

LARGE DEVIATION UPPER BOUND FOR THE LAW OF MATRIX-VALUED ORNSTEIN-UHLENBECK PROCESS

CHEN Lei

(*School of Mathematics and Statistics, Jiangsu Normal University, Xuzhou 221116, China*)

Abstract: In this paper, we study the large deviation problem for the matrixvalued Ornstein-Uhlenbeck processes. By constructing the expounded martingale, we obtain a large deviation upper bound for its empirical process, which extends the corresponding result for Hermitian Brownian motion.

Keywords: large deviation; Ornstein-Uhlenbeck process; empirical process

2010 MR Subject Classification: 60J60; 60F10

Document code: A

Article ID: 0255-7797(2016)02-0253-08

1 Introduction

Random matrix theory is focused on the asymptotic properties of random matrices as their dimension tends to infinity. The spectral characteristics of a random matrix are conveniently studied via its empirical spectral distribution. In the 1950s, motivated by numerical experiments, Wigner (see [1]) proved that the empirical spectral distribution of an $n \times n$ Hermitian matrix with on and upper diagonal entries being independent Gaussian random variables converges to the semi-circle law μ_{sc} with density

$$\rho(x) = \begin{cases} \frac{1}{2\pi} \sqrt{4 - x^2}, & |x| \leq 2; \\ 0, & |x| > 2. \end{cases}$$

It was later shown that the distribution of the matrix entries did not play a significant role and convergence to the semicircle law holds under more general conditions and holds for many other ensembles (see [2, 3]), and much of work devoted to weaken the assumptions about the ensembles (see [4]). The weakest known condition was given by Pastur (see [5]) and it was proved necessary by Girko (see [6]).

Dyson (see [7]) considered a matrix-valued random process with matrix entries are independent Brownian motions, i.e., symmetric (resp. Hermitian) Brownian motion. It can

* **Received date:** 2013-05-13

Accepted date: 2013-06-06

Foundation item: Supported by the Teachers' Scientific Research Project of Jiangsu Normal University (12XLR005).

Biography: Chen Lei (1980–), male, born at Lianyungang, Jiangsu, lecturer, major in stochastic process.

be described on the space $\mathcal{H}_{n,\gamma}$, $\gamma = 1$, symmetric (resp. $\gamma = 2$, Hermitian) matrices of dimension n , as the random process $H_{n,\gamma}$, $\gamma = 1, 2$ with entries $\{H_{n,\gamma}^{k,l}(t), t > 0\}$:

$$H_{n,\gamma}^{k,l} = \begin{cases} \frac{1}{\sqrt{n\gamma}}(B_{k,l} + i(\gamma - 1)B'_{k,l}), & \text{if } k < l; \\ \frac{\sqrt{2}}{\sqrt{n\gamma}}B_{l,l}, & \text{if } k = l, \end{cases}$$

where $(B_{k,l}, B'_{k,l})$, $1 \leq k \leq l \leq N$ is a collection of independent real valued standard Brownian motions. This leads to more effective tools, such as the method of stochastic analysis and the theory of martingale, for the study of Gaussian random matrices since a standard Gaussian variable with mean 0 and variance 1 can be seen as a standard Brownian motion at time 1. The eigenvalue processes of $H_{n,\gamma}$ corresponding to a diffusion model of an interacting particles system with electrostatic inter-particle repulsion which can be described by n coupled Itô stochastic equations:

$$d\lambda_i(t) = \frac{\sqrt{2}}{\sqrt{n\gamma}}d\beta_i(t) + \frac{1}{n} \sum_{j:j \neq i} \frac{1}{\lambda_i(t) - \lambda_j(t)} dt, \quad i = 1, \dots, n,$$

where β_i , $i = 1, \dots, n$ are independent Brownian motions. The associated empirical process

$$L_n(t) := \frac{1}{n} \sum_{i=1}^N \delta_{\lambda_i(t)}$$

is an element of $\mathcal{M}_1(\mathbb{R})$, the space of probability measure on $\mathbb{R}$, where δ_x denotes the unit mass at x . Then, under quit general assumptions, the sequence $L_n(\cdot)$ was shown to converge in law to a determinstic $\mathcal{M}_1(\mathbb{R})$ -valued process provided that $L_n(0) \rightarrow \mu$ in distribution, and thus a dynamic proof of Wigner's theorem was established (see [8]).

