \infty$) and $b \in \text{BMO}_{\mathbb{R}^n}$, then for $1 < p < p_0$, there exists a constant $C > 0$, such that

$$\|\Theta^{b}f\|_{L^p} \leq C \sum_{j=1}^{m} \|b\|_{\text{BMO}_{\mathbb{R}^n}} \|f\|_{L^p}.$$

To prove the theorems, we need the following lemmas.

**Lemma 2.1** (see [7]) Let $0 < p_0 < \infty$, $p_0 \leq p < \infty$ and $\delta > 0$. If $f$ satisfies the condition $M(|f|^\delta)^{1/\delta} \in L^{p_0}$, then exists a constant $C > 0$ such that

$$\|M(|f|^\delta)^{1/\delta}\|_{L^p} \leq C\|M_{\mathbb{R}^n}(|f|^\delta)^{1/\delta}\|_{L^p}.$$

**Lemma 2.2** (see [14]) For $1 \leq \gamma < \infty$ and $\beta > 0$, let

$$M_{\gamma, \beta}(f)(x) = \sup_{B \ni x} \left(\frac{1}{|B|^{1-\beta\gamma/n}} \int_{B} |f(y)|^\gamma dy\right)^{1/\gamma}.$$
Suppose that $\gamma < p < n/\beta$ and $1/q = 1/p - \beta/n$, then $\|M_{\gamma,\beta}(f)\|_{L^2} \leq C\|f\|_{L^p}$. 

**Remark 2.1** When $\beta = 0$, we denote $M_{\gamma,\beta} = M_\gamma$. And it is easy to see that $M_\gamma$ is boundedness on $L^p(\mathbb{R}^n)$, for $1 < r < p$.

**Lemma 2.3** (see [8]) Let $B = B(x, r)$ and $0 < r < \rho(x)$, then

$$|b_{2^k B} - b_B| \leq Ck|2^k B|^\beta \|b\|_{L^2}$$

for $k = 1, 2, \ldots$.

### 3 Proofs of Theorems 2.1–2.2

First, let us prove Theorem 2.1.

Fix a ball $B = B(x, r_0)$ and let $2B = B(x, 2r_0)$. We need only to estimate

$$\frac{1}{|B|} \int_B |\Theta^k f(y) - (\Theta^k f)_B| dy.$$

**Case I** When $0 < r_0 < \rho(x)$, using the condition $\Theta^1 f = 0$, then we have

$$\frac{1}{|B|} \int_B |\Theta^k f(y) - (\Theta^k f)_B| dy \leq \sum_{j=1}^{m} \frac{1}{|B|} \left( \int_B |T_{j,1} M_{(b-b_B)} T_{j,2} f(y) - (T_{j,1} M_{(b-b_B)} T_{j,2} f)_B| dy \right).$$

If $T_{j,1} = \nabla (-\Delta + V)^{-1/2}$, then

$$\frac{1}{|B|} \int_B |T_{j,1} M_{(b-b_B)} T_{j,2} f(y) - (T_{j,1} M_{(b-b_B)} T_{j,2} f)_B| dy$$

$$\leq \frac{2}{|B|} \int_B |T_{j,1} M_{(b-b_B)} T_{j,2} f(y) - T_{j,1} M_{(b-b_B)} T_{j,2} f(x)| dy$$

$$\leq \frac{2}{|B|} \int_B |T_{j,1} M_{(b-b_B)} T_{j,2} f(y)| dy + \frac{2}{|B|} \int_B |T_{j,1} M_{(b-b_B)} T_{j,2} f(x)| dy$$

$$= I_1 + I_2.$$

Let $\tau$ and $s$ be as in Theorem 2.1. Then using Hölder’s inequality and the $L^s$ boundedness of $T_{j,1}$ (Lemma 1.1), we have

$$I_1 \leq C \left( \frac{1}{|B|} \int_B |T_{j,1} M_{(b-b_B)} T_{j,2} f(y)|^s dy \right)^{\frac{1}{s}}$$

