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一类非线性系统的分段迭代学习控制

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摘 要: 本文提出并研究一类非线性系统的分段迭代学习控制问题. 基于 P 型学习律和 D 型学习律构建得到分段迭代学习控制律, 利用压缩映射原理, 证明这种分段迭代学习律能使得系统的输出跟踪误差沿迭代轴方向收敛. 仿真算例验证了算法的有效性.

关键词: P 型学习律; D 型学习律; 分段迭代学习控制; 非线性系统
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1 引言

迭代学习控制由 Arimoto 等人 [1] 首次提出完整的控制算法后, 已成为近年来控制理论研究的热点问题, 并引起人们的广泛关注. 在迭代学习控制设计中, 采用较多的是 D 型学习律 [1−4] 和 P 型学习律 [5−9], 根据系统所满足的性质, 在重复受控时间内, 用 D 型学习律或 P 型学习律进行控制设计. 事实上, 在不同的受控时间段内, 系统所满足的性质可能是不同的, 也就是说, 可能在一个时间段内, 系统更适合用 P 型学习律, 在另一个时间段内, 系统更适合用 D 型学习律. 如何对这类系统进行迭代学习控制设计, 并进行相应的分段控制设计, 据笔者所知, 尚无相关的研究论文.

文 [8] 针对一类具有最一般形式的非线性系统, 采用 P 型与 Newton 型迭代学习控制律相结合的方法, 对此类系统进行了控制设计; 文 [9] 基于收敛性质分析, 对文 [8] 中的 P 型迭代学习控制律, 进一步作了鲁棒优化设计, 并得到最优的学习增益取值. 对具有最一般形式的非线性系统, 如何进行控制设计, 始终是非线性系统控制理论研究深感兴趣的内容.

本文提出分段迭代学习控制的概念, 并对文 [9] 中的系统进行分段迭代学习控制设计. 根据系统在不同时间段内所满足的不同假设条件, 采用不同的学习控制律, 最终组合成整个重复受控时间内的学习控制律, 当该学习控制律作用于系统时, 系统的输出跟踪误差沿迭代轴方向收敛.

本文给出如下符号约定: 对矩阵 $A\in R^{n\times n}$, 记 $\|A\|$ 为矩阵 A 的 2 范数, 即 $\|A\|=\sqrt{\rho(A^{\mathrm T}A)}$, 其中 $\rho(A^{\mathrm T}A)$ 为矩阵 $A^{\mathrm T}A$ 的谱半径; 对向量 $x(t)\in R^n$, $t\in[\tilde T,T]$, $0\le\tilde T<T$, 定义 $x(t)$ 的上确界范数 $\|x(t)\|_s=\sup\limits_{t\in[\tilde T,T]}\|x(t)\|$, 其中 $\|x(t)\|$ 为 $x(t)$ 的 2 范数; 对给定的 $\lambda>0$, 定义 $x(t)$ 的 λ - 范数 $\|x(t)\|_\lambda=\sup\limits_{t\in[\tilde T,T]}\mathrm e^{-\lambda(t-\tilde T)}\|x(t)\|$. 由文 [8] 可知, 范数 $\|x(t)\|_s$ 与 $\|x(t)\|_\lambda$ 是等价的, 即可用其中任一种范数来证明收敛性结果.

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2 问题描述

考虑如下形式的单输入 - 单输出非线性系统 [9]:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t), \\ y(t) = g(x(t), u(t), t), \end{cases} \tag{2.1}$$

这里 $t \in [0, T]$, $x(t) \in R^n$, $u(t) \in R$, $y(t) \in R$ 分别是系统的状态, 控制输入和输出.

设系统 (2.1) 的输出 $y(t)$ 在不同的时间段内有不同的表达形式:

$$y(t) = \begin{cases} g(x(t), u(t), t), & 0 \le t \le \hat{T}, \\ g(x(t), u(\hat{T}), t) + (t - \hat{T}) \left.\dfrac{\partial g}{\partial u}\right|_{t=\hat{T}^-} \dot{u}(\hat{T}^-), & \hat{T} < t \le T, \end{cases}$$

这里 $0 < \hat{T} < T$, 而 $\left.\frac{\partial g}{\partial u}\right|_{t=\hat{T}^-}, \dot{u}(\hat{T}^-)$ 表示函数在 $t = \hat{T}$ 处的左导数.

注 1 在 $\hat{T} < t \le T$ 内, $y(t)$ 的形式保证了 $y(t)$ 在 $[0, T]$ 内的连续性和可导性.

由此系统 (2.1) 化为

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t), & 0 \le t \le T, \\ y(t) = g(x(t), u(t), t), & 0 \le t \le \hat{T}, \\ y(t) = g(x(t), u(\hat{T}), t) + (t - \hat{T}) \left.\dfrac{\partial g}{\partial u}\right|_{t=\hat{T}^-} \dot{u}(\hat{T}^-), & \hat{T} < t \le T. \end{cases} \tag{2.2}$$

参照文 [8], 记 $\Omega_1 \triangleq D \times U \times [0, \hat{T}]$, $\Omega_2 \triangleq D \times U \times (\hat{T}, T]$, 其中 D, U 分别为 R^n 和 R 中的紧集. 对系统 (2.2), 给出如下假设条件:

假设 1 对于给定的初值 $x(0)$ 及控制输入 $u(t)$, 系统 (2.2) 的解 $x(t), y(t)$ 存在唯一, 而 $f(x(t), u(t), t)$, $g(x(t), u(t), t)$ 是足够光滑函数 [10].

