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ISOPERIMETRIC ESTIMATE OF THE FIRST EIGENVALUES FOR THE WEIGHTED p-LAPLACIAN ON MANIFOLDS

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Abstract: In this paper, we estimate the lower bounds of the first eigenvalues for the weighted p-Laplacian on manifolds. By using the coarea formula, the Cavalieri principle and the Federer-Fleming theorem, we obtain the estimation of the lower bounds for the first eigenvalues by the Cheeger constant or the isoperimetric constant.

Keywords: weighted p-Laplacian; weighted manifold; isoperimetric constant; first eigenvalue; lower bound

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1 Introduction

Let Ω be a bounded domain with smooth boundary $\partial\Omega$ in an n-dimensional Riemannian manifold (M,g). The p-Laplacian is defined by

$$\begin{aligned}\Delta_p : W_0^{1,p}(\Omega) &\mapsto W^{-1,q}(\Omega),\\ u &\mapsto \Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u),\end{aligned}$$

where $W_0^{1,p}(\Omega)$ is the Sobolev space given by the closure of $C_0^\infty(\Omega)$ with norm

$$\|u\|_{1,p}^p = \int_\Omega |u|^p \mathrm{d}V + \int_\Omega |\nabla u|^p \mathrm{d}V,$$

and $W^{-1,q}(\Omega)$ is the dual space of $W_0^{1,p}(\Omega)$ and $1 < p, q < \infty$, $\frac{1}{p}+\frac{1}{q}=1$. As a generalization of the usual Laplacian, the p-Laplacian is widely used in many subjects, especially Δ_p models the non-Newtonian fluids in physics. It describes dilatant fluids when $p > 2$ and pseudoplastics when $p < 2$, whereas $p = 2$ corresponds to Newtonian fluids. The operator

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Δ_p with $p \neq 2$ also appears in many other applications, such as reaction-diffusion problems, flow through porous media, nonlinear elasticity, etc., see [14] for more details.

Let $(M, g, d\mu)$ be a weighted manifold, that is, a Riemannian manifold (M, g) endowed with a weighted volume form $d\mu = e^{-\varphi}dV$, where $\varphi \in C^\infty(M)$ and dV is the volume element induced by the metric g. With respect to the weighted measure, the weighted p-Laplacian is defined as follows

$$\Delta_{p,\varphi}u = \operatorname{div}(|\nabla u|^{p-2}\nabla u) - |\nabla u|^{(p-2)}\langle \nabla u, \nabla\varphi\rangle = \Delta_p u - |\nabla u|^{(p-2)}\langle \nabla u, \nabla\varphi\rangle.$$

We are interested in the following nonlinear eigenvalue problem (the Dirichlet eigenvalue problem)

$$\begin{cases} \Delta_{p,\varphi}u + \lambda|u|^{p-2}u = 0 \text{ in } \Omega, \\ u|_{\partial\Omega} = 0. \end{cases} \tag{1.1}$$

We recall that the first eigenvalue for the weighted p-Laplacian has the following variational characterisation

$$\lambda_{p,\varphi}(\Omega) = \min_{\substack{u \in W_0^{1,p}(\Omega) \\ u \not\equiv 0}} \frac{\displaystyle\int_\Omega |\nabla u|^p e^{-\varphi}\mathrm{d}V}{\displaystyle\int_\Omega |u|^p e^{-\varphi}\mathrm{d}V}. \tag{1.2}$$

The problems of partial differential equations involving weighted p-Laplacian have been studied by many mathematicians, see [1, 18], etc.. For more researches on eigenvalue problems, we refer the readers to [6, 15, 16] etc..

For the following nonlinear eigenvalue problem

$$\begin{cases} \Delta_p f + \lambda|f|^{p-2}f = 0 \text{ in } \Omega, \\ f|_{\partial\Omega} = 0. \end{cases}$$

The first eigenvalue associated with a Riemannian metric g on a manifold M has been extensively studied in recent mathematical literature, such as [7–11], etc.. In [8] and [9], Kawohl-Fridman and Lefton-Wei used the coarea formula and the Cavalieri principle to estimate the lower bound of the first eigenvalue about this problem by the Cheeger constant

$$\lambda_{1,p}(\Omega) \geq \left(\frac{h(\Omega)}{p}\right)^p,$$

where $h(\Omega) = \mathfrak{J}_\infty(\Omega)$ is the Cheeger constant of domain Ω. This wonderful result inspires us to estimate the lower bounds of the first eigenvalues for the weighted p-Laplacian operator eigenvalue problems by the Cheeger constant.

