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BIFURCATIONS OF GUZOWSKA-LUÍS-ELAYDI MODEL

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Abstract: In this paper, we consider a discrete time logistic competition model. The topological types of fixed points and non-hyperbolic cases are given in order to investigate bifurcations. By applying the center manifold reduction theorem we prove that transcritical bifurcation occurs at three fixed points and stable 2-periodic orbits arise through flip bifurcation which happens at two fixed points.

Keywords: logistic competition model; transcitical bifurcation; flip bifurcation; 2-periodic orbit; center manifold

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1 Introduction

Discrete competition models, including intra-specific competition models and interspecific competition models, play a important role in theoretical ecology and economics (see e.g. [1, 3, 7, 8]). Intra-specific competition refers to the competition among individuals of same species and interspecific competition to the competition between two or more species for some limiting resource. When one species is a better competitor, interspecific competition negatively influences the other species by reducing population sizes or growth rates, which in turn affects the population dynamics of the competitor. In 2011, based on the biological assumptions that each species is modeled by the logistic map, modeled species with non-overlapping generations, without interspecific competition and that one species will negatively affect the growth of the other species in the interspecific competition, Guzowska, Luís and Elaydi [5] developed the following logistic competition model

$$\begin{cases} x_{n+1} = \dfrac{ax_n(1-x_n)}{1+cy_n}, \\ \\ y_{n+1} = \dfrac{by_n(1-y_n)}{1+dx_n}, \end{cases} \tag{1.1}$$

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where $x_n \in [0,1]$ and $y_n \in [0,1]$ represent the species x density and species y density at time n respectively, the parameters $a \in (0,4]$ and $b \in (0,4]$ denote the intrinsic growth rates of species x and y respectively, and the parameters $c \in (0,1)$ and $d \in (0,1)$ are the competition parameters of species y and x respectively. As indicted in [5], system (1.1) has one extinction fixed point $E_0(0,0)$, two exclusion fixed points

$$E_1((a-1)/a, 0), \;\; E_2(0, (b-1)/b)$$

and one coexistence fixed point $E_3(x_0, y_0)$, where

$$x_0 = \frac{ab - cb + c - b}{ab - cd}, \quad y_0 = \frac{ab - da - a + d}{ab - cd}.$$

In [5] the stability of the four fixed points were investigated by center manifold theorem and Schwarzian derivative, the bifurcation scenario at E_3 is given in the parameter space, and, at fixed point E_3, fold and flip bifurcations route to chaos are exhibited via numerical simulations.

Up to now, what bifurcations happen at fixed points E_0, E_1 and E_2 is unknown. In this paper we discuss analytically these bifurcations. At first we give all topological types of the three fixed points and all non-hyperbolic cases in order to investigate bifurcations. Then, we show that system (1.1) undergoes transcritical bifurcation at E_0 as (a,b) crossing two bifurcation curves $a = 1$ or $b = 1$. At last, we prove that, at fixed point E_1 (resp. E_2), a flip bifurcation occurs for (a,b) crossing curves $a = 3$ (resp. $b = 3$) and a transcritical bifurcation happens for (a,b) crossing $b = 1 + d(a-1)/a$ (resp. $a = 1 + c(b-1)/b$).

2 Transcritical Bifurcation at E_0

System (1.1) can be described equivalently by the planar mapping $F : \mathbb{R}^2 \to \mathbb{R}^2$,

$$F(x,y) = (\frac{ax(1-x)}{1+cy}, \frac{by(1-y)}{1+dx}), \tag{2.1}$$

whose Jacobian is given by

$$JF(x,y) = \begin{pmatrix} \frac{a(1-x)-ax}{1+cy} & \frac{-acx(1-x)}{(1+cy)^2} \\ \frac{-bdy(1-y)}{(1+dx)^2} & \frac{b(1-y)-by}{1+dx} \end{pmatrix}. \tag{2.2}$$

We first give the topological types of fixed point E_0 and non-hyperbolic cases.

