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STRONG DUALITY WITH STRICT EFFICIENCY IN VECTOR OPTIMIZATION INVOLVING NONCONVEX SET-VALUED MAPS

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Abstract: This paper is diverted to the study of two strong dual problems of a primal nonconvex set-valued optimization in the sense of strict efficiency. By using the principles of Lagrange duality and Mond-Weir duality, for each dual problem, a strong duality theorem with strict efficiency is established. The conclusions can be formulated as follows: starting from a strictly efficient solution of the primal problem, it can be constructed a strictly efficient solution of the dual problem such that the corresponding objective values of both problems are equal. The results generalize the strong dual theorems in which the set-valued maps are assumed to be cone-convex.

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1 Introduction

One of the most important topics of set-valued optimization is related to proper efficiency, this is because that the range of the set of (weak) efficient solutions is often too large. In order to contract the solution range, several kinds of proper efficiency were presented. For example, Benson efficiency [1], Henig efficiency [2], Geoffrion efficiency [3], Super efficiency [4] and Strictly efficiency [5] etc. Especially, super efficiency, given by Borwein and Zhuang [4], was shown to have some desirable properties. However, the condition to guarantee its existence is rather strong. Later, weakening the existence condition, Professor Cheng and Fu [5] improved the concept of supper efficiency and introduced the concept of strict efficiency.

Since duality assertions allow to study a minimization problem through a maximization problem and to know what one can expect in the best case. At the same time, duality resulted in many applications within optimization, and it provided many unifying conceptual insights into economics and management science. So it is not surprising that duality is one of the important topics in set-valued optimization. There were many papers dedicated to duality

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theory of set-valued optimization (see [6–11]). Among results obtained in this field, we want to mention the strong duality. In vector optimization, it is often said that strong duality holds between primal and dual problems, if a weakly efficient solution of a primal problem is a weakly efficient solution of dual problem and such that the corresponding objective values of the primal and dual problems are equal. If in this problem "weakly efficient solution" is replaced by "properly efficient solution", then it is said that strong duality with proper efficiency holds between the primal and dual problems. However, strong duality with proper efficiency was considered only for the case when proper efficiency was understood in the sense of Geoffiron [10] and Benson [11].

On the other hand, it is well known that the concept of cone-convexity and its generalizations play an important role in establishing duality theorems for set-valued optimization problems. Up to now, there are many notions of generalized convexity for set-valued maps which are introduced and are proved to be useful for optimization theory and related topics. Among them, the notion of ic-cone-convexlikeness seemed to be more general one [12], and was successfully applied to strict efficiency and Henig efficiency in set-valued optimization [13–16].

Based upon the above observation, the aim of this note is to establish the strong duality theorems with strict efficiency for set-valued optimization problems under the ic-cone-convexlikeness assumptions. This paper is arranged as follows: In Section 2, some well-known definitions and results used in the sequel are recalled. In Section 3, two improved dual models are introduced, and strong duality theorems with strict efficiency are established under the assumption of ic-cone-convexlikeness, respectively.

2 Preliminaries

In this paper, let X, Y and Z be real topological spaces. Let $D \subset Y$ and $E \subset Z$ be pointed convex cones, and denoted

$$\begin{aligned} y_1 \leq y_2 \quad &\text{if} \quad y_2 - y_1 \in D\backslash\{0_Y\}, \\ y_1 \nleq y_2 \quad &\text{if} \quad y_2 - y_1 \notin D\backslash\{0_Y\}. \end{aligned}$$

Definition 2.1 Let M be a nonempty subset of Y, $\bar{y} \in M$ is called a minimize (maximize) point of M, if

$$(M - \bar{y}) \cap (-D\backslash\{0_Y\}) = \emptyset \ \ ((M - \bar{y}) \cap (D\backslash\{0_Y\}) = \emptyset).$$

The set of minimize (maximize) point of M is denoted by $\mathrm{Min}[M, D]$ ($\mathrm{Max}[M, D]$).

For a set $A \subset Y$, we write $\mathrm{cone}(A) = \{\lambda \cdot a : \ \lambda \geq 0, \ a \in A\}$. The closure and interior of set A is denoted by $\mathrm{cl}(A)$ and $\mathrm{int}(A)$. A convex subset B of a cone D is a base of D if $0_Y \notin \mathrm{cl}(B)$ and $D = \mathrm{cone}(B)$.