In this paper, we use the coarea formula, the Cavalieri principle and the Federer-Fleming Theorem to investigate the first eigenvalues of problem (1.1). We obtain the lower bounds estimations of the first eigenvalues for the weighted p-Laplace operator eigenvalue problems by the Cheeger constant and isoperimetric constant.

2 Main Results

In this section, our main goal is to estimate the lower bounds of the first eigenvalues for the weighted p-Laplacian eigenvalue problems on weighted manifolds. First, we recall some preliminary knowledge of the isoperimetric constant, Cavalieri's Principle and the coarea formula for later use.

Definition 2.1 Let M be an n-dimensional Riemannian manifold with $n \geq 2$. For each $\nu > 1$, the $\nu -$ isoperimetric constant $of\ M$, $\mathfrak{J}_\nu(M)$, is defined to be the infimum

$$\mathfrak{J}_\nu(M) = \inf_\Omega \frac{A(\partial\Omega)}{V(\Omega)^{1-\frac{1}{\nu}}},$$

where Ω varies over open submanifolds of M possessing compact closure and C^∞ boundary. If $\nu = \infty$, $\mathfrak{J}_\infty(M)$ is called the Cheeger constant, that is

$$\mathfrak{J}_\infty(M) = \inf_\Omega \frac{A(\partial\Omega)}{V(\Omega)}.$$

Remark 2.2 As stated in [3], the fact that $\mathfrak{J}_\nu(M) > 0$ is only possible for $n \leq \nu \leq \infty$. Indeed, let $\nu < n$, and consider a small geodesic ball $B(x;\epsilon)$, with center $x \in M$ and radius $\epsilon > 0$, for the isoperimetric quotient of $B(x;\epsilon)$,

$$\lim_{\epsilon\to 0} \frac{A(\partial\Omega)}{V(\Omega)^{1-\frac{1}{\nu}}} \sim \lim_{\epsilon\to 0} \text{const.}\ \epsilon^{\frac{n}{\nu}-1} = 0.$$

So it seems at first glance that one only has a discussion of isoperimetric constants for $\nu \geq n$=dim$M$.

Definition 2.3 Let M be an n-dimensional Riemannian manifold, $n \geq 2$. For each $\nu > 1$, the Sobolev constant of M, $\mathfrak{S}_\nu(M)$, is defined to be the infimum

$$\mathfrak{S}_\nu(M) = \inf_f \frac{\|\nabla f\|_1}{\|f\|_{\frac{\nu}{\nu-1}}},$$

where $f \in C_0^\infty(M)$.

The isoperimetric constant and the Sobolev constant have the following famous relationship:

Lemma 2.4 (The Federer-Fleming Theorem) The isoperimetric and Sobolev constants are equal, that is,

$$\mathfrak{J}_\nu(M) = \mathfrak{S}_\nu(M). \tag{2.1}$$

The detailed proof of the Federer-Fleming theorem can be found in [3, 4] and [12]. This elegant result was first proven in [4] by Federer and Fleming, and in [12] independently by Maz'ya in 1960.

Lemma 2.5 (see [3] The coarea Formula) Let M be a C^n Riemannian manifold, and let $\Phi : M \to \mathbb{R}$ be a C^n function. Then for any measurable function $u : M \to \mathbb{R}$ that is everywhere nonnegative or is in $L^1(M)$, one has

$$\int_M u|\nabla\Phi|\mathrm{d}V = \int_R \mathrm{d}V_1(y) \int_{\Phi^{-1}[y]} (u|_{\Phi^{-1}[y]})\mathrm{d}A.$$