假设 2 对于给定的理想轨迹 $y_r(t) \in C^1[0, T]$, 存在唯一的控制输入 $u_r(t) \in C[0, T] \cap C^1[0, \hat{T}]$, 使得

$$\begin{cases} \dot{x}_r(t) = f(x_r(t), u_r(t), t), & 0 \le t \le T, \\ y_r(t) = g(x_r(t), u_r(t), t), & 0 \le t \le \hat{T}, \\ y_r(t) = g(x_r(t), u_r(\hat{T}), t) + (t - \hat{T}) \left.\dfrac{\partial g}{\partial u_r}\right|_{t=\hat{T}^-} \dot{u}_r(\hat{T}^-), & \hat{T} < t \le T \end{cases}$$

成立.

假设 3 系统的初始定位条件为 $x_k(0) = x_r(0)$, $k = 0, 1, 2, \cdots$.

设动态系统 (2.2) 在有限区间 $t \in [0, T]$ 内是可重复的, 在迭代学习过程中, 重写系统 (2.2) 为

$$\begin{cases} \dot{x}_k(t) = f(x_k(t), u_k(t), t), & 0 \le t \le T, \\ y_k(t) = g(x_k(t), u_k(t), t), & 0 \le t \le \hat{T}, \\ y_k(t) = g(x_k(t), u_k(\hat{T}), t) + (t - \hat{T}) \left.\dfrac{\partial g}{\partial u_k}\right|_{t=\hat{T}^-} \dot{u}_k(\hat{T}^-), & \hat{T} < t \le T. \end{cases} \tag{2.3}$$

学习控制的目的是寻找适当的学习律, 使得迭代学习序列 $y_k(t)$ 一致收敛于理想的输出 $y_r(t)$, 即

$$\lim_{k\to\infty}\|e_k(t)\|_s=0,$$

其中 $e_k(t)=y_r(t)-y_k(t)$.

设系统 (2.2) 在不同时间段内满足不同的假设条件, 在 $t\in[0,\hat{T}]$ 内, 有

假设 4 非线性函数 $f(x,u,t)$ 在 Ω_1 内对变量 x,u 是全局 Lipschitz 连续的, 即

$$\|f(x_1,u_1,t)-f(x_2,u_2,t)\|\le L_f(\|x_1-x_2\|+|u_1-u_2|),$$

其中 L_f 是未知的 Lipschitz 常数.

假设 5 $\forall(x,u,t)\in\Omega_1$, 有 $0<\alpha_1\le\frac{\partial g}{\partial u}\le\alpha_2, \left\|\left(\frac{\partial g}{\partial x}\right)^{\mathrm{T}}\right\|\le\beta_1, \left\|\left(\frac{\partial^2 g}{\partial u\partial x}\right)^{\mathrm{T}}\right\|\le\beta_2, \left|\frac{\partial^2 g}{\partial u^2}\right|\le\beta_3$ 成立, 其中 α_1,α_2 是已知的常数, β_1,β_2,β_3 是未知的常数. 记 $D=\{a|\,\alpha_1\le a\le\alpha_2\}$.

注 2 假设 4, 5 使得系统 (2.2) 在 $t\in[0,\hat{T}]$ 内适合用 P 型学习律.

在 $t\in(\hat{T},T]$ 内, 对系统 (2.2) 提出如下的假设条件:

假设 6 $\forall(x,u,t)\in\Omega_2, i=1,2,\cdots,n$, 有 $\|f\|\le\beta_4, \left\|\frac{\partial f}{\partial x}\right\|\le\beta_5, 0<\alpha_{i1}\le\left(\frac{\partial f}{\partial u}\right)_i\le\alpha_{i2}$ 成立, 其中 α_{i1},α_{i2} 是已知的常数, β_4,β_5 是未知的常数, 而 $\left(\frac{\partial f}{\partial u}\right)_i$ 表示向量 $\frac{\partial f}{\partial u}$ 的第 i 个分量. 记 $\bar{D}=\{(a_1,a_2,\cdots,a_n)|\,\alpha_{i1}\le a_i\le\alpha_{i2}, i=1,2,\cdots,n\}$.

假设 7 $\forall(x,u(\hat{T}),t)\in\Omega_2, j=1,2,\cdots,n$, 有 $\left|\frac{\partial^2 g}{\partial t\partial u(\hat{T})}\right|\le\beta_6, \left\|\frac{\partial^2 g}{\partial x^2}\right\|\le\beta_7, \left\|\left(\frac{\partial^2 g}{\partial t\partial x}\right)^{\mathrm{T}}\right\|\le\beta_8, \left\|\left(\frac{\partial^2 g}{\partial x\partial u(\hat{T})}\right)^{\mathrm{T}}\right\|\le\beta_9, 0<\gamma_{j1}\le\left(\frac{\partial g}{\partial x}\right)_j^{\mathrm{T}}\le\gamma_{j2}$ 成立, 其中 γ_{j1},γ_{j2} 是已知的常数, β_6, β_7, β_8, β_9 是未知的常数, 而 $\left(\frac{\partial g}{\partial x}\right)_j^{\mathrm{T}}$ 表示向量 $\left(\frac{\partial g}{\partial x}\right)^{\mathrm{T}}$ 的第 j 个分量. 记

$$\hat{D}=\{(\hat{a}_1,\hat{a}_2,\cdots,\hat{a}_n)|\,\gamma_{j1}\le\hat{a}_j\le\gamma_{j2}, j=1,2,\cdots,n\}.$$

注 3 假设 6, 7 使得系统 (2.2) 在 $t\in[\hat{T},T]$ 内适合用 D 型学习律.

引理 1 [11] 设 $\{a_k\}$, $\{b_k\}$ 是满足

$$a_{k+1}\le\rho a_k+b_k,\ \ 0\le\rho<1$$

的非负实数列, 如有 $\lim\limits_{k\to\infty}b_k=0$, 则有 $\lim\limits_{k\to\infty}a_k=0$.

3 主要结果

对系统 (2.2) 构建分段学习律

$$\begin{cases}u_{k+1}(t)=u_k(t)+qe_k(t), & t\in[0,\hat{T}],\\ u_{k+1}(t)=u_k(t)+p\dot{e}_k(t)+qe_k(\hat{T})-p\dot{e}_k(\hat{T}), & t\in(\hat{T},T],\end{cases}\tag{3.1}$$

其中 $q,p>0$ 为学习增益.

