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Hölder 范数下关于 Brown 运动增量的泛函局部收敛速度

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摘 要: 本文研究了 Brown 运动在 Hölder 范数与容度下的泛函极限问题. 利用大偏差小偏差方法, 获得了 Brown 运动增量局部泛函极限的收敛速度, 推广了文 [4] 中的结果.

关键词: Brown 运动; 收敛速度; Hölder 范数; 容度
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1 引言

考虑经典的 Wiener 空间 (B, H, μ) , 设 $D^{r,p}$ 是 Wiener 泛函的 Sobolev 空间, 即

$$D^{r,p} = (1-\mathcal{L})^{-\frac{r}{2}}\mathbf{L}^p, \|F\|_{r,p} = \|(1-\mathcal{L})^{r/2}F\|_p, F \in \mathbf{L}^p, r \geqslant 0, 1 \leqslant p < \infty,$$

其中 $\mathbf{L}^p$ 记为 (B,μ) 上的实值函数的 L^p 空间, $\mathcal{L}$ 是 (B,H,μ) 上的 Ornstein-Uhlenbeck 算子. 对 $r \geqslant 0, p > 1$, (r,p) - 容度定义如下

$$C_{r,p}(O) = \inf\{\|F\|_{r,p}^p; F \in D_{r,p}, F \geqslant 1, \mu-\text{a.s. 在}O\text{上}\}, \quad \text{对开集 } O \subset B,$$

且对任意集合 $A \subset B$ 有, $C_{r,p}(A) = \inf\{C_{r,p}(O); A \subset O \subset B,\ O \text{ 是开集}\}$. 设 $\mathcal{C}^d$ 为从 $[0,1]$ 到 $\mathbb{R}^d$ 的连续函数空间, 赋予上确界范数 $\|f\| = \sup\limits_{0\leqslant t\leqslant 1}|f(t)|$. 记 $\mathcal{C}_0^d = \{f \in \mathcal{C}^d; f(0) = 0\}$, $\mathcal{H}^d = \{f \in \mathcal{C}_0^d; f(t) = \int_0^t \dot{f}(s)ds, \|f\|_{\mathcal{H}^d}^2 = \int_0^1 |\dot{f}(t)|^2 dt < \infty\}$, $\mathcal{H}^d$ 是一定义如下内积的 Hilbert 空间, $\langle r_1, r_2\rangle_{\mathcal{H}^d} = \int_0^1 (\dot{r}_1(s), \dot{r}_2(s))ds$. 设 μ 是 $\mathcal{C}_0^d$ 上的 Wiener 测度, $(\mathcal{C}_0^d, \mathcal{H}^d, \mu)$ 是一经典 Wiener 空间. 下面考虑如下两个 Banach 空间

$$\mathcal{C}^\alpha = \{f \in \mathcal{C}_0^d; \|f(\cdot)\|_\alpha = \sup_{\substack{s,\ t\in[0,1]\\ s\neq t}} \frac{|f(t)-f(s)|}{|t-s|^\alpha} < \infty\},$$

$$\mathcal{C}^{\alpha,\,0} = \{f \in \mathcal{C}^\alpha; \lim_{\delta\to 0}\sup_{\substack{s,\ t\in[0,1]\\ 0<|t-s|<\delta}} \frac{|f(t)-f(s)|}{|t-s|^\alpha} = 0\},$$

其中 $0 < \alpha < \frac{1}{2}$. 则有 $(\mathcal{C}^{\alpha,0}, \mathcal{H}^d, \mu)$ 也是一经典 Wiener 空间 (见文 [2] 定理 2.4). 设 $w \in \mathcal{C}^{\alpha,\,0}$ 为一标准 Brown 运动, 记 $K = \{f \in \mathcal{H}^d; I(f) \leqslant 1\}$, 其中 $I: B \to [0,\infty]$ 定义为若 $z \in \mathcal{H}^d$, $I(z) = \|z\|_{\mathcal{H}^d}^2/2$; 否则 $I(z) = \infty$.