Lemma 1 Fixed point E_0 is non-hyperbolic if and only if $a = 1$ or $b = 1$. Otherwise, E_0 is one of the types in Table 1.

Proof By (2.2) the Jacobian evaluated at the fixed point $E_0(0,0)$ is given by

$$JF(0,0) = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \tag{2.3}$$

Table 1: The topological types of fixed point E_0

conditions		types	cases
$0 < a < 1$	$0 < b < 1$	stable node	E_0-I
	$b > 1$	saddle	E_0-II
$1 < a < 4$	$0 < b < 1$	saddle	E_0-III
	$b > 1$	unstable node	E_0-IV

which has eigenvalues $\lambda_1 = a$ and $\lambda_2 = b$. Hence it is easy to obtain the results in Table 1 by [4] (see p. 194–200).

From the lemma it is obvious that the bifurcations occur at the fixed point E_0 if $a = 1$ or $b = 1$.

Theorem 1 If a (resp. b) crosses 1 and $b \neq 1$ (resp. $a \neq 1$), then the map F undergoes a transcritical bifurcation at fixed point $E_0(0, 0)$.

Proof We prove one case that a crosses 1 and $b \neq 1$. The proof of the other case is similar. From $0 < c < 1, 0 < d < 1, 0 < x < 1$ and $0 < y < 1$, map (2.1) can expand the following form at $a = 1:$

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ by \end{pmatrix} - \begin{pmatrix} x^2 + cxy + \mathcal{O}(3) \\ by^2 + bdxy + \mathcal{O}(3) \end{pmatrix}, \tag{2.4}$$

where $\mathcal{O}(3)$ is a function with order at least 3 in the variables. We choose $\epsilon = a - 1$ as a bifurcation parameter to study the bifurcation of the mapping F at the fixed point $E_0(0, 0)$, where $|\epsilon| \ll 1$. We consider a perturbation of (2.4) as follows:

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ by \end{pmatrix} - \begin{pmatrix} x^2 - x\epsilon + cxy + \mathcal{O}(3) \\ by^2 + bdxy + \mathcal{O}(3) \end{pmatrix}. \tag{2.5}$$

System (2.4) can be rewritten in the following suspended form

$$\begin{pmatrix} x \\ \epsilon \\ y \end{pmatrix} \to \begin{pmatrix} x \\ \epsilon \\ by \end{pmatrix} - \begin{pmatrix} x^2 - x\epsilon + cxy + \mathcal{O}(3), \\ 0 \\ by^2 + bdxy + \mathcal{O}(3), \end{pmatrix}. \tag{2.6}$$

By the center manifold theory (see p. 33–35 in [2]) the center manifold of system (2.6) can expressed locally as follows:

$$W^c(O) = \{(x, y, \varepsilon) \in \mathbb{R}^3 | y = h(x, \epsilon), h(0, 0) = Dh(0, 0) = 0, |x| < \varepsilon, |\epsilon| < \delta\},$$

where ε and δ are sufficient small positives. Assume that $h(x, \epsilon)$ has the following form

$$y = h(x, \epsilon) = c_1 x^2 + c_2 x\epsilon + c_3 \epsilon^2 + \mathcal{O}(3), \tag{2.7}$$

which must satisfy

$$h\left(x - x^2 + x\epsilon - cxh(x, \epsilon) + \mathcal{O}(3), \epsilon\right) = bh(x, \epsilon) - bh^2(x, \epsilon) - bdxh(x, \epsilon) + \mathcal{O}(3) \tag{2.8}$$

by the center manifold theorem. Comparing coefficients of $x^2, x\epsilon$ and ϵ^2 in (2.8) we obtain that