Lemma 2.6 (see [3] Cavalieri's Principle) Let ν be a measure on Borel sets in $[0,\infty]$, ϕ its indefinite integral, given by

$$\phi(t)=\nu([0,t))<+\infty,\ \forall\ t>0,$$

(Ω,Σ,μ) a measure space, and u a nonnegative Σ-measurable function on Ω. Then

$$\int_\Omega \phi(u(x))\mathrm{d}\mu(x)=\int_0^\infty \mu(u>t)\mathrm{d}\nu(t)$$

or equivalently

$$\int_\Omega \mathrm{d}\mu(x)\int_0^{u(x)}\mathrm{d}\nu(t)=\int_0^\infty \mathrm{d}\nu(t)\int_\Omega I_{\{u>t\}}\mathrm{d}\mu.$$

Using the coarea formula and the Cavalieri principle, we can get the following lower bound estimation of the first eigenvalue for the weighted p-Laplacian on weighted Riemannian manifold by the Cheeger constant.

Theorem 2.7 Let Ω be a connected domain with smooth boundary $\partial\Omega$ in an n-dimensional weighted Riemannian manifold $(M,g,d\mu)$. Assume $\lambda_{p,\varphi}(\Omega)$ is the first eigenvalue of problem (1.1) for $\varphi\in C^\infty(\Omega)$. Then

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega)\geq\frac{1}{p}\left(h(\Omega)-C_\varphi\right), \tag{2.2}$$

where $C_\varphi=\max\limits_{x\in\Omega}|\nabla\varphi|$ and $h(\Omega)=\mathfrak{J}_\infty(\Omega)$ are the the Cheeger constant of domain Ω.

Proof For any $u\in C_0^\infty(\Omega)$, set

$$\Omega(t)=\{x\in\Omega:|u|^pe^{-\varphi}>t\}$$

and

$$\mathrm{V}(t)=\mathrm{V}(\Omega(t)),\ \mathrm{A}(t)=\mathrm{A}(\partial\Omega(t)).$$

It follows from the Hölder inequality that

$$\begin{aligned}&\int_\Omega|\nabla(u^pe^{-\varphi})|\mathrm{d}V\leq p\int_\Omega|u|^{p-1}|\nabla u|e^{-\varphi}\mathrm{d}V+\int_\Omega|u|^p|\nabla\varphi|e^{-\varphi}\mathrm{d}V\\ \leq& p\left\{\int_\Omega|u|^pe^{-\varphi}\mathrm{d}V\right\}^{\frac{p-1}{p}}\left\{\int_\Omega|\nabla u|^pe^{-\varphi}\mathrm{d}V\right\}^{\frac{1}{p}}+\int_\Omega|u|^p|\nabla\varphi|e^{-\varphi}\mathrm{d}V.\end{aligned} \tag{2.3}$$

From the coarea formula, the Cavalieri principle and the definition of Cheeger constant, we can get

$$\begin{aligned}&\int_\Omega |\nabla(u^p e^{-\varphi})|\mathrm{d}V = \int_0^\infty \mathrm{A}(t)\mathrm{d}t\\ =&\int_0^\infty \frac{\mathrm{A}(t)}{\mathrm{V}(t)}\mathrm{V}(t)\mathrm{d}t \geq h(\Omega)\int_0^\infty \mathrm{V}(t)\mathrm{d}t\\ =&h(\Omega)\int_0^\infty \mathrm{d}t\int_\Omega I_{\{|u|^p e^{-\varphi}>t\}}\mathrm{d}V\\ =&h(\Omega)\int_\Omega \mathrm{d}V\int_0^{|u|^p e^{-\varphi}}\mathrm{d}t\\ =&h(\Omega)\int_\Omega |u|^p e^{-\varphi}\mathrm{d}V,\end{aligned}$$

since $C_0^\infty(\Omega)$ is dense in $W_0^{1,p}(\Omega)$, the above relation holds also for any $u \in W_0^{1,p}(\Omega)$, which together with (2.3) implies

$$\frac{\{\int_\Omega |\nabla u|^p e^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}}{\{\int_\Omega |u|^p e^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}} \geq \frac{1}{p}\left(h(\Omega) - C_\varphi\right),$$

this inequality and (1.2) imply

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) = \min_{\substack{u\in W_0^{1,p}(\Omega)\\ u\not\equiv 0}} \frac{\{\int_\Omega |\nabla u|^p e^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}}{\{\int_\Omega |u|^p e^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}} \geq \frac{1}{p}\left(h(\Omega) - C_\varphi\right),$$

which completes the proof.