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自从 Yoshida [1] 首先得到了关于容度 $C_{r,p}$ 意义的大偏差结论, 近年来关于 Brown 运动在 Hölder 范数下的拟必然泛函极限理论开始受到关注. Baldi 与 Roynette [2] 利用大偏差得到了 Brown 运动在 Hölder 范数下的收敛速度, 并证明了存在 $k = k(\alpha) \geq 0$, 使得

$$\lim_{\varepsilon \to 0} \varepsilon^{2/(1-2\alpha)} \log P\{\|w\|_\alpha \leq \varepsilon\} = -k. \tag{1}$$

进一步, 对任意 $f \in K$ 与 $\gamma = (1-2\alpha)/2$ 有

$$\lim_{\varepsilon \to 0} \varepsilon^{1/\gamma} \log P\left(\left\|w - \frac{f}{\varepsilon^{1/(2\gamma)}}\right\|_\alpha \leq r\varepsilon\right) = -I(f) - \frac{k}{r^{1/\gamma}}. \tag{2}$$

后来, Chen 与 Balakrishnan [3] 得到 Brown 运动在 Hölder 范数与容度 $C_{r,p}$ 意义下泛函重对数律的极限定理. 本文利用 Hölder 范数下的大偏差小偏差, 得到了 Brown 运动增量在 Hölder 范数下, 关于容度 $C_{r,p}$ 意义下局部泛函极限的收敛速度. 主要结果如下.

定理 1 设 r_u 定义为从 R^+ 到 R^+ 的单调不减函数, 满足 $0 < r_u \leq u$ 且 u/r_u 单调不减. 记 $\sigma_u = \log \frac{u \log u}{r_u}$, 若 $\lim\limits_{u \to \infty} \frac{\log(u/r_u)}{\log u} = \infty$, 则在 $C_{r,p}$-q.s. 意义下有

$$\lim_{u \to \infty} (\sigma_u)^{1-\alpha} \inf_{t \in [0, 1-\frac{r_u}{u}]} \|\left(r_u \sigma_u\right)^{-1/2} \left(w(ut + r_u \cdot) - w(ut)\right)\|_\alpha = k^\gamma,$$

其中 $\gamma = (1-2\alpha)/2$, $k > 0$ 如式 (1) 中所定义.

定理 1 的证明可由引理 5 与引理 6 得到, 在此之前叙述已有的相关结果.

2 定理 1 的证明

引理 1 (见文献 [3] 定理 2.1) 设 $\{S_\varepsilon\}_{\varepsilon > 0}$ 是 $\mathcal{C}^{\alpha,0}$ 上一双射线性算子, 使得对任意 $\varepsilon > 0$ 及 $A \subset \mathcal{C}^{\alpha,0}$ 有 $\mu(S_\varepsilon^{-1}A) = \mu(\varepsilon^{-1/2}A)$, 则对 $(r,p) \in [0,\infty) \times (1,\infty)$, 有

$$-\inf_{f \in \mathring{A}} I(f) \leqslant \varliminf_{\varepsilon \to 0} \varepsilon \log C_{r,p}(S_\varepsilon^{-1}A) \leqslant \varlimsup_{\varepsilon \to 0} \varepsilon \log C_{r,p}(S_\varepsilon^{-1}A) \leqslant -\inf_{f \in \bar{A}} I(f).$$

引理 2 (见文献 [6] 引理 2.1) 设 $k \in \mathrm{N}$, $q_1, q_2 \in (1,\infty)$ 满足 $1/p = 1/q_1 + 1/q_2$, 则存在常数 $c = c(k,p,q_1,q_2) > 0$ 使得对任意 $-\infty < a_i < b_i < \infty$, $\delta \in (0,1)$, 及 $F_i \in D^{k,kq_1}$ 有下式成立

$$\begin{aligned} & C_{k,p}\left(\bigcap_{i=1}^n \left\{a_i < \tilde{F}_i(z) < b_i\right\}\right)^{1/p} \\ \leq \quad & c\left(\frac{n}{\delta}\right)^k \left(1 + \max_{1 \leq i \leq n} \|F_i\|_{k,kq_1}\right)^k \mu\left(\bigcap_{i=1}^n \{a_i - \delta < F_i(z) < b_i + \delta\}\right)^{1/q_2}, \end{aligned}$$

其中 $\tilde{F}_i$ 为 F_i 的拟连续修正.