$$c_1 = c_2 = c_3 = 0,$$

and (2.7) has the determinative form

$$y = h(x, \epsilon) = \mathcal{O}(3). \tag{2.9}$$

Substituting (2.9) into the first two equations in (2.6) yields

$$\begin{pmatrix} x \\ \epsilon \end{pmatrix} \to \begin{pmatrix} x - x^2 + x\epsilon + \mathcal{O}(3) \\ \epsilon \end{pmatrix},$$

which defines a one-dimensional mapping $(x, \epsilon) \to f_1(x, \epsilon)$ by

$$f_1(x, \epsilon) = x - x^2 + x\epsilon + \mathcal{O}(3).$$

From

$$\left.\frac{\partial^2 f_1}{\partial x \partial \epsilon}\right|_{(x,\epsilon)=(0,0)} = 1 \tag{2.10}$$

and

$$\left.\frac{\partial^2 f_1}{\partial x^2}\right|_{(x,\epsilon)=(0,0)} = -2,$$

we get that the map F undergoes a transcritical bifurcation on the center manifold at E_0 (see p. 504–507 in [10]). This completes the proof.

3 Flip Bifurcation and Transcritical Bifurcation at E_1 and E_2

In order that E_1 has biological significance, we have $a > 1$. By (2.2) the Jacobian evaluated at the fixed point E_1 is given by

$$JF((a-1)/a, 0) = \begin{pmatrix} 2-a & -\frac{c}{a}(a-1) \\ 0 & \frac{ab}{ad+a-d} \end{pmatrix}, \tag{3.1}$$

whose eigenvalues are $\lambda_1 = 2 - a$ and $\lambda_2 = ab/(ad + a - d)$. Hence we have the following results.

Lemma 2 The fixed point E_1 is not hyperbolic if and only if $a = 3$ or $b = 1+d(a-1)/a$. Otherwise, E_1 is one of the types in Table 2.

Proof Solving $|\lambda_1| = |2 - a| < 1$ yields $1 < a < 3$. Obviously $\lambda_2 > 0$, from $\lambda_2 = ab/(ad + a - d) < 1$ we get $0 < b < 1 + d(a - 1)/a$. Hence E_1 is stable node for $1 < a \le 3$ and $0 < b < 1 + d(a - 1)/a$ (refers to case E_1-I). Similarly, we can obtain the other three cases in Table 2. This completes the proof.

From the lemma, it is obvious that the bifurcation occurs at the fixed point E_1 if $a = 3$ or $b = 1 + d(a - 1)/a$. Let $u = x - (a - 1)/a$ and $v = y$. Then we get map $\tilde{F} : \mathbb{R}^2 \to \mathbb{R}^2$,

$$\begin{pmatrix} u \\ v \end{pmatrix} \to \begin{pmatrix} (2-a)u - \frac{c}{a}(a-1)v - au^2 - c(2-a)uv + \frac{c^2}{a}(a-1)v^2 + \mathcal{O}_1(3) \\ \frac{ab}{a+d(a-1)}v - \frac{a^2bd}{(a+d(a-1))^2}uv - \frac{ab}{a+d(a-1)}v^2 + \mathcal{O}_2(3) \end{pmatrix}. \tag{3.2}$$

Table 2: The topological types of fixed point E_1

conditions		types	cases
$1 < a < 3$	$0 < b < 1 + d(a-1)/a$	stable node	E_1-I
	$1 + d(a-1)/a < b < 4$	saddle	E_1-II
$3 < a < 4$	$0 < b < 1 + d(a-1)/a$	saddle	E_1-III
	$1 + d(a-1)/a < b < 4$	unstable node	E_1-IV