Using the Federer-Fleming theorem, we can get the following lower bound estimation of the first eigenvalue by the isoperimetric constant.

Theorem 2.8 Assume that Ω satisfies the conditions of Theorem 2.7, and the isoperimetric constant $\mathfrak{J}_\nu(\Omega)$ is positive for some $\nu > 1$. Then

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) \geq \frac{1}{p}\left(\frac{\mathfrak{J}_\nu(\Omega)}{V(\Omega)^{\frac{1}{\nu}}} - C_\varphi\right). \tag{2.4}$$

Proof For any $u \in W_0^{1,p}(\Omega)$, let $f(u) = |u|^{p-1}ue^{-\varphi}$, then, we first have by the Hölder inequality that

$$\int_\Omega |f|\mathrm{d}V \leq \left\{\int_\Omega |f|^{\frac{\nu}{\nu-1}}\mathrm{d}V\right\}^{\frac{\nu-1}{\nu}} \left\{\int_\Omega 1\mathrm{d}V\right\}^{\frac{1}{\nu}} = \left\{\int_\Omega |f|^{\frac{\nu}{\nu-1}}\mathrm{d}V\right\}^{\frac{\nu-1}{\nu}} V(\Omega)^{\frac{1}{\nu}}. \tag{2.5}$$

According to the Federer-Fleming theorem (2.1) and the definition of the sobolev constant, we deduce

$$\mathfrak{J}_\nu(\Omega)\left\{\int_\Omega |f|^{\frac{\nu}{\nu-1}}\mathrm{d}V\right\}^{\frac{\nu-1}{\nu}} \leq \int_\Omega |\nabla f|\mathrm{d}V,$$

which together with (2.5) gives us

$$\int_\Omega |f|\mathrm{d}V \le \frac{V(\Omega)^{\frac{1}{\nu}}}{\mathfrak{I}_\nu(\Omega)}\int_\Omega |\nabla f|\mathrm{d}V. \tag{2.6}$$

Again, by the Hölder inequality, we have

$$\begin{aligned}\int_\Omega |\nabla f|\mathrm{d}V &= p\int_\Omega |u|^{p-1}|\nabla u|e^{-\varphi}\mathrm{d}V + \int_\Omega |u|^p|\nabla\varphi|e^{-\varphi}\mathrm{d}V\\ &\le p\left\{\int_\Omega |u|^pe^{-\varphi}\mathrm{d}V\right\}^{\frac{p-1}{p}}\left\{\int_\Omega |\nabla u|^pe^{-\varphi}\mathrm{d}V\right\}^{\frac{1}{p}} + \int_\Omega |u|^p|\nabla\varphi|e^{-\varphi}\mathrm{d}V. \end{aligned}\tag{2.7}$$

The combination of (2.6) and (2.7) can yield

$$\begin{aligned}&\int_\Omega |u|^pe^{-\varphi}\mathrm{d}V\\ \le&\frac{V(\Omega)^{\frac{1}{\nu}}}{\mathfrak{I}_\nu(\Omega)}\left\{p\left\{\int_\Omega |u|^pe^{-\varphi}\mathrm{d}V\right\}^{\frac{p-1}{p}}\left\{\int_\Omega |\nabla u|^pe^{-\varphi}\mathrm{d}V\right\}^{\frac{1}{p}} + \int_\Omega |u|^p|\nabla\varphi|e^{-\varphi}\mathrm{d}V\right\},\end{aligned}$$

this inequality implies

$$\frac{\{\int_\Omega |\nabla u|^pe^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}}{\{\int_\Omega |u|^pe^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}} \ge \frac{1}{p}\left(\frac{\mathfrak{I}_\nu(\Omega)}{V(\Omega)^{\frac{1}{\nu}}} - C_\varphi\right),$$

from this inequality and (1.2), it is obvious that

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) = \min_{\substack{u\in W_0^{1,p}(\Omega)\\ u\not\equiv 0}}\frac{\{\int_\Omega |\nabla u|^pe^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}}{\{\int_\Omega |u|^pe^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}} \ge \frac{1}{p}\left(\frac{\mathfrak{I}_\nu(\Omega)}{V(\Omega)^{\frac{1}{\nu}}} - C_\varphi\right),$$

which completes the proof.