引理 3 设 k, p, q_1, q_2 如引理 2 中定义. 对任意 $\varepsilon > 0, t_i \geq 0,\ h_i > 0,\ i = 1, 2, \cdots, n$, 及 $f \in K$, 设

$$F_\varepsilon^{(i)}(w) = \left\|\varepsilon\left(\frac{w(t_i + h_i \cdot) - w(t_i)}{\sqrt{h_i}}\right) - f\right\|_\alpha,$$

则存在一常数 $c=c(k,p,q_1,f)>0$, 对任意 $\delta\in(0,1]$, $\varepsilon\in(0,1]$, 有

$$C_{k,p}\left(\bigcap_{i=1}^{n}\{z:a_i<F_\varepsilon^{(i)}(z)<b_i\}\right)^{\frac{1}{p}}\le c\,\delta^{-2k^2-k}n^k\mu\left(\bigcap_{i=1}^{n}\{z:a_i-\delta<F_\varepsilon^{(i)}(z)<b_i+\delta\}\right)^{\frac{1}{q_2}}.$$

证 利用引理 2, 类似文献 [6] 中引理 2.2 易证.

引理 4 设 $0<\alpha<\frac{1}{2}$, $\gamma=\frac{1}{2}-\alpha$, $t\ge0$, $f\in K$, $k>0$ 如 (1) 中所定义, 则对任意 $\tau>0$ 有

$$\lim_{\varepsilon\to0}\varepsilon^{1/\gamma}\log C_{r,p}\left(\left\|\frac{w(t+h\cdot)-w(t)}{\sqrt{h}}-\frac{f}{\varepsilon^{1/(2\gamma)}}\right\|_\alpha\le\varepsilon\tau\right)=-\frac{k}{\tau^{1/\gamma}}-I(f).$$

证 考虑到容度具有性质 $C_{r,p}(\cdot)\ge\mu(\cdot)$, 结合 (2) 式只需证明

$$\lim_{\varepsilon\to0}\varepsilon^{1/\gamma}\log C_{r,p}\left(\left\|\frac{w(t+h\cdot)-w(t)}{\sqrt{h}}-\frac{f}{\varepsilon^{1/(2\gamma)}}\right\|_\alpha\le\varepsilon\tau\right)\le-\frac{k}{\tau^{1/\gamma}}-I(f).$$

对任意 $1>\delta>0$, $c_0>0$, 令 $k=[r]+1$, 根据引理 3, 有

$$\begin{aligned}&C_{r,p}\left(\left\|\frac{w(t+h\cdot)-w(t)}{\sqrt{h}}-\frac{f}{\varepsilon^{1/(2\gamma)}}\right\|_\alpha\le\varepsilon\tau\right)^{1/p}\\ \le\;& c_0(\varepsilon^{1/(2\gamma)+1}\delta)^{-2k^2-k}\mu\left(\left\|\varepsilon^{1/(2\gamma)}\frac{w(t+h\cdot)-w(t)}{\sqrt{h}}-f\right\|_\alpha\le\varepsilon^{1/(2\gamma)+1}(\tau+\delta)\right)^{1/q_2},\end{aligned}$$

再根据 (2) 式得

$$\begin{aligned}&\lim_{\varepsilon\to0}\varepsilon^{1/\gamma}\log C_{r,p}\left(\left\|\frac{w(t+h\cdot)-w(t)}{\sqrt{h}}-\frac{f}{\varepsilon^{1/(2\gamma)}}\right\|_\alpha\le\varepsilon\tau\right)\\ \le\;&\frac{p}{q_2}\lim_{\varepsilon\to0}\varepsilon^{1/\gamma}\log\mu\left(\left\|w(\cdot)-\frac{f}{\varepsilon^{1/(2\gamma)}}\right\|_\alpha\le\varepsilon(\tau+\delta)\right)\\ =\;&\frac{p}{q_2}(-k(\tau+\delta)^{-\frac{1}{\gamma}}-I(f)).\end{aligned}$$

最后令 $\delta\to0,q_2\to p$, 引理 4 获证.