注 4 由 (3.1) 式可知选取 $u_0(t)\in C[0,T]$, 则 $u_k(t)\in C[0,T]$, $k=1,2,3,\cdots$, 由此保证了系统 (2.3) 解的存在性.

注 5 由 (2.3), (3.1) 式及假设 1 中 $f(x(t),u(t),t),g(x(t),u(t),t)$ 的足够光滑性可知选取 $u_0(t)\in C[0,T]\cap C^1[0,\hat{T}]$, 则 $y_k(t)\in C^1[0,T]$, 再由假设 2, 有 $e_k(t)\in C^1[0,T]$.

由此给出如下定理:

定理 1 假设 1–7 成立, 如果有

$$\rho=\max\left\{\max_{\substack{\frac{\partial g}{\partial u}\in D\\ t\in[0,\hat{T}]}}\left|1-q\frac{\partial g}{\partial u}\right|,\ \max_{\substack{\frac{\partial f}{\partial u}\in\tilde{D}\\ \left(\frac{\partial g}{\partial x}\right)^{\mathrm{T}}\in\hat{D}\\ t\in(\hat{T},T]}}\left|1-p\frac{\partial g}{\partial x}\frac{\partial f}{\partial u}\right|\right\}<1, \tag{3.2}$$

则系统 (2.2) 在分段学习律 (3.1) 作用下是收敛的, 即 $\lim\limits_{k\to\infty}\|e_k(t)\|_s=0$.

证 因证明过程需多次用到泰勒展开公式, 为简便起见, 用 $*$ 表示公式中各个不同的中值, 记 $\delta x_k=x_{k+1}(t)-x_k(t),\ \ \delta u_k=u_{k+1}(t)-u_k(t)$. 由于系统的分段性质, 证明分两部分进行.

(1) 对 $t\in[0,\hat{T}]$. 应用泰勒展开公式, 由 (2.3), (3.1) 式有

$$\begin{aligned}y_{k+1}(t)&=g(x_{k+1},u_{k+1},t)=g(x_k+\delta x_k,u_k+\delta u_k,t)\\&=g(x_k,u_k,t)+\left.\frac{\partial g}{\partial u}\right|_*\delta u_k+\left.\frac{\partial g}{\partial x}\right|_*\delta x_k\\&=y_k(t)+q\left.\frac{\partial g}{\partial u}\right|_*e_k(t)+\left.\frac{\partial g}{\partial x}\right|_*\delta x_k,\end{aligned}$$

由此有

$$\begin{aligned}e_{k+1}(t)&=e_k(t)+y_k(t)-y_{k+1}(t)\\&=e_k(t)-q\left.\frac{\partial g}{\partial u}\right|_*e_k(t)-\left.\frac{\partial g}{\partial x}\right|_*\delta x_k\\&=\left[1-q\left.\frac{\partial g}{\partial u}\right|_*\right]e_k(t)-\left.\frac{\partial g}{\partial x}\right|_*\delta x_k.\end{aligned}$$

上式两端取范数, 由假设 5 及 (3.2) 式有

$$|e_{k+1}(t)|\le\rho\,|e_k(t)|+\beta_1\,\|x_{k+1}(t)-x_k(t)\|\,.$$

取 λ - 范数, 有

$$\begin{aligned}\|e_{k+1}(t)\|_\lambda&=\max_{t\in[0,\hat{T}]}\mathrm{e}^{-\lambda t}\,|e_{k+1}(t)|\\&\le\max_{t\in[0,\hat{T}]}\mathrm{e}^{-\lambda t}\,\{\rho\,|e_k(t)|+\beta_1\,\|x_{k+1}(t)-x_k(t)\|\}\\&\le\rho\max_{t\in[0,\hat{T}]}\mathrm{e}^{-\lambda t}\,|e_k(t)|+\beta_1\max_{t\in[0,\hat{T}]}\mathrm{e}^{-\lambda t}\,\|x_{k+1}(t)-x_k(t)\|\\&=\rho\|e_k(t)\|_\lambda+\beta_1\|x_{k+1}(t)-x_k(t)\|_\lambda.\end{aligned}\tag{3.3}$$

由系统 (2.3) 及假设 3, 4 可知

$$\begin{aligned}\|x_{k+1}(t)-x_k(t)\| &= \left\|\int_0^t \left(f(x_{k+1},u_{k+1},\tau)-f(x_k,u_k,\tau)\right)d\tau\right\| \\ &\le L_f\int_0^t (\|x_{k+1}-x_k\|+|u_{k+1}-u_k|)d\tau.\end{aligned}$$

由 (3.1) 式可得

$$\|x_{k+1}(t)-x_k(t)\|\le L_f\int_0^t (\|x_{k+1}-x_k\|+q\,|e_k|)d\tau.$$

应用 Gronwall 引理, 有

$$\begin{aligned}\|x_{k+1}(t)-x_k(t)\| &\le L_f q\int_0^t (\mathrm{e}^{L_f(t-\tau)}\,|e_k|)d\tau \\ &\le L_f q\mathrm{e}^{L_f\hat{T}}\int_0^t |e_k|d\tau = L_f q\mathrm{e}^{L_f\hat{T}}\int_0^t \mathrm{e}^{\lambda\tau}\mathrm{e}^{-\lambda\tau}\,|e_k|d\tau \\ &\le L_f q\mathrm{e}^{L_f\hat{T}}\int_0^t \mathrm{e}^{\lambda\tau}d\tau\|e_k\|_\lambda = L_f q\mathrm{e}^{L_f\hat{T}}\frac{\mathrm{e}^{\lambda t}-1}{\lambda}\|e_k\|_\lambda,\end{aligned}$$