$$\leq C \left( \frac{1}{|2B|^{1+\beta s\tau}} \int_{2B} |b(y) - b_B|^s dy \right)^{\frac{1}{s^\tau}} \left( \frac{1}{|2B|^{1-\beta s\tau}} \int_{2B} |T_{j,2} f(y)|^{s\tau} dy \right)^{\frac{1}{s^\tau}}$$

$$\leq C \|b\|_{L^2} M_{s,\tau,0}(T_{j,2} f)(x).$$
Let’s estimate $I_2$. From (1.4), it follows that

$$\left| T_{j,1}[(b - b_B)\chi_{R^n\setminus 2B}T_{j,2}f](y) - T_{j,1}[(b - b_B)\chi_{R^n\setminus 2B}T_{j,2}f](x) \right|$$

$$= \left| \int_{2B^c} (b(z) - b_B)T_{j,2}f(z)(K(y, z) - K(x, z))dz \right|$$

$$\leq C\sum_{k=1}^{\infty} \int_{2^kr_0 < |z-x| \leq 2^{k+1}r_0} \frac{|y-x|^\delta}{|z-x|^{n+\delta}} |b(z) - b_B||T_{j,2}f(z)|dz$$

$$\leq C\sum_{k=1}^{\infty} \int_{2^kr_0 < |z-x| \leq 2^{k+1}r_0} |b(z) - b_{2^{k+1}B}||T_{j,2}f(z)|dz$$

$$+ C\sum_{k=1}^{\infty} \int_{2^kr_0 < |z-x| \leq 2^{k+1}r_0} |b_{2^{k+1}B} - b_B||T_{j,2}f(z)|dz$$

$$=: H_1 + H_2.$$ For $H_1$, since $\delta > 0$, by Hölder’s inequality, we have

$$H_1 \leq C\sum_{k=1}^{\infty} 2^{-k\delta} \left( \frac{1}{|2^{k+1}B|^{1-\beta(s\tau)'}} \int_{2^{k+1}B} |b(y) - b_{2^{k+1}B}|^{(s\tau)'} dy \right)^{1/(s\tau)'} \times \left( \frac{1}{|2^{k+1}B|^{1-3s\tau}} \int_{2^{k+1}B} |T_{j,2}f(y)|^{s\tau} dy \right)^{1/s\tau}$$

$$\leq C\|b\|_{A^\beta} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

From Lemma 2.3 and Hölder's inequality, it follows that

$$H_2 \leq C\|b\|_{A^\beta} \sum_{k=1}^{\infty} k2^{-k\delta} \left( \frac{1}{|2^{k+1}B|^{1-\beta}} \int_{2^{k+1}B} |T_{j,2}f(y)|dy \right)$$

$$\leq C\|b\|_{A^\beta} \sum_{k=1}^{\infty} k2^{-k\delta} \left( \frac{1}{|2^{k+1}B|^{1-3s\tau}} \int_{2^{k+1}B} |T_{j,2}f(y)|^{s\tau} dy \right)^{1/s\tau}$$

$$\leq C\|b\|_{A^\beta} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

Thus

$$I_2 \leq C\|b\|_{A^\beta} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

So when $T_{j,1} = \nabla(-\Delta + V)^{-1/2}$, we conclude that

$$\frac{1}{|B|} \int_B |T_{j,1}M_{(b-b_B)}T_{j,2}f(y) - (T_{j,1}M_{(b-b_B)}T_{j,2}f)_B|dy \leq C\|b\|_{A^\beta} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

If $T_{j,1} = \pm I$, it is obvious that

$$\frac{2}{|B|} \int_B |T_{j,1}M_{(b-b_B)}\chi_{R^n\setminus 2B}T_{j,2}f(y)|dy = 0.$$
Thus using the above formula and Hölder’s inequality, we conclude
\[
\frac{1}{|B|} \int_B |T_{j,1} M(b - b_0) T_{j,2} f(y) - (T_{j,1} M(b - b_0) T_{j,2} f)_B| dy \\
\leq \frac{2}{|B|} \int_B |T_{j,1} M(b - b_0) T_{j,2} f(y)| dy \\
\leq C \left( \frac{1}{|B|^{1 + \beta(s\tau)'}} \int_B |b(y) - b_B|^{s\tau}' dy \right)^{\frac{1}{s\tau}'} \left( \frac{1}{|B|^{1 - \beta s\tau}} \int_B |T_{j,2} f(y)|^{s\tau} dy \right)^{\frac{1}{s\tau}} \\
\leq C \|b\|_{A^\delta_{\mathcal{L}}} \Lambda_{s\tau,n\beta}(T_{j,2} f)(x).
\]