引理 5 设 r_u,σ_u 如命题 1 所定义, 则对 $s\in[0,1]$ 在 $C_{r,p}$-q.s. 意义下有

$$\liminf_{u\to\infty}(\sigma_u)^{1-\alpha}\inf_{t\in[0,1-\frac{r_u}{u}]}\|(r_u\sigma_u)^{-1/2}(w(ut+r_us)-w(ut))\|_\alpha\ge k^\gamma.$$

证 设 $l(u)=r_u(\sigma_u)^{-\frac{2}{1-2\alpha}}$. 对于 $1<\theta<(1-\varepsilon)^{-2}$, 记 $u_n=\theta^n$. 选取适当 $\delta_1>0$, 使得 $\delta_2=1/(\theta^{1/2}(1-\varepsilon))^{1/\gamma}-\delta_1>1$. 设 $k_n=[\frac{u_{n+1}}{l(u_n)}]$, $t_i=il(u_n),i=0,1,2,\cdots,k_n$. 则有

$$\begin{aligned}&\min_{0<i\le k_n}\|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(t_i+r_u\cdot)-w(t_i))\|_\alpha\\ \le&\max_{0\le i\le k_n}\sup_{0\le S\le l(u_n)}\|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(S+(t_i+r_u\cdot))-w(t_i+r_u\cdot))\|_\alpha\\ &+\inf_{t\in[0,1-\frac{r_{u_{n+1}}}{u_{n+1}}]}\|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(u_{n+1}t+r_u\cdot)-w(u_{n+1}t))\|_\alpha.\end{aligned}\tag{3}$$

由引理 4, 对任意 $0<\varepsilon<1$, 当 n 充分大时有

$$\begin{aligned}
&C_{r,p}\left(\sigma_{u_{n+1}}^{1-\alpha}\min_{0\le i\le k_n}\left\|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(t_i+r_u\cdot)-w(t_i))\right\|_\alpha)\le(1-\varepsilon)k^\gamma\right)\\
\le&\sum_{0\le i\le k_n}C_{r,p}\left(\left\|\frac{w(t_i+r_u\cdot)-w(t_i)}{\sqrt{\sigma_{u_{n+1}}}\sqrt{r_u}}\right\|_\alpha\le\theta^{1/2}\frac{k^\gamma(1-\varepsilon)}{(\sigma_{u_{n+1}})^{1-\alpha}}\right)\\
\le&(1+k_n)\exp\left\{-\sigma_{u_{n+1}}\delta_2\right\}\le\frac{u_{n+1}+l(u_n)}{l(u_n)}\left(\frac{r_{u_{n+1}}}{u_{n+1}\log u_{n+1}}\right)^{\delta_2},
\end{aligned}$$

因此根据 Borel-Cantelli 引理得到在 $C_{r,p}$-q.s. 意义下有

$$\liminf_{n\to\infty}\sigma_{u_{n+1}}^{1-\alpha}\min_{0\le i\le k_n}\|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(t_i+r_u\cdot)-w(t_i))\|_\alpha\ge k^\gamma. \tag{4}$$

另一方面, 对任意 $\eta>0$ 有

$$\begin{aligned}
&C_{r,p}\left\{\sigma_{u_{n+1}}^{1-\alpha}\sup_{0\le i\le k_n}\sup_{0\le s\le l(u_n)}\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}\|w(s+t_i+r_u\cdot)-w(t_i+r_u\cdot)\|_\alpha>\eta\right\}\\
=&C_{r,p}\left\{\frac{\sigma_{u_{n+1}}^{1-\alpha}}{\sqrt{r_{u_{n+1}}\sigma_{u_{n+1}}}}\sup_{0\le i\le k_n}\sup_{0\le z\le 1}\|w(zl(u_n)+t_i+r_u\cdot)-w(t_i+r_u\cdot)\|_\alpha>\eta\right\}\\
\le&\sum_{j=0}^{[\frac{ru}{l(un)}]}\sum_{i=0}^{k_n}C_{r,p}\left\{\frac{\sigma_{u_{n+1}}^{1-\alpha}}{\sqrt{r_{u_{n+1}}\sigma_{u_{n+1}}}}\frac{r_u^\alpha}{(l(u_n))^\alpha}T(j,n)>\eta\right\}\\
\le&\sum_{j=0}^{[\frac{ru}{l(un)}]}\sum_{i=0}^{k_n}C_{r,p}\left\{\frac{1}{(\sigma_{u_{n+1}})^{\frac12+\alpha}}\theta^{\alpha+\frac32}\frac{1}{\sqrt{l(u_n)}}T(j,n)>\eta\right\},
\end{aligned}$$