Chan (see [9]) studied the role of the Wigner semi-circle law from the point of view of symmetric matrix which entries are independent Ornstein-Uhlenbeck processes, it can be described on the space of n -dimension symmetric matrices as the random process X_t :

$$dX_t = -\frac{1}{2}X_t dt + \frac{1}{2\sqrt{n}}(dB + dB^T), \quad (1.1)$$

where B is a standard matrix-valued Brownian motion and B^T denotes the transpose of B . He also derived a system of Itô stochastic equations for the eigenvalues of X_t which corresponds to a diffusion model of an interacting particles system with linear drift towards the origin and electrostatic inter-particle repulsion, i.e.,

$$d\lambda_i(t) = \frac{1}{\sqrt{n}}d\beta_i(t) + \frac{1}{2n} \sum_{k \neq i} \frac{1}{\lambda_i(t) - \lambda_k(t)} dt - \frac{1}{2}\lambda_i(t)dt, \quad i = 1, \dots, n, \quad (1.2)$$

where β_i , $i = 1, \dots, n$ are independent Brownian motions. Its associated empirical process converges weakly to a measure-valued process which characterized by a weak solution of a deterministic ordinary differential equation and the Wigner semi-circle law is one of the equilibrium points of this limiting equation.

The purpose of this paper is to study the large deviation properties of the law of the empirical process associate with X defined by (1.1). This problem was studied by several authors, Dawson et al. (see [10]) studied it from the McKean-Vlasov limit, Duvillard et al. (see [11]) studied the same problem for symmetric (Hermitian) Brownian motion and it was completed by Guionnet et al. (see [12]).

2 Main Result and the Proof

We first state the assumptions and some notations for our result.

H1 The initial values $\mu_n(0) := L_n(0) = \frac{1}{n} \sum_{i=1}^n \delta_{\lambda_i(0)}$ are weak convergence to some limiting measure μ , where $\lambda_1(0), \dots, \lambda_n(0)$ are the initial condition of (1.2).

H2 $\sup \langle \mu_n(0), \log(1+x^2) \rangle < +\infty$.

Let $C([0, T], \mathbb{R})$ be the space of continuous function on $[0, T]$ equipped with the uniform convergence topology and $C([0, T], \mathcal{M}_1(\mathbb{R}))$ be the space of continuous measure-valued processes furnished with the topology generated by the weak topology on $\mathcal{M}_1(\mathbb{R})$ and the uniform convergence topology on $[0, T]$. Denote

$$\mathcal{D} = \left\{ f : f \in C_b^{2,1}(\mathbb{R} \times [0, T]) \text{ and } x \partial_x f(x, t) \text{ is bounded} \right\}$$

and

$$\langle f, v \rangle = \int f(x) v(dx), \quad f \in \mathcal{D}$$

for any measure v .

Next, we introduce a candidate for a rate function on $C([0, T], \mathcal{M}_1(\mathbb{R}))$. By Itô formula and (1.2) for any $f \in C_b^{2,1}(\mathbb{R} \times [0, T])$,

$$\begin{aligned} f(t, \lambda_i(t)) &= f(0, \lambda_i(0)) + \int_0^t \partial_s f(s, \lambda_i(s)) ds \\ &\quad + \int_0^t \partial_x f(s, \lambda_i(s)) d\lambda_i(s) + \frac{1}{2} \int_0^t \partial_x^2 f(s, \lambda_i(s)) d\langle \lambda_i(s) \rangle \\ &= f(0, \lambda_i(0)) + \int_0^t \partial_s f(s, \lambda_i(s)) ds + \frac{1}{\sqrt{n}} \int_0^t \partial_x f(s, \lambda_i(s)) d\beta_i \\ &\quad + \int_0^t \partial_x f(s, \lambda_i(s)) \left(\frac{1}{2n} \sum_{j:j \neq i} \frac{1}{\lambda_i - \lambda_j} - \frac{1}{2} \lambda_i \right) ds + \frac{1}{2n} \int_0^t \partial_x^2 f(s, \lambda_i(s)) ds. \end{aligned}$$