Throughout this paper, it is always assumed that the pointed convex cone $D \subset Y$ has a base B.

Definition 2.2 [5, 13] Let M be a nonempty subset of Y, $\bar{y} \in M$ is called a strictly minimize point of M with respect to B, if there is a neighbourhood U of 0_Y such that

$$\mathrm{cl}\big[\mathrm{cone}(M - \bar{y})\big] \cap (U - B) = \emptyset. \tag{2.1}$$

The set of strictly efficient point of M with respect to B is denoted by Strmin$[M, B]$.

Remark 2.1 [5, 13] (1) With respect to the definition of strictly minimize points, equality (2.1) is equivalent to

$$\mathrm{cone}(M - \bar{y}) \cap (U - B) = \emptyset. \tag{2.2}$$

Moreover, if necessary, the neighbourhood U of 0_Y can be chosen to be open, convex or balanced.

(2) Strmin$[M, B] \subset$ Min$[M, D]$.

(3) Similarly, $\bar{y} \in M$ is called a strictly maximize point of M with respect to B, if there is a neighbourhood V of 0_Y such that

$$\mathrm{cl}\big[\mathrm{cone}(M - \bar{y})\big] \cap (B - V) = \emptyset.$$

Remark 2.2 In Definition 2.2, if equality (2.1) holds, then

$$\mathrm{cl}\big[\mathrm{cone}(M - \bar{y}) + D\big] \cap (U - B) = \emptyset. \tag{2.3}$$

In fact, if not, there exist $\lambda > 0$, $m \in M$, $d \in D \backslash \{0_Y\}$, $u \in U$ and $b \in B$, such that $\lambda(m - \bar{y} + d) = u - b$. Since B is the base of D, there exist $\mu > 0$ and $b_1 \in B$ such that $d = \mu \cdot b_1$. Since B is convex set, we get that

$$\lambda(m - \bar{y}) = u - (b + \lambda\mu \cdot b_1) \in U - B.$$

Therefore, we can get

$$\frac{\lambda}{1+\lambda\mu}(m - \bar{y}) = \frac{1}{1+\lambda\mu} \cdot u - (\frac{1}{1+\lambda\mu} b + \frac{\lambda\mu}{1+\lambda\mu} \cdot b_1) \in U - B,$$

which contradicts equality (2.1).

Definition 2.3 [12] The set-valued map $F : \ X \to 2^Y$ is called ic-D-convexlike if $\mathrm{int}(\mathrm{cone}(\mathrm{im}(F) + D))$ is convex and

$$\mathrm{cone}(\mathrm{im}(F) + D) \subset \mathrm{cl}\big[\mathrm{int}(\mathrm{cone}(\mathrm{im}(F) + D))\big],$$

where $\mathrm{im}(F)$ is the image of F, and that is

$$\mathrm{im}(F) = \bigcup \big\{F(x) : \ x \in X\big\}.$$

Assume that $F : \ X \to 2^Y$ and $G : \ X \to 2^Z$ are set-valued maps. This note considers the following set-valued optimization problem (SOP):

$$(\mathrm{SOP}) \qquad \begin{cases} \text{minimize} & F(x), \\ \text{subject to} & G(x) \cap (-E) \neq \emptyset, \\ & x \in X. \end{cases}$$

The set of feasible solution of (SOP) is denoted by Ω, that is

$$\Omega = \big\{x \in X:\ G(x) \cap (-E) \neq \emptyset\big\}.$$

Definition 2.4 If $\bar{x} \in S$ and $\bar{y} \in F(\bar{x}) \cap \mathrm{Strmin}\big[F(S), B\big]$, then we say that $(\bar{x}, \bar{y})$ is a strictly efficient solution of problem (SOP).

Let $L(X,Y)$ be the family of (single-valued) linear continuous maps from X into Y. Let

$$L_+(Z,Y) = \big\{T \in L(Z,Y):\ T(E) \subset D\big\}.$$

Definition 2.5 [13] Let $F:\ X \to 2^Y$ be a set-valued map, $\bar{x} \in X$ and $\bar{y} \in F(\bar{x})$. A map $T \in L(X,Y)$ is said to be a strict subgradient of F at $(\bar{x}, \bar{y})$ if

$$\bar{y} - T(\bar{x}) \in \mathrm{Strmin}\big[\mathrm{im}(F - T), B\big].$$

The set of all strict subgradients of F at $(\bar{x}, \bar{y})$ is denoted by $\partial_{\mathrm{str}} F(\bar{x}, \bar{y})$.