其中 $T(j,n)=\sup\limits_{0\le z\le1}\|w(zl(u_n)+t_i+jl(u_n)+l(u_n)\cdot)-w(t_i+jl(u_n)+l(u_n)\cdot)\|_\alpha$. 由于

$$\begin{aligned}
&\mu\left\{\frac{1}{(\sigma_{u_{n+1}})^{\frac12+\alpha}}\theta^{\alpha+\frac32}\frac{1}{\sqrt{l(u_n)}}T(j,n)>\eta\right\}\\
=&\mu\left\{\frac{\sqrt2\theta^{\alpha+3/2}}{\left(\sigma_{u_{n+1}}\right)^{\frac12+\alpha}}\sup_{0\le z\le1}\left\|w(\frac z2+\frac12\cdot)-w(\frac12\cdot)\right\|_\alpha>\eta\right\}\\
=&\mu\left\{\frac{\sqrt2\theta^{\alpha+3/2}}{\left(\sigma_{u_{n+1}}\right)^{\frac12+\alpha}}w\in Q\right\},
\end{aligned}$$

其中 $Q=\{f\in\mathcal{C}^{\alpha,0};\sup\limits_{0\le z\le1}\|f(\frac12z+\frac12\cdot)-f(\frac12\cdot)\|_\alpha\ge\eta\}$, 且 $\inf\limits_{f\in A}I(f)\ge\frac{\eta^2}{32}$, 故由引理 1 知当 n 充分大时有

$$\begin{aligned}
C_{r,p}\left\{\frac{\theta^{\alpha+\frac32}}{(\sigma_{u_{n+1}})^{\frac12+\alpha}}\frac{1}{\sqrt{l(u_n)}}T(j,n)>\eta\right\}&\le\exp\left(-\frac{\eta^2}{128\theta^{\alpha+3/2}}\left(\sigma_{u_{n+1}}\right)^{2\alpha}\left(\sigma_{u_{n+1}}\right)\right)\\
&=\left(\frac{r_{u_{n+1}}}{u_{n+1}\log u_{n+1}}\right)^{\frac{\eta^2}{128\theta^{\alpha+3/2}}\left(\sigma_{u_{n+1}}\right)^{2\alpha}}.
\end{aligned}$$

考虑到当 $n \to \infty$, $\sigma_{u_{n+1}} \to \infty$, 因此

$$\sum_n \frac{r_{u_{n+1}}}{l(u_n)} \frac{u_{n+1}+l(u_n)}{l(u_n)} \left(\frac{r_{u_{n+1}}}{u_{n+1}\log u_{n+1}}\right)^{\frac{\eta^2}{128\theta^{\alpha+3/2}}\left(\sigma_{u_{n+1}}\right)^{2\alpha}} < \infty.$$

再次利用 Borel-Cantelli 引理可得在 $C_{r,p}$-q.s. 意义下有

$$\limsup_{n\to\infty} \sigma_{u_{n+1}}^{1-\alpha} \sup_{0\le i\le k_n} \sup_{0\le s\le l(u_n)} \left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2} \|w(s+t_i+r_u\cdot)-w(t_i+r_u\cdot)\|_\alpha = 0. \tag{5}$$

联合 (3), (4) ,(5) 式得

$$\liminf_{n\to\infty} \sigma_{u_{n+1}}^{1-\alpha} \inf_{t\in[0,1-\frac{r_{u_{n+1}}}{u_{n+1}}]} \|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2} \left(w(u_{n+1}t+r_u\cdot)-w(u_{n+1}t)\right)\|_\alpha \ge k^\gamma, \tag{6}$$