由此有

$$\begin{aligned}\|x_{k+1}(t)-x_k(t)\|_\lambda &= \max_{t\in[0,\hat{T}]}\mathrm{e}^{-\lambda t}\,\|x_{k+1}(t)-x_k(t)\| \\ &\le \max_{t\in[0,\hat{T}]} L_f q\mathrm{e}^{L_f\hat{T}}\frac{1-\mathrm{e}^{-\lambda t}}{\lambda}\|e_k\|_\lambda \\ &= L_f q\mathrm{e}^{L_f\hat{T}}\frac{1-\mathrm{e}^{-\lambda\hat{T}}}{\lambda}\|e_k\|_\lambda.\end{aligned}$$

记 $O_1(\lambda^{-1})=L_f q\mathrm{e}^{L_f\hat{T}}\frac{1-\mathrm{e}^{-\lambda\hat{T}}}{\lambda}$, 则有

$$\|x_{k+1}(t)-x_k(t)\|_\lambda\le O_1(\lambda^{-1})\|e_k\|_\lambda. \tag{3.4}$$

显然, 当 λ 足够大时, 能使 $O_1(\lambda^{-1})$ 任意小.

将 (3.4) 式代入 (3.3) 式, 有

$$\|e_{k+1}(t)\|_\lambda\le(\rho+\beta_1 O_1(\lambda^{-1}))\|e_k(t)\|_\lambda.$$

因为 $\rho<1$, 所以取足够大的 λ, 能使得 $\rho+\beta_1 O_1(\lambda^{-1})<1$ 成立, 由压缩映射原理可知

$$\lim_{k\to\infty}\|e_k(t)\|_\lambda=0,$$

再由范数的等价性, 有

$$\lim_{k\to\infty}\|e_k(t)\|_s=0, t\in[0,\hat{T}], \tag{3.5}$$

由此有

$$\lim_{k\to\infty}\|\dot{e}_k(t)\|_s=0, t\in[0,\hat{T}],$$

特别地

$$\lim_{k\to\infty}\left|e_k(\hat{T})\right|=0,\lim_{k\to\infty}\left|\dot{e}_k(\hat{T})\right|=\lim_{k\to\infty}\left|\dot{e}_k(\hat{T}^-)\right|=0. \tag{3.6}$$

由假设 1, 2, 有

$$\begin{aligned}
&\lim_{k\to\infty}\left|y_r(\hat{T})-y_k(\hat{T})\right|=0\\
\Rightarrow\ &\lim_{k\to\infty}\left\|x_r(\hat{T})-x_k(\hat{T})\right\|=0\\
\Rightarrow\ &\lim_{k\to\infty}\left\|x_k(\hat{T})-x_{k+1}(\hat{T})\right\|=0,
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
&\lim_{k\to\infty}\|e_k(t)\|_s=0(t\in[0,\hat{T}])\\
\Rightarrow\ &\lim_{k\to\infty}\|u_r(t)-u_k(t)\|_s=0(t\in[0,\hat{T}])\\
\Rightarrow\ &\lim_{k\to\infty}\|\dot{u}_r(t)-\dot{u}_k(t)\|_s=0(t\in[0,\hat{T}])\Rightarrow\lim_{k\to\infty}\left|\dot{u}_r(\hat{T}^-)-\dot{u}_k(\hat{T}^-)\right|=0\\
\Rightarrow\ &\dot{u}_k(\hat{T}^-)\text{有界}\Leftrightarrow\exists\beta_{10}>0,\left|\dot{u}_k(\hat{T}^-)\right|\le\beta_{10}(k=0,1,2,\cdots).
\end{aligned} \tag{3.8}$$

(2) 对 $t\in(\hat{T},T]$. 由 (2.3) 式有

$$\begin{aligned}
&\dot{e}_{k+1}(t)=\dot{e}_k(t)+\dot{y}_k(t)-\dot{y}_{k+1}(t)\\
=&\dot{e}_k(t)+\frac{\partial g(x_k,u_k(\hat{T}),t)}{\partial x_k}\dot{x}_k(t)-\frac{\partial g(x_{k+1},u_{k+1}(\hat{T}),t)}{\partial x_{k+1}}\dot{x}_{k+1}(t)\\
&+\frac{\partial g(x_k,u_k(\hat{T}),t)}{\partial t}-\frac{\partial g(x_{k+1},u_{k+1}(\hat{T}),t)}{\partial t}+\left.\frac{\partial g}{\partial u_k}\right|_{t=\hat{T}^-}\dot{u}_k(\hat{T}^-)-\left.\frac{\partial g}{\partial u_{k+1}}\right|_{t=\hat{T}^-}\dot{u}_{k+1}(\hat{T}^-)\\
=&\dot{e}_k(t)+\frac{\partial g(x_k,u_k(\hat{T}),t)}{\partial x_k}f(x_k,u_k,t)-\frac{\partial g(x_{k+1},u_{k+1}(\hat{T}),t)}{\partial x_{k+1}}f(x_k,u_k,t)\\
&+\frac{\partial g(x_{k+1},u_{k+1}(\hat{T}),t)}{\partial x_{k+1}}f(x_k,u_k,t)-\frac{\partial g(x_{k+1},u_{k+1}(\hat{T}),t)}{\partial x_{k+1}}f(x_{k+1},u_{k+1},t)\\
&+\frac{\partial g(x_k,u_k(\hat{T}),t)}{\partial t}-\frac{\partial g(x_{k+1},u_{k+1}(\hat{T}),t)}{\partial t}+\left.\frac{\partial g}{\partial u_{k+1}}\right|_{t=\hat{T}^-}(\dot{u}_k(\hat{T}^-)-\dot{u}_{k+1}(\hat{T}^-))\\
&+\left(\left.\frac{\partial g}{\partial u_k}\right|_{t=\hat{T}^-}-\left.\frac{\partial g}{\partial u_{k+1}}\right|_{t=\hat{T}^-}\right)\dot{u}_k(\hat{T}^-)\\
=&\dot{e}_k(t)+(x_k-x_{k+1})^{\mathrm{T}}\left.\left(\frac{\partial^2 g}{\partial x^2}\right)^{\mathrm{T}}\right|_*f(x_k,u_k,t)+(u_k(\hat{T})-u_{k+1}(\hat{T}))\left.\frac{\partial^2 g}{\partial x_k\partial u(\hat{T})}\right|_*f(x_k,u_k,t)\\
&+\frac{\partial g(x_{k+1},u_{k+1}(\hat{T}),t)}{\partial x_{k+1}}\left(\left.\frac{\partial f}{\partial x}\right|_*(x_k-x_{k+1})+\left.\frac{\partial f}{\partial u}\right|_*(u_k-u_{k+1})\right)+\left.\frac{\partial^2 g}{\partial t\partial x}\right|_*(x_k-x_{k+1})\\
&+\left.\frac{\partial^2 g}{\partial t\partial u(\hat{T})}\right|_*(u_k(\hat{T})-u_{k+1}(\hat{T}))+\left.\frac{\partial g}{\partial u_{k+1}}\right|_{t=\hat{T}^-}(\dot{u}_k(\hat{T}^-)-\dot{u}_{k+1}(\hat{T}^-))\\
&+\left(\left.\frac{\partial^2 g}{\partial u\partial x}\right|_*(x_k(\hat{T})-x_{k+1}(\hat{T}))+\left.\frac{\partial^2 g}{\partial u^2}\right|_*(u_k(\hat{T})-u_{k+1}(\hat{T}))\right)\dot{u}_k(\hat{T}^-).
\end{aligned}$$