Note that there isn't the term u^3 in $\mathcal{O}_1(3)$ and the term v^3 in $\mathcal{O}_2(3)$. Its Jacobian evaluated at the O, $J\tilde{F}(0,0)$, is equal to $JF((a-1)/a, 0)$. One can easily see that the matrix $J\tilde{F}(0,0)$ has eigenvectors $(1,0)^T$ and

$$\left(\frac{ad+a-b}{(2-a)(ad+a-d)-ab}, \frac{c}{a}(a-1)\right)^T$$

corresponding to $\lambda_1 = 2-a$ and $\lambda_2 = ab/(ad+a-d)$, respectively, where T denotes the transpose of matrices. One can check that $\lambda_1 \neq \lambda_2$ if $a = 3$ or $b = 1 + d(a-1)/a$. Hence the matrix $J\tilde{F}(0,0)$ can be diagonalized by the change of variables $(u,v)^T = H_1(\xi,\eta)^T$, where

$$H_1 = \begin{pmatrix} 1 & \frac{ad+a-b}{(2-a)(ad+a-d)-ab} \\ & \\ 0 & \frac{c}{a}(a-1) \end{pmatrix},$$

and therefore the map $\tilde{F}$ can be changed into the mapping $G: \mathbb{R}^2 \to \mathbb{R}^2$,

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} \to \begin{pmatrix} h_{10}\xi \\ g_{01}\eta \end{pmatrix} + \begin{pmatrix} h_{20}\xi^2 + h_{11}\xi\eta + h_{02}\eta^2 + \mathcal{O}_1(3) \\ g_{11}\xi\eta + g_{02}\eta^2 + \mathcal{O}_2(3) \end{pmatrix}, \tag{3.3}$$

where

$$\begin{aligned}
h_{10} &:= 2-a, \ g_{01} := \frac{ab}{ad+a-d}, \ h_{20} := -a, \ g_{11} = -a^2bd/(ad+a-d)^2, \\
h_{11} &:= (-11a^4bd - 17a^2bd^2 + 6d^2ab + 19a^3bd + 2a^5bd + 17a^3bd^2 - a^3b^2d + a^2b^2d \\
&\quad -7a^4bd^2 - 10a^2bd - 4a^4d^2b + 7a^3d^2b - 6a^4db + 8a^3db - 6a^2d^2b - 4a^2db \\
&\quad -2a^3b^2 + 2a^5db + a^5b + 12a^2d + a^6d^3 + a^5b - 4a^4b + 4a^3b + 3a^6d^2 \\
&\quad +3a^6d - 2a^4b + 2a^3b + 2d^2ab + 42a^2d^2 - 12ad^2 - 60a^3d^2 - 30a^3d \\
&\quad +21a^4d^3 + 45a^4d^2 - 35a^3d^3 - 7a^5d^3 - 18a^5d^2 + 34a^2d^3 + 30a^4d \\
&\quad -15a^5d - 18ad^3 - 4a^3 - 4a^5 + 6a^4 + 4d^3 + a^6 + a^5bd^2 + a^4b^2d + a^4b^2 \\
&\quad +a^5d^2b - 3a^3b^2d + 2da^2b^2)a/((ad+a-d)(-3ad - 2a + 2d \\
&\quad +a^2d + a^2 + ab)(-3ad - 2a + 2d + a^2d + a^2 + ab)(a-1)),
\end{aligned}$$