对于 $u\in[u_n,u_{n+1})$, 再设 $\phi_{t,u}(s)=\left(r_u\sigma_u\right)^{-1/2}\left(w(ut+r_us)-w(ut)\right)$, 从而有

$$\begin{aligned}
&\sigma_u^{1-\alpha} \inf_{t\in[0,1-\frac{r_u}{u}]} \|\phi_{t,u}\|_\alpha \\
\ge\ & \sigma_u^{1-\alpha} \inf_{t\in[0,1-\frac{r_{u_{n+1}}}{u_{n+1}}]} \frac{\left(r_u\sigma_u\right)^{-1/2}}{\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}} \left\|\phi_{t,u_{n+1}}\left(\frac{r_u}{r_{u_{n+1}}}\cdot\right)\right\|_\alpha \\
\ge\ & \sigma_{u_n}^{1-\alpha} \inf_{t\in[0,1-\frac{r_{u_{n+1}}}{u_{n+1}}]} \left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2} \|w(u_{n+1}t+r_u\cdot)-w(u_{n+1}t)\|_\alpha \\
\ge\ & \frac{1}{\theta^{2(1-\alpha)}}\sigma_{u_{n+1}}^{1-\alpha} \inf_{t\in[0,1-\frac{r_{u_{n+1}}}{u_{n+1}}]} \left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2} \|w(u_{n+1}t+r_u\cdot)-w(u_{n+1}t)\|_\alpha.
\end{aligned} \tag{7}$$

令 $\theta\to1$, 由式 (6)、(7) 证得在 $C_{r,p}$-q.s. 意义下有

$$\liminf_{u\to\infty} \sigma_u^{1-\alpha} \inf_{t\in[0,1-\frac{r_u}{u}]} \|\phi_{t,u}\|_\alpha \ge k^\gamma.$$

引理 6 设 r_u,σ_u 如引理 5 中所定义, 若 $\lim\limits_{u\to\infty}\frac{\log ur_u^{-1}}{\log\log u}=\infty$, 则在 $C_{r,p}$-q.s. 意义下有

$$\limsup_{u\to\infty} \sigma_u^{1-\alpha} \inf_{t\in[0,u-r_u]} \|\left(r_u\sigma_u\right)^{-1/2}\left(w(t+r_u\cdot)-w(t)\right)\|_\alpha \le k^\gamma.$$

证 由于 $\lim\limits_{u\to\infty}\frac{\log\frac{u}{r_u}}{\log\log u}=\infty$, 故存在子列 $\{u_n\}$, 满足 $\frac{u_n}{r_{u_n}}=n^{p_0}$, $p_0>1$. 显然 $\{u_n\}$ 单调递增, 且当 $n\to\infty$ 时 $u_n\to\infty$. 设 $t_i=ir_{u_{n+1}}$, $i=0,1,2,\cdots$, $k_n=\left[\frac{u_n}{r_{u_{n+1}}}\right]-1$, 设 $g(n)=\frac{\log\frac{u_n}{r_{u_n}}}{\log\log u_n}=\frac{\log n^{p_0}}{\log\log u_n}$, 则有 $u_n=\exp\left(n^{\frac{p_0}{g(n)}}\right)$, 且当 $n\to\infty$, $g(n)\to\infty$. 进一步, 对任意 $\theta>0$, 当 $n\to\infty$, $\frac{n^\theta}{\log u_n}\to\infty$, $1\le\frac{u_{n+1}}{u_n}=\exp\{(n+1)^{\frac{p_0}{g(n+1)}}-n^{\frac{p_0}{g(n)}}\}\le\exp\{n^{\frac{p_0}{g(n)}-1}\}\to1$. 选取 $\delta'>0$ 使得 $\delta''=\frac{1}{(1+\varepsilon)^{1/\gamma}}+\delta'<1$. 令 $k=[r]+1$, 根据引理 3 得

$$\begin{aligned}
&C_{r,p}\left(\sigma_{u_{n+1}}^{1-\alpha} \inf_{t\in[0,u_n-r_{u_{n+1}}]} \|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}\left(w(t+r_u\cdot)-w(t)\right)\|_\alpha \ge k^\gamma(1+2\varepsilon)\right)^{1/p} \\
\le\ & C_{r,p}\left(\min_{0\le i\le k_n} \|\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}\left(w(t_i+r_u\cdot)-w(t_i)\right)\|_\alpha \ge \frac{k^\gamma(1+2\varepsilon)}{\left(\sigma_{u_{n+1}}\right)^{1-\alpha}}\right)^{1/p} \\
=\ & (1+k_n)^k\left(\frac{\sigma_{u_{n+1}}^{1-\alpha}}{k^\gamma\varepsilon}\right)^{2k^2+k} \mu\left(\pi_i \ge \frac{k^\gamma(1+\varepsilon)}{\left(\sigma_{u_{n+1}}\right)^{1-\alpha}}\right)^{\frac{(1+kn)}{q_2}},
\end{aligned}$$