将 (3.1) 式代入上式, 得

$$
\begin{aligned}
\dot{e}_{k+1}(t) =& \left(1 - p\frac{\partial g(x_{k+1}, u_{k+1}(\hat{T}), t)}{\partial x_{k+1}}\frac{\partial f}{\partial u}\bigg|_*\right)\dot{e}_k(t) \\
& - \frac{\partial g(x_{k+1}, u_{k+1}(\hat{T}), t)}{\partial x_{k+1}}\frac{\partial f}{\partial u}\bigg|_*(qe_k(\hat{T}) - p\dot{e}_k(\hat{T})) + (x_k - x_{k+1})^{\mathrm{T}}\left(\frac{\partial^2 g}{\partial x^2}\right)^{\mathrm{T}}\bigg|_* f(x_k, u_k, t) \\
& -qe_k(\hat{T})\frac{\partial^2 g}{\partial x_k \partial u(\hat{T})}\bigg|_* f(x_k, u_k, t) + \frac{\partial g(x_{k+1}, u_{k+1}(\hat{T}), t)}{\partial x_{k+1}}\frac{\partial f}{\partial x}\bigg|_*(x_k - x_{k+1}) \\
& +\frac{\partial^2 g}{\partial t \partial x}\bigg|_*(x_k - x_{k+1}) - qe_k(\hat{T})\frac{\partial^2 g}{\partial t \partial u(\hat{T})}\bigg|_* - q\dot{e}_k(\hat{T})\frac{\partial g}{\partial u_{k+1}}\bigg|_{t=\hat{T}^-} \\
& + \left(\frac{\partial^2 g}{\partial u \partial x}\bigg|_*(x_k(\hat{T}) - x_{k+1}(\hat{T})) - qe_k(\hat{T})\frac{\partial^2 g}{\partial u^2}\bigg|_*\right)\dot{u}_k(\hat{T}^-),
\end{aligned}
$$

上式取范数, 由 (3.2), (3.8) 式及假设 5, 6, 7, 可得

$$
\begin{aligned}
|\dot{e}_{k+1}(t)| \leq\ & \rho\,|\dot{e}_k(t)| + \left(\beta_4\beta_7 + \beta_5\sqrt{\sum_{j=1}^n \gamma_{j2}^2} + \beta_8\right)\|x_k - x_{k+1}\| \\
& +\beta_2\beta_{10}\left\|x_k(\hat{T}) - x_{k+1}(\hat{T})\right\| \\
& +q\left(\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\sqrt{\sum_{j=1}^n \gamma_{j2}^2} + \beta_4\beta_9 + \beta_6 + \beta_3\beta_{10}\right)\left|e_k(\hat{T})\right| \\
& + \left(p\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\sqrt{\sum_{j=1}^n \gamma_{j2}^2} + q\alpha_2\right)\left|\dot{e}_k(\hat{T})\right| \\
=\ & \rho\,|\dot{e}_k(t)| + b\,\|x_k - x_{k+1}\| + F_k(\hat{T}), t \in (\hat{T}, T],
\end{aligned}
\tag{3.9}
$$

其中 $b = \beta_4\beta_7 + \beta_5\sqrt{\sum\limits_{j=1}^n \gamma_{j2}^2} + \beta_8$,

$$
\begin{aligned}
F_k(\hat{T}) =& \beta_2\beta_{10}\left\|x_k(\hat{T}) - x_{k+1}(\hat{T})\right\| + q\left(\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\sqrt{\sum_{j=1}^n \gamma_{j2}^2} + \beta_4\beta_9 + \beta_6 + \beta_3\beta_{10}\right)\left|e_k(\hat{T})\right| \\
& + \left(p\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\sqrt{\sum_{j=1}^n \gamma_{j2}^2} + q\alpha_2\right)\left|\dot{e}_k(\hat{T})\right| \geq 0,
\end{aligned}
$$