$$\begin{aligned} h_{02} \;:=\;& a(-ca^4d^2b+3ca^3d^2b-2ca^4db+4ca^3db-3ca^2d^2b-2ca^2db-6ca^2bd+ca^5bd^2\\ &+2ca^5bd+ca^4b^2+9ca^3bd^2-7ca^2bd^2-5ca^4bd^2-9ca^4bd-2ca^3b^2-2ca^3b^2d\\ &+ca^2b^2d+ca^4b^2d+ca^3b^3+2cd^2ab+13ca^3bd-8a^2bd+a^3b^3+a^5b+3ca^4\\ &-ca^5+2cd^3-2ca^3-4a^4b-4a^3b^2+2a^4b^2+a^5bd^2+2a^5bd+ca^5b-4ca^4b\\ &-2ca^3b^2+ca^4b^2+4ca^3b-ca^4b+ca^3b+cd^2ab+6ca^2d+21ca^2d^2-6cad^2\\ &-27ca^3d^2-15ca^3d+6ca^4d^3+15ca^4d^2-14ca^3d^3-ca^5d^3-3ca^5d^2\\ &+16ca^2d^3+12ca^4d-3ca^5d-9cad^3+13a^3bd^2-12a^2bd^2-6a^4bd^2\\ &-10a^4bd-6a^3b^2d+4a^2b^2d+2a^4b^2d+4d^2ab+16a^3bd-3ca^3b^2d+2cda^2b^2\\ &+ca^4b^2d+4a^3b)/(c(-3ad-2a+2d+a^2d+a^2+ab)(a-1)(-3ad-2a\\ &+2d+a^2d+a^2+ab)^2),\\ g_{02} \;:=\;& a^2b(dca-dc-3ad-2a+2d+a^2d+a^2+ab)/(c(a-1)(-3ad-2a\\ &+2d+a^2d+a^2+ab)(ad+a-d)). \end{aligned}$$

Theorem 2 If $1<a<4$, then the map F undergoes flip bifurcation at the fixed point E_1 as a crossing 3 and $b\neq 1+d(a-1)/a$. More concretely, for the restriction of mapping F to a one-dimensional center manifold, a stable 2-periodic orbit emerges near the fixed point E_1 for $a-3>0$ small.

Proof We choose a as bifurcation parameter. Rewrite system (3.3) in the suspended form

$$\begin{pmatrix}\xi\\ a\\ \eta\end{pmatrix}\to\begin{pmatrix}h_{10}\xi\\ -a\\ g_{01}\eta\end{pmatrix}+\begin{pmatrix}h_{20}\xi^2+h_{11}\xi\eta+h_{02}\eta^2+\mathcal{O}_1(3)\\ 0\\ g_{11}\xi\eta+g_{02}\eta^2+\mathcal{O}_2(3)\end{pmatrix},\tag{3.4}$$

so as to involve the parameter a explicitly in the discussion. The suspended system (3.4) has a two-dimensional center manifold

$$W_1^c(O)=\{(\xi,\eta,a)\in\mathbb{R}^3:\eta=h_1(\xi,a),h_1(0,3)=Dh_1(0,3)=0,|\xi|<\varepsilon_1,|a-3|<\delta_1\},\tag{3.5}$$

where ε_1 and δ_1 are sufficient small positives. Assume that $h_1(\xi,a)$ has the following form

$$\eta=h_1(\xi,a)=b_1\xi^2+b_2\xi a+b_3a^2+\mathcal{O}_1(3),\tag{3.6}$$

which must satisfy

$$\begin{aligned}&h_1(h_{10}\xi+h_{20}\xi^2+h_{11}\xi h_1(\xi,a)+h_{02}h_1^2(\xi,a)+\mathcal{O}(3),a)\\ =&g_{01}h_1(\xi,a)+g_{11}\xi h_1(\xi,a)+g_{02}h_1^2(\xi,a)\end{aligned}\tag{3.7}$$

by the center manifold theorem (see p. 33–35 in [2]). Comparing coefficients of $\xi^2, a\xi$ and a^2 in (3.7) we obtain that $b_1=b_2=b_3=0$, and (3.6) has the determinative form

$$y=h_1(\xi,a)=\mathcal{O}_1(3).\tag{3.8}$$

Substituting (3.14) into the first two equations in (3.4) yields

$$\begin{pmatrix} \xi \\ a \end{pmatrix} \to \begin{pmatrix} (2-a)\xi - a\xi^2 + \mathcal{O}_1(4) \\ -a \end{pmatrix},$$

which defines a two-dimensional mapping $(\xi, a) \to f_2(\xi, a)$ by $f_2(\xi, a) = (2-a)\xi - a\xi^2 + \mathcal{O}_1(4)$. From that there isn't the term u^3 in $\mathcal{O}_1(3)$ in system (3.2), it is not difficult to follow that there isn't the term ξ^3 in $\mathcal{O}_1(4)$. One can check that