Assumption (A) [12] In problem (SOP), let $\bar{x} \in S$, $\bar{y} \in F(\bar{x})$ and $\bar{z} \in G(\bar{x}) \cap (-E)$. It is said that Assumption (A) is satisfied if there exists $\beta \in [0,1)$ such that the set-valued map $H_\beta := (F - \bar{x}) \times (G - \beta \cdot \bar{z}):\ X \to 2^{Y \times Z}$ is ic-$D \times E$-convexlike.

Definition 2.6 [12] It is said that condition (CQ) holds if $\mathrm{cl}\big[\mathrm{cone}(\mathrm{im}G + E)\big] = Z$.

Lemma 2.7 [13] Let $\bar{x} \in S$, $\bar{y} \in F(\bar{x})$ and $\bar{z} \in G(\bar{x}) \cap (-E)$. Let Assumption (A) and condition (CQ) be satisfied. If $(\bar{x}, \bar{y})$ is a strictly efficient solution of problem (SOP), then there exists $\bar{T} \in L_+(Z,Y)$ such that $\bar{T}(\bar{z}) = 0_Y$ and

$$0 \in \partial_{\mathrm{str}}(F + \bar{T}G)(\bar{x}, \bar{y} + \bar{T}(\bar{z})). \tag{2.4}$$

3 Strong Duality

3.1 Lagrange-Wolfe Strong Duality

We first rewrite the Lagrange dual problem in the form similar to the Wolfe dual problem [17], which is denoted by problem (LWD) as follows:

$$\begin{aligned} \text{(LWD)}\quad \text{maximize}\quad & u + T(v), \\ \text{subject to}\quad & (u,v) \in F(\xi) \times G(\xi),\quad \xi \in X, && (3.1)\\ & 0 \in \partial_{\mathrm{str}}(F + TG)(\xi, u + T(v)), && (3.2)\\ & T \in L_+(Z,Y). && (3.3) \end{aligned}$$

Denote by Q_1 the set of all feasible points of (LWD), i.e., the set of points $(\xi, u, v, T) \in X \times Y \times Z \times L(Z,Y)$ satisfying (3.1)–(3.3). Let S_1 be the set of all points $u + T(v)$ such that there exists $\xi \in X$ with $(\xi, u, v, T) \in Q_1$.

Definition 3.1 If $(\xi, u, v, T) \in Q_1$, and $u + T(v) \in \mathrm{Strmax}\big[S_,B\big]$, then we say that (ξ, u, v, T) is a strictly efficient solution of problem (LWD).

Theorem 3.2 (Weak Duality) If $x \in \Omega$ and $(\xi, u, v, T) \in Q_1$, then

$$\mathrm{cl}\big[\mathrm{cone}(F(x) - (u + T(v)))\big] \cap (U - B) = \emptyset. \tag{3.4}$$

Proof Since $x \in \Omega$, it holds that $G(x) \cap (-E) \neq \emptyset$. So we can take a point $v' \in G(x) \cap (-E)$ such that $-T(v') \in T(E) \subset D$. Hence

$$\begin{aligned} F(x) - (u + T(v)) &= F(x) + T(v') - (u + T(v)) - T(v') \\ &\subset F(x) + T(v') - (u + T(v)) + D \\ &\subset F(x) + T \circ G(x) - (u + T(v)) + D \\ &\subset \mathrm{im}(F + T \circ G) - (u + T(v)) + D. \end{aligned}$$

On the other hand, (3.2) shows that there exists a neighbourhood U of 0_Y such that

$$\mathrm{cl}\big[\mathrm{cone}(\mathrm{im}(F + T \circ G) - (u + T(v)))\big] \cap (U - B) = \emptyset.$$

It follows from Remark 2.2 that

$$\mathrm{cl}\big[\mathrm{cone}(\mathrm{im}(F + TG) - (u + T(v)) + D)\big] \cap (U - B) = \emptyset.$$

So we get (3.4), as desired.

Remark 3.1 In weak duality Theorem 3.2, it follows from (3.4) and Remark 2.1 that $u + T(v) \in \min\big[F(x), D\big]$. This leads to

$$(F(x) - (u + T(v))) \cap (-D\backslash\{0_Y\}) = \emptyset,$$

so (3.4) means that $y \not\leq u + T(v),\ \ \forall y \in F(x)$, which is the sense of general weak duality in literatures [6–8].