由 (3.6), (3.7) 式, 有

$$
\lim_{k\to\infty} F_k(\hat{T}) = 0, \tag{3.10}
$$

由 (2.3) 式及注 5, 可知 $\dot{e}_k(t), \dot{e}_{k+1}(t), x_k(t) - x_{k+1}(t)$ 在 $t \in [0, T]$ 上连续, 由此 (3.9) 式在 $t \in [\hat{T}, T]$ 成立, 即

$$
|\dot{e}_{k+1}(t)| \leq \rho\,|\dot{e}_k(t)| + b\,\|x_k - x_{k+1}\| + F_k(\hat{T}), t \in [\hat{T}, T]. \tag{3.11}
$$

对 (3.11) 式取 λ - 范数, 则有

$$\begin{aligned}\|\dot e_{k+1}(t)\|_\lambda &= \max_{t\in[\hat T,T]} \mathrm{e}^{-\lambda(t-\hat T)}\,|\dot e_{k+1}(t)| \\ &\le \rho \max_{t\in[\hat T,T]} \mathrm{e}^{-\lambda(t-\hat T)}\,|\dot e_k(t)| + b\max_{t\in[\hat T,T]} \mathrm{e}^{-\lambda(t-\hat T)}\,\|x_k-x_{k+1}\| + \max_{t\in[\hat T,T]} \mathrm{e}^{-\lambda(t-\hat T)} F_k(\hat T) \\ &= \rho\|\dot e_k(t)\|_\lambda + b\|x_k-x_{k+1}\|_\lambda + F_k(\hat T). \end{aligned} \tag{3.12}$$

同样由连续性, 对 $t\in[\hat T,T]$, 由方程 (2.3) 有

$$x_{k+1}(t)-x_k(t) = x_{k+1}(\hat T)-x_k(\hat T) + \int_{\hat T}^t \left(f(x_{k+1},u_{k+1},\tau)-f(x_k,u_k,\tau)\right)d\tau.$$

取范数, 由假设 6 及 (3.1) 式, 有

$$\begin{aligned}&\|x_{k+1}(t)-x_k(t)\| \\ \le\ & \left\|x_{k+1}(\hat T)-x_k(\hat T)\right\| + \left\|\int_{\hat T}^t \left(\left.\frac{\partial f}{\partial x}\right|_*(x_{k+1}-x_k) + \left.\frac{\partial f}{\partial u}\right|_*(u_{k+1}-u_k)\right)d\tau\right\| \\ \le\ & \left\|x_{k+1}(\hat T)-x_k(\hat T)\right\| + \int_{\hat T}^t \left(\beta_5\,\|x_{k+1}-x_k\| + \sqrt{\sum_{i=1}^n \alpha_{i2}^2}(p\,|\dot e_k| + q\left|e_k(\hat T)\right| + p\left|\dot e_k(\hat T)\right|)\right)d\tau \\ \le\ & \left\|x_{k+1}(\hat T)-x_k(\hat T)\right\| + (T-\hat T)\left(q\left|e_k(\hat T)\right| + p\left|\dot e_k(\hat T)\right|\right)\sqrt{\sum_{i=1}^n \alpha_{i2}^2} \\ & + \int_{\hat T}^t \left(\beta_5\,\|x_{k+1}-x_k\| + p\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\,|\dot e_k|\right)d\tau. \end{aligned}$$

再次应用 Gronwall 引理, 有

$$\begin{aligned}\|x_{k+1}(t)-x_k(t)\| &\le G_k(\hat T) + p\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\int_{\hat T}^t (\mathrm{e}^{\beta_5(t-\tau)}\,|\dot e_k|)d\tau \\ &\le G_k(\hat T) + p\mathrm{e}^{\beta_5(T-\hat T)}\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\int_{\hat T}^t |\dot e_k|d\tau \\ &= G_k(\hat T) + p\mathrm{e}^{\beta_5(T-\hat T)}\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\int_{\hat T}^t \mathrm{e}^{\lambda(\tau-\hat T)}\mathrm{e}^{-\lambda(\tau-\hat T)}\,|\dot e_k|d\tau \\ &\le G_k(\hat T) + p\mathrm{e}^{\beta_5(T-\hat T)}\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\int_{\hat T}^t \mathrm{e}^{\lambda(\tau-\hat T)}d\tau\|\dot e_k\|_\lambda \\ &= G_k(\hat T) + p\mathrm{e}^{\beta_5(T-\hat T)}\sqrt{\sum_{i=1}^n \alpha_{i2}^2}\frac{\mathrm{e}^{\lambda(t-\hat T)}-1}{\lambda}\|\dot e_k\|_\lambda, \end{aligned}$$

其中

$$G_k(\hat{T}) = \mathrm{e}^{\beta_5(T-\hat{T})}\left(\left\|x_{k+1}(\hat{T}) - x_k(\hat{T})\right\| + (T-\hat{T})(q\left|e_k(\hat{T})\right| + p\left|\dot{e}_k(\hat{T})\right|)\sqrt{\sum_{i=1}^{n}\alpha_{i2}^2}\right) \geq 0,$$

并有

$$\lim_{k\to\infty} G_k(\hat{T}) = 0, \tag{3.13}$$

由此有

$$\begin{aligned}\|x_{k+1}(t) - x_k(t)\|_\lambda &= \max_{t\in[\hat{T},T]} \mathrm{e}^{-\lambda(t-\hat{T})}\|x_{k+1}(t) - x_k(t)\| \\ &\leq \max_{t\in[\hat{T},T]} \mathrm{e}^{-\lambda(t-\hat{T})}G_k(\hat{T}) + \max_{t\in[\hat{T},T]} p\mathrm{e}^{\beta_5(T-\hat{T})}\sqrt{\sum_{i=1}^{n}\alpha_{i2}^2}\frac{1-\mathrm{e}^{-\lambda(t-\hat{T})}}{\lambda}\|\dot{e}_k\|_\lambda \\ &= G_k(\hat{T}) + p\mathrm{e}^{\beta_5(T-\hat{T})}\sqrt{\sum_{i=1}^{n}\alpha_{i2}^2}\frac{1-\mathrm{e}^{-\lambda(T-\hat{T})}}{\lambda}\|\dot{e}_k\|_\lambda.\end{aligned}$$