其中 $\pi_i = \| \left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(t_i+r_u\cdot)-w(t_i))\|_\alpha$. 由 (2) 式, 当 n 充分大时,

$$\begin{aligned}
&C_{r,p}\left(\sigma_{u_{n+1}}^{1-\alpha}\inf_{t\in[0,u_n-r_{u_{n+1}}]}\| \left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(t+r_u\cdot)-w(t))\|_\alpha \geq k^\gamma(1+2\varepsilon)\right)^{1/p}\\
\leq\ & (1+k_n)^k\left(\frac{\left(\sigma_{u_{n+1}}\right)^{1-\alpha}}{k^\gamma\varepsilon}\right)^{2k^2+k}\left(1-\left(\frac{r_{u_{n+1}}}{u_{n+1}\log u_{n+1}}\right)^{\delta''}\right)^{\frac{1+kn}{q_2}}\\
\leq\ & cn^{kp_0}\left(\log(n+1)\right)^{(2k^2+k)(1-\alpha)}\exp\left\{-\frac{1}{q_2}\left(\frac{r_{u_{n+1}}}{u_{n+1}\log u_{n+1}}\right)^{\delta''}\left[\frac{u_n}{r_{u_{n+1}}}\right]\right\},
\end{aligned}$$

其中常数 $c>0$. 选取适当的 p_0 可得到

$$\sum_{n=1}^{\infty}cn^{pkp_0}\left(\log(n+1)\right)^{p(2k^2+k)(1-\alpha)}\exp\left\{-\frac{p}{q_2}\left(\frac{r_{u_{n+1}}}{u_{n+1}\log u_{n+1}}\right)^{\delta''}\left[\frac{u_n}{r_{u_{n+1}}}\right]\right\}<\infty,$$

由 Borel-Cantelli 引理, 从而证得存在一递增序列 $u_n, u_n\to\infty$, 使得在 $C_{r,p}$-q.s. 意义下有

$$\limsup_{n\to\infty}\sigma_{u_{n+1}}^{1-\alpha}\inf_{t\in[0,u_n-r_{u_{n+1}}]}\| \left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}(w(t+r_u\cdot)-w(t))\|_\alpha\leq k^\gamma, \tag{8}$$

再设 $\psi_{t,u}(s)=(r_u\sigma_u)^{-1/2}(w(t+r_us)-w(t))$, 经推导有 $\psi_{t,u}(s)=\frac{(r_u\sigma_u)^{-1/2}}{\beta_{u_{n+1}}}\psi_{t,u_{n+1}}\left(\frac{r_u}{r_{u_{n+1}}}s\right)$, 其中 $u\in(u_n,u_{n+1})$, 从而有

$$\begin{aligned}
&\inf_{t\in[0,u-r_u]}\|(r_u\sigma_u)^{-1/2}(w(t+r_us)-w(t))\|_\alpha\\
\leq\ & \inf_{t\in[0,u_n-r_{u_{n+1}}]}\frac{\left(r_{u_n}\sigma_{u_n}\right)^{-1/2}}{\left(r_{u_{n+1}}\sigma_{u_{n+1}}\right)^{-1/2}}\left\|\psi_{t,u_{n+1}}\left(\frac{r_u}{r_{u_{n+1}}}\cdot\right)\right\|_\alpha,
\end{aligned}$$

由 (8) 式, 考虑到当 $n\to\infty$ 时 $\frac{r_{un}\sigma_{un}}{r_{u_{n+1}}\sigma_{u_{n+1}}}\to 1$, 引理 6 得证.

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THE RATE OF LOCAL FUNCTIONAL CONVERGENCE FOR BROWNIAN MOTION'S INCREMENTS IN HÖLDER NORM

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Abstract: In this paper, the local functional limit theorem for increments of a Brownian motion is derived. With large and small deviations, the local functional convergence rate for increments of Brownian motion in Hölder norm with respect to capacity is estimated, and the result in [4] is generalized.

Keywords: Brownian motion; convergence rate; Hölder norm; capacity

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