$$\left.\frac{\partial^2 f_2}{\partial\xi\partial a}\right|_{(\xi,a)=(0,3)} = -1$$

and

$$\left.\left[\frac{1}{2}\left(\frac{\partial^2 f_2}{\partial\xi^2}\right)^2 + \frac{1}{3}\left(\frac{\partial^3 f_2}{\partial\xi^3}\right)\right]\right|_{(\xi,a)=(0,3)} = 18 > 0.$$

Hence the transversality condition and non-degeneracy condition of Theorem 4.3 in [9] are satisfied, which implies that a flip bifurcation occurs at $\xi = 0$ as a crossing 3 and a stable cycle of period two arises in system (3.3). So the map F undergoes flip bifurcation at the fixed point E_1 on the center manifold if a crosses 3 and $b \neq 1 + d(a-1)/a$.

Theorem 3 If $a \neq 3$ and (a, b) crosses $b = 1 + d(a-1)/a$, then system (2.1) undergoes a transcritical bifurcation at the fixed point E_1.

Proof We choose b as bifurcation parameter. Rewrite system (3.3) in the suspended form

$$\begin{pmatrix} \xi \\ b \\ \eta \end{pmatrix} \to \begin{pmatrix} h_{10}\xi \\ b \\ g_{01}\eta \end{pmatrix} + \begin{pmatrix} h_{20}\xi^2 + h_{11}\xi\eta + h_{02}\eta^2 + \mathcal{O}_1(3) \\ 0 \\ g_{11}\xi\eta + g_{02}\eta^2 + \mathcal{O}_2(3) \end{pmatrix}. \tag{3.9}$$

The suspended system (3.9) has a two-dimensional center manifold

$$\begin{aligned} W_2^c(O) = \big\{&(\xi, \eta, b) \in \mathbb{R}^3 : \xi = h_2(\eta, b), h_2(0, b_0) = Dh_2(0, b_0) = 0, \\ &|\eta| < \varepsilon_2, |b - b_0)| < \delta_2\big\}, \end{aligned} \tag{3.10}$$

where ε_2 and δ_2 are sufficient small positives and $b_0 = 1 + d(a-1)/a$. Assume that $h_2(\eta, b)$ has the following form

$$\xi = h_2(\eta, b) = a_1\eta^2 + a_2\eta b + a_3 b^2 + \mathcal{O}(3), \tag{3.11}$$

which must satisfy

$$h_2(g_{01}\eta + g_{11}\eta h_2(\eta, b) + g_{02}\eta^2 + \mathcal{O}_2(3), b) \tag{3.12}$$

$$= h_{10}h_2(\eta, b) + h_{20}h_2^2(\eta, b) + h_{11}\eta h_2(\eta, b) + h_{02}\eta^2 + \mathcal{O}_1(3) \tag{3.13}$$

by the center manifold theorem (see p. 33–35 in [2]). Comparing coefficients of $\eta^2, b\eta$ and b^2 in (3.12) we obtain that $a_1 g_{01}^2 = h_{02} + a_1 h_{01}$, $a_2 g_{01} = a_2 h_{10}$, $a_3 = a_3 h_{10}$, from which we find $a_2 = a_3 = 0$ and

$$a_1 = \frac{h_{02}}{g_{01}^2 - h_{10}}.$$

Hence (3.11) has the determinative form

$$\xi = h_2(\eta, b) = \frac{h_{02}}{g_{01}^2 - h_{10}}\eta^2 + \mathcal{O}(3). \tag{3.14}$$