Theorem 3.3 (Strong Duality) Let $\bar{x} \in X$, $\bar{y} \in F(\bar{x})$ and $\bar{z} \in G(\bar{x}) \cap (-E)$. Let Assumption (A) and condition (CQ) be satisfied. If $(\bar{x}, \bar{y})$ is a strictly efficient solution of problem (SOP), then there exists $\bar{T} \in L_+(Z, Y)$ such that $\bar{T}(\bar{z}) = 0$, $(\bar{x}, \bar{y}, \bar{z}, \bar{T})$ is a strictly efficient solution of (LWD), and the corresponding objective values of (SOP) and (LWD) are equal.

Proof It yields from Lemma 2.7 that there exists $\bar{T} \in L_+(Z, Y)$ such that $\bar{T}(\bar{z}) = 0$ and $(\bar{x}, \bar{y}, \bar{z}, \bar{T}) \in Q_1$. It remains to prove that $\bar{y} = \bar{y} + \bar{T}(\bar{z}) \in \mathrm{Strmax}[S_1, B]$. In fact, otherwise there exist the neighbourhood U_0 of 0_Y such that

$$\mathrm{cl}\big[\mathrm{cone}(S_1 - \bar{y})\big] \cap (B - U_0) \neq \emptyset.$$

Hence, there exist $b_0 \in (B - U_0)$, $\lambda > 0$ and $\hat{u} + T(\hat{v}) \in S_1$ such that $b_0 = \lambda(\hat{u} + T(\hat{v}) - \bar{y})$ or, equivalently,

$$-b_0 = \lambda(\bar{y} - (\hat{u} + T(\hat{v}))) \in \mathrm{cl}\big[\mathrm{cone}(F(\bar{x}) - (\hat{u} + T(\hat{v})))\big].$$

This indicates that

$$(U_0 - B) \cap \mathrm{cl}\big[\mathrm{cone}(F(\bar{x}) - (\hat{u} + T(\hat{v})))\big] \neq \emptyset,$$

a contradiction to the weak duality property (3.4) with $x = \bar{x}$.

3.2 Mond-Weir Strong Duality

This subsection is devoted to construct another duality problem on the basis of the idea of Mond-Weir [18], called the Mond-Weir duality problem (MWD), and establish a strong duality result between (SOP) and (MWD). The next problem is named the Mond-Weir dual problem of (SOP) and is denoted by (MWD):

$$\begin{aligned}
\text{(MWD)} \quad & \text{maximize} && u, \\
& \text{subject to} && (u, v) \in F(\xi) \times G(\xi), && (3.5)\\
& && 0 \in \partial_{\mathrm{str}}(F + TG)(\xi, u + T(v)), && (3.6)\\
& && T \in L_+(Z, Y), && (3.7)\\
& && T(v) \in D. && (3.8)
\end{aligned}$$

Denote by Q_2 the set of all feasible points of (MWD), i.e., the set of points $(\xi, u, v, T) \in X \times Y \times Z \times L(Z, Y)$ satisfying (3.5)–(3.8). Let S_2 be the set of all points u such that there exists $(\xi, v, T) \in X \times Z \times L(Z, Y)$ with $(\xi, u, v, T) \in Q_2$.

Lemma 3.4 It holds that $Q_2 \subset Q_1$ and $S_2 \subset S_1 - D$.

Proof According to the definitions of Q_1 and Q_2, it is obviously that $Q_2 \subset Q_1$ is satisfied. So it is to prove the second one only. Let $u \in S_2$. Then there exists $(\xi, v, T) \in X \times Z \times L(Z, Y)$ such that $(\xi, u, v, T) \in Q_2 \subset Q_1$ is satisfied. We get that

$$u = u + T(v) - T(v) \in S_1 - T(v) \subset S_1 - D.$$

Thus, $u \in S_1 - D$. This completes proof.