记 $O_2(\lambda^{-1}) = p\mathrm{e}^{\beta_5(T-\hat{T})}\sqrt{\sum\limits_{i=1}^{n}\alpha_{i2}^2}\frac{1-\mathrm{e}^{-\lambda(T-\hat{T})}}{\lambda}$, 则有

$$\|x_{k+1}(t) - x_k(t)\|_\lambda \leq G_k(\hat{T}) + O_2(\lambda^{-1})\|\dot{e}_k\|_\lambda, \tag{3.14}$$

将 (3.14) 式代入 (3.12) 式, 有

$$\|\dot{e}_{k+1}(t)\|_\lambda \leq (\rho + bO_2(\lambda^{-1}))\|\dot{e}_k(t)\|_\lambda + bG_k(\hat{T}) + F_k(\hat{T}). \tag{3.15}$$

因为 $\rho < 1$, 所以取足够大的 λ, 能使得 $\rho + bO_2(\lambda^{-1}) < 1$ 成立, 而由 (3.10), (3.13) 式有 $\lim\limits_{k\to\infty}(bG_k(\hat{T}) + F_k(\hat{T})) = 0$, 对 (3.15) 式利用引理 1, 得 $\lim\limits_{k\to\infty}\|\dot{e}_k(t)\|_\lambda = 0$. 再由范数的等价性, 有

$$\lim_{k\to\infty}\|\dot{e}_k(t)\|_s = 0, t\in[\hat{T},T],$$

由 (3.6) 式可得

$$\begin{aligned}\lim_{k\to\infty}\|e_k(t)\|_s &= \lim_{k\to\infty}\left\|e_k(\hat{T}) + \int_{\hat{T}}^{t}\dot{e}_k(\tau)d\tau\right\|_s \leq \lim_{k\to\infty}\left|e_k(\hat{T})\right| + \lim_{k\to\infty}\left\|\int_{\hat{T}}^{t}\dot{e}_k(\tau)d\tau\right\|_s \\ &= \lim_{k\to\infty}\left\|\int_{\hat{T}}^{t}\dot{e}_k(\tau)d\tau\right\|_s \leq (T-\hat{T})\lim_{k\to\infty}\|\dot{e}_k(t)\|_s = 0,\end{aligned}$$

由此可知

$$\lim_{k\to\infty}\|e_k(t)\|_s = 0, t\in[\hat{T},T]. \tag{3.16}$$

综上, 由 (3.5), (3.16) 式得 $\lim\limits_{k\to\infty}\|e_k(t)\|_s = 0, t\in[0,T]$.

注 6 由假设 5, 6, 7, 选取学习增益 q, 使其满足 $0 < q < \frac{2}{\alpha_2}$; 选取学习增益 p, 使其满足 $0 < p < \frac{2}{\sum\limits_{i=1}^{n}(\alpha_{i2}\gamma_{i2})}$, 则收敛性条件 (3.2) 式就能成立.

4 仿真算例

构建如下非线性系统

$$\begin{cases} \dot{x}(t) = u(t) - \dfrac{1}{2}\arctan(\dfrac{1}{2}u(t)) + \sin(x(t)), & 0 \le t \le 2, \\ y(t) = 2x(t) + \cos(x(t)) + u(t), & 0 \le t \le 1, \\ y(t) = 2x(t) + \cos(x(t)) + u(1) + (t-1)\dot{u}(1), & 1 < t \le 2. \end{cases} \tag{4.1}$$

理想轨迹为 $y_r(t) = t^2$, $t \in [0,2]$. 通过验证可知系统 (4.1) 满足假设条件 1 $\sim$ 7 (见附录). 由系统 (4.1), 有 $\frac{\partial g}{\partial u} = 1$, $\frac{\partial f}{\partial u} \in [0.75, 1]$, $\frac{\partial g}{\partial x} \in [1,3]$, $\forall u, x \in R$, 即 $\alpha_1 = 1$, $\alpha_2 = a_{12} = \gamma_{11} = 1$, $a_{11} = 0.75$, $\gamma_{12} = 3$.

于是由注 6 可知 $q < \frac{2}{\alpha_2} = 2, p < \frac{2}{\alpha_{12}\gamma_{12}} = \frac{2}{3}$, 选取学习增益为 $q = 0.5, p = 0.3$, 此时 $\rho = \max\{|1-0.5\cdot 1|, |1-0.3\cdot 0.75|\} = 0.775 < 1$, 满足定理 1 的条件. 将 p, q 的值代入 (3.1) 式, 可得系统 (4.1) 的分段学习律为

$$\begin{cases} u_{k+1}(t) = u_k(t) + 0.5e_k(t), & t \in [0,1], \\ u_{k+1}(t) = u_k(t) + 0.3\dot{e}_k(t) + 0.5e_k(1) - 0.3\dot{e}_k(1), & t \in (1,2]. \end{cases}$$

取 $u_0(t) = 0, x_k(0) = 0, k = 0, 1, 2, \cdots$, 运用 Matlab 的 Simulink 模块进行仿真可得仿真结果如图 1 $\sim$ 2 所示. 从图中可以看出, 随着迭代次数的增加, 跟踪误差在分段学习律下呈较好的收敛趋势.