Thus for \(0 < r_0 < \rho(x)\), we conclude that
\[
M^\delta_{A_{\mathcal{L}}}(\Theta^b f)(x) \leq C \sum_{j=1}^m \|b\|_{A_{\mathcal{L}}} M_{s\tau,n\beta}(T_{j,2} f)(x).
\]

**Case II** When \(r_0 > \rho(x)\), we have
\[
\frac{1}{|B|} \int_B |\Theta^b f(y)| dy \leq \sum_{j=1}^m \left( \frac{1}{|B|} \int_B |T_{j,1} M_{b\chi_{\mathcal{L}} B} T_{j,2} f(y)| dy \right) + \frac{1}{|B|} \int_B |T_{j,1} M_{b\chi_{\mathcal{L}} \setminus \{B\}} T_{j,2} f(y)| dy.
\]

If \(T_{j,1} = \nabla(-\Delta + V)^{-1/2}\), then for \(1 < \tau < \infty\) and \(1 < s < p_0\) are as in Theorem 2.1, we have
\[
\frac{1}{|B|} \int_B |T_{j,1} M_{b\chi_{\mathcal{L}} B} T_{j,2} f(y)| dy \\
\leq C \left( \frac{1}{|B|} \int_B |T_{j,1} b \chi_{\mathcal{L}} B T_{j,2} f(y)|^s dy \right)^{\frac{1}{s}} \\
\leq \left( \frac{1}{|B|} \int_{2B} |b(y)|^s |T_{j,2} f(y)|^s dy \right)^{\frac{1}{s}} \\
\leq C \left( \frac{1}{|2B|^{1 + \beta(s\tau)'}} \int_{2B} |b(y)|^{s\tau}' dy \right)^{\frac{1}{s\tau}'} \left( \frac{1}{|2B|^{1 - \beta s\tau}} \int_{2B} |T_{j,2} f(y)|^{s\tau} dy \right)^{\frac{1}{s\tau}} \\
\leq C \|b\|_{A^\delta_{\mathcal{L}}} \Lambda_{s\tau,n\beta}(T_{j,2} f)(x).
\]

Since when \(y \in 2B\) and \(z \in 2^{k+1}B \setminus 2^k B\), we have \(|x - z| \sim |y - z|\). Then taking \(s, \tau\) as above, we get
\[
|T_{j,1} M_{b\chi_{\mathcal{L}} \setminus \{B\}} T_{j,2} f(y)| \leq \int_{(2B)^c} |b(z)||T_{j,2} f(z)||K(y, z)| dz \\
\leq C \sum_{k=1}^\infty \frac{1}{2^k} \int_{2^k r_0 < |z-x| \leq 2^{k+1} r_0} \left( 1 + |z-x| m(x, V) \right) |z-x|^\rho |b(z)||T_{j,2} f(z)| dz \\
\leq C \sum_{k=1}^\infty \frac{1}{2^k} \int_{2^k r_0 < |z-x| \leq 2^{k+1} r_0} \left( 1 + |z-x| m(x, V) \right) |z-x|^\rho |b(z)||T_{j,2} f(z)| dz \\
\leq C \|b\|_{A^\delta_{\mathcal{L}}} \Lambda_{s\tau,n\beta}(T_{j,2} f)(x).
\]
Thus
\[ \frac{1}{|B|} \int_B |T_{j,1}M_{b \chi_{\mathbb{R}^n \setminus B}} T_{j,2}f(y)| dy \leq C \| b \|_{\Lambda_2^s} \mathcal{M}_{\tau,n} \mathcal{M}_{T_{j,2}f}(x). \]