Theorem 3.5 (Weak Duality) If $x \in \Omega$ and $(\xi, u, v, T) \in Q_2$, then there exists a neighbourhood U of 0_Y such that

$$\mathrm{cl}\big[\mathrm{cone}(F(x) - u)\big] \cap (U - B) = \emptyset. \tag{3.9}$$

Proof By Lemma 3.4, we obtain that $Q_2 \subset Q_1$. Again, we get from Theorem 3.2 that there exists a neighbourhood U of 0_Y such that

$$\mathrm{cl}\big[\mathrm{cone}(F(x) - (u + T(v)))\big] \cap (U - B) = \emptyset.$$

Hence it follows from Remark 2.2 that

$$\mathrm{cl}\big[\mathrm{cone}(F(x) - (u + T(v) + D))\big] \cap (U - B) = \emptyset. \tag{3.10}$$

On the other hand, it yields from (3.8) that

$$F(x) - u = F(x) - (u + T(v) + T(v) \subset F(x) - (u + T(v)) + D.$$

Combing above inquality with (3.10) yields (3.9), as required.

In order to formulating the strong duality between (SOP) and (MWD), we need propose the following Lemma 3.6.

Lemma 3.6 If $(\bar{\xi}, \bar{u}, \bar{v}, \bar{T})$ is a strictly efficient solution of (LWD) and $\bar{T}(\bar{v}) = 0$, then $(\bar{\xi}, \bar{u}, \bar{v}, \bar{T})$ is a strictly efficient solution of (MWD) and the corresponding objective values of both problems are equal.

Proof Because $(\bar{\xi}, \bar{u}, \bar{v}, \bar{T})$ is a strictly efficient solution of (LWD), it follows from the definition of set S_1 that there exists a neighbourhood U of 0_Y such that

$$\mathrm{cl}\big[\mathrm{cone}(S_1 - (\bar{u} + \bar{T}(\bar{v})))\big] \cap (B - U) = \emptyset.$$

Therefore, we get from Remark 2.2 that

$$\mathrm{cl}\big[\mathrm{cone}(S_1 - (\bar{u} + \bar{T}(\bar{v})) - D)\big] \cap (B - U) = \emptyset. \tag{3.11}$$

On the other hand, according to Lemma 3.4, we have $S_2 \subset S_1 - D$. Then we derive from $\bar{T}(\bar{v}) = 0$ that

$$S_2 - \bar{u} \subset S_1 - (\bar{u} + \bar{T}(\bar{v})) - D. \tag{3.12}$$

Together (3.11) with (3.12), it is clear that $\mathrm{cl}\big[\mathrm{cone}(S_2 - \bar{u})\big] \cap (B - U) = \emptyset$, which is the desired result.

Theorem 3.7 (Strong Duality) Let $\bar{x} \in X$, $\bar{y} \in F(\bar{x})$ and $\bar{z} \in G(\bar{x}) \cap (-E)$. Let Assumption (A) and condition (CQ) be satisfied. If $(\bar{x}, \bar{y})$ is a strictly efficient solution of problem (SOP), then there exists $\bar{T} \in L_+(Z, Y)$ such that $\bar{T}(\bar{z}) = 0$, $(\bar{x}, \bar{y}, \bar{z}, \bar{T})$ is a strictly efficient of (MWD), and the corresponding objective values of (SOP) and (MWD) are equal.

Proof It follows from Lemma 2.7 that there exists $\bar{T} \in L_+(Z, Y)$ such that $\bar{T}(\bar{z}) = 0$ and $(\bar{x}, \bar{y}, \bar{z}, \bar{T}) \in Q_2 \subset Q_1$. Hence, we get from the strong duality Theorem 3.3 between (SOP) and (LWD) that $(\bar{x}, \bar{y}, \bar{z}, \bar{T})$ is a strictly efficient solution of (LWD) and the corresponding objective values of (SOP) and (LWD) are equal. Therefore, it yields from Lemma 3.6 that $(\bar{x}, \bar{y}, \bar{z}, \bar{T})$ is also a strictly efficient of (MWD) and the corresponding objective values of (LWD) and (MWD) are equal. This can obtain the desired results.

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非凸集值优化问题严有效解的强对偶定理

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摘要: 本文研究了非凸集值向量优化的严有效解在两种对偶模型的强对偶问题. 利用Lagrange对偶和Mond-Weir对偶原理, 获得了如下结果: 原集值优化问题的严有效解, 在一些条件下是对偶问题的强有效解, 并且原问题和对偶问题的目标函数值相等; 推广了集值优化对偶理论在锥-凸假设下的相应结果.

关键词: 严有效性; 强对偶; 集值优化; 生成锥内部凸-锥类凸性

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