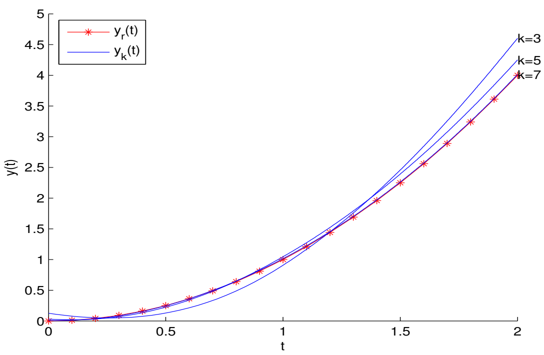

图 1: 输出曲线

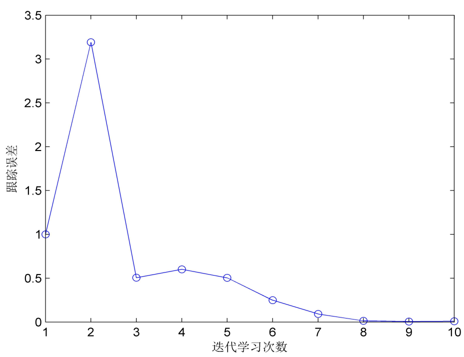

图 2: 跟踪误差

5 结论

本文提出了分段迭代学习控制的概念, 并对一类满足通常假设条件的非线性系统进行分段迭代学习控制设计. 根据系统在不同时间段内所满足的不同假设条件, 采用不同的学习控制律. 在前段时间段内, 采用 P 型学习控制律, 在后段时间段内, 采用 D 型学习控制律, 通过两个时间段的合理衔接, 最终组合成整个重复受控时间内的学习控制律, 当该学习控制律作用于系统时, 系统在整个重复受控时间内的输出跟踪误差沿迭代轴方向收敛. 仿真算例验证了算法的有效性. 文 [9] 及本文研究的系统均为单输入单输出, 如何将本文的结论推广到多输入多输出系统, 有待作进一步的研究.

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PIECEWISE ITERATIVE LEARNING CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

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Abstract: The problem of piecewise iterative learning control for a class of nonlinear systems is proposed and studied. Piecewise iterative learning scheme is constructed based on P-type learning scheme and D-type learning scheme. By using the contraction mapping method, it is shown that the piecewise iterative learning scheme can guarantee the output tracking error of the whole system converges along the iteration axis. The effectiveness of the proposed algorithm is verified through simulation.

Keywords: P-type learning scheme; D-type learning scheme; piecewise iterative learning control; nonlinear systems

2010 MR Subject Classification: 93C10; 68T05

附录

对给定的输入 $u(t)$ 及初值 $x(0)$, 容易验证 $f(x,u,t)$ 满足 Lipschitz 条件, 根据微分方程定性理论 [12] 可知假设 1 成立.

由

$$\begin{aligned}|f(x_1,u_1,t)-f(x_2,u_2,t)| &\le |u_1-u_2|+\frac{1}{2}\left|\arctan(\frac{1}{2}u_1)-\arctan(\frac{1}{2}u_2)\right|+|\sin x_1-\sin x_2|\\ &\le |u_1-u_2|+\frac{1}{2}\cdot\frac{1}{2}|u_1-u_2|+|x_1-x_2|\\ &\le 2(|x_1-x_2|+|u_1-u_2|)\end{aligned}$$

可知假设 4 成立.

下验证假设 2: 令 $h_1(x)=2x+\cos x$, $h_2(u)=u-\frac{1}{2}\arctan(\frac{u}{2})$, 因为 $h'_1,h'_2>0$, 所以 h_1,h_2 均为 R 上的连续单调增函数且均存在连续的导函数, 从而存在连续的单调增反函数 h_1^{-1},h_2^{-1}, 且它们的导函数也连续. 于是系统 (4.1) 化为

$$\begin{cases}\dot{x}(t)=h_2(u)+\sin x, & 0\le t\le 2,\\ y(t)=h_1(x)+u(t), & 0\le t\le 1,\\ y(t)=h_1(x)+u(1)+(t-1)\dot{u}(1), & 1<t\le 2.\end{cases}\tag{A.1}$$

对于给定的理想轨迹 $y_r(t)=t^2$, 当 $t\in[0,1]$ 时, $u(t)=t^2-(2x+\cos x)$, 结合假设 4 可知

$$\begin{aligned}|f(x_1,u(x_1,t),t)-f(x_2,u(x_2,t),t)| &\le 2(|x_1-x_2|+|u(x_1,t)-u(x_2,t)|)\\ &\le 2(|x_1-x_2|+2|x_1-x_2|+|\cos x_1-\cos x_2|)\\ &\le 2(3|x_1-x_2|+|x_1-x_2|)=8|x_1-x_2|,\end{aligned}$$

即 $f(x,u(x,t),t)$ 满足 Lipschitz 条件, 由文献 [12] 可知微分方程 $\dot{x}(t)=h_2(t^2-2x-\cos x)+\sin x$ 在给定初值 $x(0)=0$ 时, 存在唯一的解 $x_r(t)$, 继而存在唯一的控制输入 $u_r(t)\in C^1[0,1]$, 并且由 (A.1) 的第一式可得 $u_r(1)=h_2^{-1}(\dot{x}_r(1)-\sin(x_r(1)))$, 由 (A.1) 的第二式可得 $x_r(1)=h_1^{-1}(1-u_r(1))$.

当 $t\in(1,2]$ 时, 解 (A.1) 式可得唯一解

$$\begin{cases}x_r(t)=h_1^{-1}(t^2-u_r(1)-(t-1)\dot{u}_r(1)),\\ u_r(t)=h_2^{-1}(\dot{x}_r(t)-\sin(x_r(t))),\end{cases}$$

因此 $x(t)$ 在 $t\in(1,2]$ 上连续且有连续导函数, 从而 $\lim\limits_{t\to1^+}x_r(t)=h_1^{-1}(1-u_r(1))=x_r(1)$, $\lim\limits_{t\to1^+}u_r(t)=h_2^{-1}(\dot{x}_r(1)-\sin x_r(1))=u_r(1)$, 即 $u_r(t)$ 在 [0,2] 上连续, 由此可知假设 2 成立.

可选取初值, 使假设 3 成立. 假设 $5\sim7$ 可通过计算直接验证.