If \( T_{j,1} = \pm I \), then by Hölder’s inequality, we obtain
\[ \frac{1}{|B|} \int_B |T_{j,1}M_{b \chi_{\mathbb{R}^n \setminus B}} T_{j,2}f(y)| dy \leq C \left( \frac{1}{|B|^{1+\beta(s\tau)}} \int_B |b(y)|^{(s\tau)'} dy \right)^{\frac{1}{s\tau}} \left( \frac{1}{|B|^{1-\beta s\tau}} \int_B |T_{j,2}f(y)|^{s\tau} dy \right)^{\frac{1}{s\tau}} \]
\[ \leq C \| b \|_{\Lambda_2^s} \mathcal{M}_{\tau,n} \mathcal{M}_{T_{j,2}f}(x). \]

And
\[ \frac{1}{|B|} \int_B |T_{j,1}M_{b \chi_{\mathbb{R}^n \setminus B}} T_{j,2}f(y)| dy = 0. \]

Thus for \( r_0 > \rho(x) \),
\[ M_{E}^f(\Theta^b f)(x) \leq C \sum_{j=1}^{m} \| b \|_{\Lambda_2^s} \mathcal{M}_{\tau,n} \mathcal{M}_{T_{j,2}f}(x). \]

So whenever \( 0 < r_0 < \rho(x) \) or \( r_0 > \rho(x) \), we have
\[ M_{E}^f(\Theta^b f)(x) \leq C \sum_{j=1}^{m} \| b \|_{\Lambda_2^s} \mathcal{M}_{\tau,n} \mathcal{M}_{T_{j,2}f}(x). \]

Now, let us turn to prove Theorem 2.2.

Let \( s, \tau \) be as in Theorem 2.1 and satisfy \( 1 < s\tau < p \). Then applying Theorem 2.1, Lemma 2.1 and Lemma 2.2, we know that
\[ \| \Theta^b f \|_{L^p} \leq C \| M_{E}^f(\Theta^b f) \|_{L^p} \leq C \sum_{j=1}^{m} \| b \|_{\Lambda_2^s} \| \mathcal{M}_{\tau,n} \mathcal{M}_{T_{j,2}f} \|_{L^p} \]
\[ \leq C \sum_{j=1}^{m} \| b \|_{\Lambda_2^s} \| T_{j,2}f \|_{L^p} \leq C \sum_{j=1}^{m} \| b \|_{\Lambda_2^s} \| f \|_{L^p}. \]

Thus we complete the proof of Theorems 2.1–2.1.

4 Proof of Theorem 2.3

It is obvious that \( \Lambda_0^s = BMO_{\mathbb{C}} \). Thus from the proof of Theorem 2.1, we have
\[ M_{E}^f(\Theta^b f)(x) \leq C \sum_{j=1}^{m} \| b \|_{BMO_{\mathbb{C}}} \| \mathcal{M}_{\tau} \mathcal{M}_{T_{j,2}f} \|_{L^p} \]

Since \( \mathcal{M}_{\tau} \) is boundedness on \( L^p(\mathbb{R}^n) \), then
\[ \| \Theta^b f \|_{L^p} \leq C \| M_{E}^f(\Theta^b f) \|_{L^p} \leq C \sum_{j=1}^{m} \| b \|_{BMO_{\mathbb{C}}} \| \mathcal{M}_{\tau} \mathcal{M}_{T_{j,2}f} \|_{L^p} \leq C \sum_{j=1}^{m} \| b \|_{BMO_{\mathbb{C}}} \| f \|_{L^p}. \]

Therefore, we complete the proof of Theorem 2.3.
References


由Campanato型函数和与薛定谔算子相关的Riesz变换生成的
Toeplitz算子的有界性

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摘要：本文研究了由Campanato型函数及与Schrödinger算子相关的Riesz变换生成的Toeplitz算子的有界性. 利用Sharp极大函数估计得到了Toeplitz算子$\Theta^p$在Lebesgue空间的有界性, 拓广了已有交换子的结果.

关键词：交换子; Campanato型函数; Riesz变换; Schrödinger算子

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