Thus

$$\begin{aligned} &\langle f(t, \cdot), L_n(t) \rangle \\ &= \langle f(0, \cdot), L_n(0) \rangle + \int_0^t \langle \partial_s f(s, \cdot), L_n(s) \rangle ds + M_f^n(t) + \frac{1}{2n} \int_0^t \langle \partial_x^2 f(s, \cdot), L_n(s) \rangle ds \\ &\quad + \int_0^t \left\{ \frac{1}{4n^2} \sum_{j \neq i} \frac{\partial_x f(s, \lambda_i(s)) - \partial_x f(s, \lambda_j(s))}{\lambda_i - \lambda_j} - \frac{1}{2n} \sum_{i=1}^n \partial_x f(s, \lambda_i(s)) \lambda_i(s) \right\} ds \end{aligned}$$

$$\begin{aligned}
&= \langle f(0, \cdot), L_n(0) \rangle + \int_0^t \langle \partial_s f(s, \cdot), L_n(s) \rangle ds + M_f^n(t) + \frac{1}{2n} \int_0^t \langle \partial_x^2 f(s, \cdot), L_n(s) \rangle ds \\
&\quad - \frac{1}{2} \int_0^t \langle x \partial_x f(s, \cdot), L_n(s) \rangle ds + \frac{1}{4} \int_0^t \iint_{x \neq y} \frac{\partial_x f(s, x) - \partial_y f(s, y)}{x - y} dL_n(s)(x) dL_n(s)(y) ds \\
&= \langle f(0, \cdot), L_n(0) \rangle + \int_0^t \langle \partial_s f(s, \cdot), L_n(s) \rangle ds + M_f^n(t) + \frac{1}{4n} \int_0^t \langle \partial_x^2 f(s, \cdot), L_n(s) \rangle ds \\
&\quad - \frac{1}{2} \int_0^t \langle x \partial_x f(s, \cdot), L_n(s) \rangle ds + \frac{1}{4} \int_0^t \iint \frac{\partial_x f(s, x) - \partial_y f(s, y)}{x - y} dL_n(s)(x) dL_n(s)(y) ds,
\end{aligned}$$

where $M_f^n(t)$, $t \leq T$ is a martingale given by

$$M_f^n(t) = \frac{1}{n^{3/2}} \sum_{i=1}^n \int_0^t \partial_x f(s, \lambda_i(s)) d\beta_i.$$

For any $f, g \in \mathcal{D}$, $s \leq t$ and $v \in C([0, T], \mathcal{M}_1(\mathbb{R}))$, set

$$\begin{aligned}
S^{s,t}(v, f) &= \int f(x, t) dv_t(x) - \int f(x, s) dv_s(x) - \int_s^t \int \partial_u f(x, u) dv_u(x) du \\
&\quad + \frac{1}{2} \int_0^t \int x \partial_x f(x, u) dv_u(x) du - \frac{1}{4} \int_s^t \iint \frac{\partial_x f(x, u) - \partial_y f(y, u)}{x - y} dv_u(x) dv_u(y) du, \\
\langle f, g \rangle_v^{s,t} &= \int_s^t \int \partial_x f(x, u) \partial_x g(x, u) dv_u(x) du
\end{aligned} \tag{2.1}$$

and

$$\bar{S}^{s,t}(v, f) = S^{s,t}(v, f) - \frac{1}{2} \langle f, f \rangle_v^{s,t}.$$

Our main result is as follows:

Theorem 2.1 Under the Assumption H1 and H2, $\{L_n(t), t \in [0, T]\}_{n \geq 1}$ obeys a large deviation upper bound on $C([0, T], \mathcal{M}_1(\mathbb{R}))$ with speed n^2 and with good rate function

$$S(v) = \begin{cases} S^{0,T}(v) := \sup_{f \in \mathcal{D}} \sup_{0 \leq s \leq t \leq T} \bar{S}^{s,t}(v, f), & \text{if } v_0 = \mu; \\ \infty, & \text{otherwise.} \end{cases}$$

Thanks to the exponential tightness result established in [9], to prove Theorem 2.1, by the usual scheme (see [13]), we only need to show that the rate function is good and a weak large deviation upper bound holds. We first recall the exponential tightness result (see [9] Theorem 3.3):

Lemma 2.2 Under the Assumption H1 for any $L > 0$, there exists a compact set $\mathcal{K}_L \subset \mathcal{M}_1(\mathbb{R})$ such that

$$\limsup_{n \rightarrow \infty} \frac{1}{n^2} \log P(L_n \notin \mathcal{K}_L) \leq -L.$$

Next, we show that S is good and a weak large deviation upper bound holds for $\{L_n(t), t \in [0, T]\}_{n \geq 1}$.