Substituting (3.14) into the last two equations in (3.9) yields

$$\begin{pmatrix} b \\ \\ \eta \end{pmatrix} \to \begin{pmatrix} b \\ \\ g_{01}\eta + g_{02}\eta^2 + \frac{g_{11}h_{02}}{g_{01}^2 - h_{01}}\eta^3 + \mathcal{O}(4) \end{pmatrix},$$

which defines a two-dimensional mapping $(\eta, b) \to f_3(\eta, b)$ by

$$f_3(\eta, b) = g_{01}\eta + g_{02}\eta^2 + \frac{g_{11}h_{02}}{g_{01}^2 - h_{01}}\eta^3 + \mathcal{O}(4).$$

By $a > 1$ and $d > 0$ we obtain that

$$\frac{\partial^2 f_3}{\partial\eta\partial b}|_{(\eta,b)=(0,b_0)} = \frac{a}{(a-1)d + a} > 0 \tag{3.15}$$

and

$$\left.\frac{\partial^2 f_3}{\partial\eta^2}\right|_{(\eta,b)=(0,b_0)} = \frac{2a^2\left[1 + \frac{d}{a}(a-1)\right]g_1(a)}{c(a-1)[(a-1)d+a]g_2(a)} > 0, \tag{3.16}$$

where

$$g_1(a) = (1+d)a^2 + (-2d + dc - 1)a - d(-1+c)$$

and

$$g_2(a) = (1+d)a^2 + (-2d-1)a + d.$$

In fact, from $a > 1, 0 < d < 1$ and $0 < c < 1$, one can check that $g_1(1) = g_2(1) = 0$,

$$g_1'(a) = (2a-1) + 2d(a-1) + dc > 0$$

and

$$g_2'(a) = (2a-1) + 2d(a-1) > 0,$$

which imply that $g_1(a) > 0$ and $g_2(a) > 0$. Hence (3.16) is true, and the mapping G undergoes a transcritical bifurcation on the center manifold at E_1 if $a > 1$, $a \neq 3$ and $b = 1 + (a-1)b/a$. The proof is completed.

Using the same arguments we have the following results.

Theorem 4 If $1 < b < 4$, then the map F undergoes flip bifurcation at the fixed point E_2 when b crosses 3 and $a \neq 1 + c(b-1)/b$. More concretely, the bifurcation is supercritical and a stable 2-periodic orbit emerges near the fixed point E_2 when $b > 3$. If $b \neq 3$ and (a, b) crosses $a = 1 + c(b-1)/b$, the transcritical bifurcation occurs at the fixed point E_2 in system (2.1).

Sometimes flip bifurcation is also called period-doubling bifurcation (see p.114 in [9]). Theorem 2 (resp. Theorem 4) shows that a 2-periodic oscillation of the population sizes in species x (resp. y) emerges near the equilibrium $(a-1)/a$ (resp. $(b-1)/b$).

4 Conclusion

In this paper we only discuss the codimension 1 local bifurcations at fixed points E_0, E_1 and E_2. In fact, if $a = 1$ and $b = 1$ in Theorem 1, the map F has a double multiplier 1, which implies that 1:1 resonance may occur at the fixed point E_0 (see p. 410–415 in [9]). If $a = 3$ (resp. $b = 3$) and $b = 1 + d(a-1)/a$ (resp. $a = 1 + c(b-1)/a$), the map F has eigenvalues -1 and 1. A fold-flip bifurcation may occur at the fixed point E_1 (resp. E_2) in the system (2.1) (see e.g. [6]). All of these codimension 2 bifurcations will involve more complicated computation. We leave these to our next work.

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关于Guzowska-Luís-Elaydi模型的分岔

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摘要: 本文考虑了一个离散的Logistic竞争模型. 为了讨论分岔, 给出了不动点的拓扑类型及非双曲的情况. 应用中心流行约化定理, 证明了跨临界分岔会在三个不动点上发生. 本文还证明了在两个不动点处, 跳跃分岔会发生, 同时稳定的周期2轨会出现.

关键词: Logistic竞争模型; 跨临界分岔; 跳跃分岔; 周期2轨; 中心流行

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