Remark 2.9 It is obvious that, if we take $\nu=\infty$, then from (2.4) we have

$$\begin{aligned}\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) &= \min_{\substack{u\in W_0^{1,p}(\Omega)\\ u\not\equiv 0}}\frac{\{\int_\Omega |\nabla u|^pe^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}}{\{\int_\Omega |u|^pe^{-\varphi}\mathrm{d}V\}^{\frac{1}{p}}}\\ &\ge \lim_{\nu\to\infty}\frac{1}{p}\left(\frac{\mathfrak{I}_\nu(\Omega)}{V(\Omega)^{\frac{1}{\nu}}} - C_\varphi\right) = \frac{1}{p}\left(h(\Omega) - C_\varphi\right).\end{aligned}$$

Corollary 2.10 Let Ω be a connected domain with smooth boundary $\partial\Omega$ in the Euclidean space $\mathbb{R}^n$. Then

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) \ge \frac{n}{p}\left(\left(\frac{\omega_n}{V(\Omega)}\right)^{\frac{1}{n}} - nC_\varphi\right), \tag{2.8}$$

where ω_n denotes the volume of the unit ball in R^n.

Proof It is well known that

$$\mathfrak{I}_n(\Omega) = n\omega_n^{\frac{1}{n}}$$

for any domain $\Omega \subseteq \mathbb{R}^n$, where ω_n denotes the volume of the unit ball in $\mathbb{R}^n$. From this fact and (2.4), we can get

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) \geq \frac{n}{p}\left(\left(\frac{\omega_n}{V(\Omega)}\right)^{\frac{1}{n}} - nC_\varphi\right),$$

which completes the proof.

Example 1 If $\Omega = B_n(R)$ is a ball in $\mathbb{R}^n$ with radius R, then the volume of Ω is $V(\Omega) = \omega_n R^n$, and we can get

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) \geq \frac{n}{p}\left(\frac{1}{R} - nC_\varphi\right)$$

directly by (2.8). Since any ball is trivial Cheeger set (see [2]), by simply calculation, we can obtain

$$h(\Omega) = \frac{A(\partial\Omega)}{V(\Omega)} = \frac{n}{R}$$

from inequality (2.2), thus, we can get the same inequality as above.

Example 2 Let S^n be a unit sphere with sectional curvature 1, and $\Omega \subseteq S^n$ (small enough) be a relatively compact domain with smooth boundary $\partial\Omega$. Then the Ricci curvature of S^n is $n-1$. From [17, Theorem 1.4], we know that for any connected domain $\Omega \subset S^n$, $n = 2, 3, 4, 5$,

$$\frac{A(\partial\Omega)}{V(\Omega)^{1-\frac{1}{n}}} \geq n\omega_n^{\frac{1}{n}}\left(1 - \tau V(\Omega)^{\frac{2}{n}}\right)^{\frac{1}{n}},$$

where $\tau = \frac{n(n-1)}{2(n+2)\omega_n^{\frac{2}{n}}}$. According to Definition 2.1, we derive

$$\mathfrak{I}_n(\Omega) \geq n\omega_n^{\frac{1}{n}}\left(1 - \tau V(\Omega)^{\frac{2}{n}}\right)^{\frac{1}{n}}.$$

Then from (2.4), we have

$$\lambda_{p,\varphi}^{\frac{1}{p}}(\Omega) \geq \frac{1}{p}\left(\frac{n\omega_n^{\frac{1}{n}}\left(1 - \tau V(\Omega)^{\frac{2}{n}}\right)^{\frac{1}{n}}}{V(\Omega)^{\frac{1}{n}}} - C_\varphi\right).$$

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加权流形上加权p-Laplace特征值问题的第一特征值下界估计

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摘要: 本文研究了加权流形上加权p-Laplacian特征值问题的第一特征值下界估计的问题. 利用余面积公式、Cavalieri原理以及Federer-Fleming定理, 获得了由Cheeger常数或等周常数确定的第一特征值的下界估计.

关键词: 加权p-Laplacian; 加权流形; 等周常数; 第一特征值; 下界

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