Lemma 2.3 $\bar{S}$ is a good rate function, i.e., S is a non-negative function and for any $l \geq 0$, the level set $\{v \in C([0, T], \mathcal{M}_1(\mathbb{R})) : S(v) \leq l\}$ is compact.

Proof First, we have

$$\begin{aligned} S^{0,T}(v) &= \sup_{f \in \mathcal{D}} \sup_{0 \leq s \leq t \leq T} \left(S^{s,t}(v, f) - \frac{1}{2} \langle f, f \rangle_v^{s,t} \right) \\ &= \sup_{f \in \mathcal{D}} \sup_{0 \leq s \leq t \leq T} \sup_{\lambda \in \mathbb{R}} \left(S^{s,t}(v, \lambda f) - \frac{1}{2} \langle \lambda f, \lambda f \rangle_v^{s,t} \right) = \sup_{f \in \mathcal{D}} \sup_{0 \leq s \leq t \leq T} \frac{(S^{s,t}(v, f))^2}{2 \langle f, f \rangle_v^{s,t}}. \end{aligned} \quad (2.2)$$

Hence, S is non-negative for $v_0 = \mu$, and as a supremum of continuous functions on $C([0, T], \mathcal{M}_1(\mathbb{R}))$, S is lower semi-continuous. Thus, we only need to show that the level set of S is compact. By Lemma 5.4 in [10], it is suffice to show that it is contained in a compact set of the form:

$$\mathcal{K} = \left\{ v \in C([0, T], \mathcal{M}_1(\mathbb{R})), v_t \in \mathfrak{C}, \forall t \in [0, T] \right\} \cap \left(\bigcap_{n \geq 0} \left\{ t \rightarrow v_t(g_n) \in \mathfrak{C}_n \right\} \right), \quad (2.3)$$

where $\mathfrak{C}$ and $\mathfrak{C}_n$ are compact subsets of $\mathcal{M}_1(\mathbb{R})$ and $C([0, T], \mathbb{R})$, respectively, $(g_n)_{n \geq 0}$ is a family of bounded continuous functions dense in $C_c(\mathbb{R})$.

According to Prohorov's theorem, $\mathfrak{C}$ in (2.3) can be taken as

$$\mathfrak{C}(\ell) = \bigcap_{m \in \mathbb{N}} \left\{ v \in \mathcal{M}_1(\mathbb{R}) : v(|x| \geq \ell_m) \leq \frac{1}{m} \right\}$$

with a positive real valued sequence $\ell = (\ell_m)_{m \in \mathbb{N}}$. Moreover, by Arzela-Ascoli's theorem and the fact that $t \rightarrow v_t(g_n)$ ($\forall n \in \mathbb{N}$) is uniformly bounded on $C([0, T], \mathcal{P}(\mathbb{R}))$ for g_n is bounded, we can take $\mathfrak{C}_n$ in (2.3) of the form

$$\mathfrak{C}(\delta) = \bigcap_{m \in \mathbb{N}} \left\{ h \in C([0, T], \mathbb{R}), \sup_{|t-s| \leq \delta_m} |h(t) - h(s)| \leq \frac{1}{m} \right\}$$

with a positive sequence $\delta = (\delta_m)_{m \in \mathbb{N}}$. Thus, to finish the proof, we need to show that for any $v \in \{S(v) \leq l\}$ ($\forall l > 0$) and any integer m ,

- there is a positive real number ℓ_m^l such that

$$\sup_{0 \leq s \leq T} v_s(|x| \geq \ell_m^l) \leq \frac{1}{m}. \quad (2.4)$$

- there exists a positive real number δ_m^l such that

$$\sup_{|t-s| \leq \delta_m^l} |v_t(f) - v_s(f)| \leq \frac{1}{m}, \quad \forall f \in \mathcal{D}. \quad (2.5)$$

For $0 < \varepsilon \leq 1$, set $f_\varepsilon(x) = \log(1 + x^2(1 + \varepsilon x^2)^{-1}) \in \mathcal{D}$, then

$$\alpha := \sup_{0 < \varepsilon \leq 1} \|\partial_x f_\varepsilon\|_\infty + \sup_{0 < \varepsilon \leq 1} \|x \partial_x f_\varepsilon\|_\infty + \sup_{0 < \varepsilon \leq 1} \|\partial_x^2 f_\varepsilon\|_\infty < +\infty,$$

and

$$\left| \frac{\partial_x f_\varepsilon(x) - \partial_y f_\varepsilon(y)}{x - y} \right| \leq \alpha.$$

Taking $f = f_\varepsilon$ in the supremum of (2.2), we have that for any $t \in [0, T]$ and any $v \in \{S(\cdot) \leq l\}$,

$$(S^{s,t}(v, f_\varepsilon))^2 \leq 2l \langle f_\varepsilon, f_\varepsilon \rangle_v^{s,t} \leq 2\alpha^2 lt. \quad (2.6)$$

Thus by (2.1), we have

$$v_t(f_\varepsilon) \leq v_0(f_\varepsilon) + \frac{5}{4}\alpha t + \alpha\sqrt{2lt} \leq v_0(f_\varepsilon) + 2\alpha t + 2\alpha\sqrt{lt},$$

let $\varepsilon \downarrow 0$, by monotone convergence theorem, we have

$$\sup_{t \in [0, T]} v_t(1 + x^2) \leq \langle \mu, \log(1 + x^2) \rangle + 2\alpha(T + \sqrt{lT}) = C_0 + 2\alpha(T + \sqrt{lT}).$$

Then by Chebyshev's inequality and Assumption H2, for any $v \in \{S(\cdot) \leq l\}$ and any $K \in \mathbb{R}^+$,

$$\begin{aligned} \sup_{t \in [0, T]} v_t(|x| \geq K) &= \sup_{t \in [0, T]} v_t(x^2 \geq K^2) \\ &\leq \frac{\sup_{t \in [0, T]} v_t(1 + x^2)}{1 + K^2} \leq \frac{C_0 + 2\alpha(T + \sqrt{lT})}{1 + K^2}. \end{aligned}$$

So (2.4) is proved. Again by (2.6), we have for any $f \in \mathcal{D}$,

$$S^{s,t}(v, f) \leq \sqrt{2l \langle f, f \rangle_v^{s,t}} \leq \sqrt{2l} \|f'\|_\infty \sqrt{|t - s|}.$$

Thus by (2.1), for any $v \in \{S(\cdot) \leq l\}$ and $0 \leq s \leq t \leq T$,

$$|\langle f, v_t - v_s \rangle| \leq \sqrt{2l} \|f'\|_\infty \sqrt{|t - s|} + \frac{1}{4} \|f''\|_\infty |t - s| + \frac{1}{2} \|xf'(x)\|_\infty |t - s|.$$

This complete the proof of (2.5).

To end the proof of the main result, we would only need to show that $\{L_n(t), t \in [0, T]\}_{n \geq 1}$ obeys the weak upper bound of large deviation.

Lemma 2.4 For $v \in C([0, T], \mathcal{M}_1(\mathbb{R}))$,

$$\lim_{r \rightarrow 0} \limsup_{n \rightarrow \infty} \frac{1}{n^2} \log P(L_n \in B(v, r)) \leq -S(v),$$

where $B(v, r)$ denotes the open ball with center v and radius r .

Proof First, since $L_n(0)$ is deterministic and converges to μ , if $v_0 \neq \mu$,

$$\lim_{r \rightarrow 0} \limsup_{n \rightarrow \infty} \frac{1}{n^2} \log P(L_n \in B(v, r)) = -\infty.$$

Second, take $v \in C([0, T], \mathcal{M}_1(\mathbb{R}))$ and $f \in \mathcal{D}$, by (2.1), we have that $S^{s,t}(L_n, f) - \frac{1}{n} \varepsilon(f)_{L_n}^{s,t}$, $s \leq t$ is a martingale for the filtration of the Brownian motion β , and equals to

$$\frac{1}{n^{3/2}} \sum_{i=1}^n \int_s^t \partial_x f(s, \lambda_i(s)) d\beta_i(s),$$

its bracket is $\langle f, f \rangle_{L_n}^{s,t}$, where

$$\varepsilon(f)_v^{s,t} = \frac{1}{4} \int_s^t \langle \partial_x^2 f(s, \cdot), v_s \rangle ds.$$

Since f' is uniformly bounded, we have for $v \in C([0, T], \mathcal{M}_1(\mathbb{R}))$,

$$M(v, f)(t) = \exp \left\{ n^2 S^{s,t}(v, f) - \frac{n^2}{2} \langle f, f \rangle_v^{s,t} + n \varepsilon(f)_v^{s,t} \right\}$$

is a martingale. Moreover, $C([0, T], \mathbb{R}) \ni v \rightarrow \bar{S}^{s,t}(v, f) = S^{s,t} - \frac{1}{2} \langle f, f \rangle_v^{s,t}$ is continuous as f and its two derivatives are bounded continuous whereas the function

$$v \rightarrow \int_s^t \int \partial_x^2 f(s, x) dv_u(x) du$$

is uniformly bounded by $T \|\partial_x^2 f\|_\infty$. Therefore, for any $v \in C([0, T], \mathcal{M}_1(\mathbb{R}))$ and $r > 0$, we have for $s \leq t \leq T$,

$$\begin{aligned} P(L_n \in B(v, r)) &= E \left(\frac{M(L_n, f)}{M(L_n, f)} 1_{L_n \in B(v, r)} \right) \\ &\leq \exp \left\{ -n^2 \inf_{\mu \in B(v, r)} \left(S^{s,t}(\mu, f) - \frac{1}{2} \langle f, f \rangle_\mu^{s,t} \right) + \sup_{\mu \in B(v, r)} n \varepsilon(f)_\mu^{s,t} \right\} \\ &= \exp \left\{ -n^2 \bar{S}^{s,t}(v, f) + \sup_{\mu \in B(v, r)} n \varepsilon(f)_\mu^{s,t} \right\}, \end{aligned}$$

where we have used the fact that $E(M(L_n, f)(t)) = 1$ since the process $\{M(L_n, f)(t), s \leq t\}$ is a martingale. Hence, for any $f \in \mathcal{D}$,

$$\lim_{r \rightarrow 0} \limsup_{n \rightarrow \infty} \frac{1}{n^2} \log P(L_n \in B(v, r)) \leq -\bar{S}^{s,t}(v, f),$$

optimizing over f gives

$$\lim_{r \rightarrow 0} \limsup_{n \rightarrow \infty} \frac{1}{n^2} \log P(L_n \in B(v, r)) \leq -\sup_{f \in \mathcal{D}} \sup_{0 \leq s \leq t \leq T} \bar{S}^{s,t}(v, f) \leq -S^{0,T}(v, f).$$

Thus the proof is completed.

References

- [1] Wigner E P. On the distribution of the roots of certain symmetric matrices[J]. Ann. Math., 1958, 67(2): 325–327.
- [2] Arnold L. On the asymptotic distribution of the eigenvalues of random matrices[J]. J. Math. Anal. Appl., 1967, 20(2): 262–268.
- [3] Bai Z D, Silverstein J W. Spectral analysis of large dimensional random matrices[M]. Berlin: Springer-Verlag, 2010.
- [4] Tao T, Vu V. Random matrices: the four moment theorem for wigner ensembles[J]. 2011, arXiv preprint arXiv: 1112.1976.
- [5] Pastur L A. On the spectrum of random matrices[J]. The. Math. Phys., 1972, 10(1): 67–74.
- [6] Girko V L. Spectral theory of random matrices[J]. Uspekhi Matematicheskikh Nauk, 1985, 40(1): 67–106.
- [7] Dyson F J. A Brownian-motion model for the eigenvalues of a random matrix[J]. J. Math. Phys., 1962, 3: 1191–1198.
- [8] Anderson G W, Guionnet A, Zeitouni O. An introduction to random matrices[M]. London: Cambridge University Press, 2010.
- [9] Chan T. The Wigner semi-circle law and eigenvalues of matrix-valued diffusions[J]. Prob. The. Rel. Fiel., 1992, 93(2): 249–272.
- [10] Dawsont D A, Gärtner J. Large deviations from the McKean-Vlasov limit for weakly interacting diffusions[J]. Stoch.: Intern. J. Prob. Stoch. Proc., 1987, 20(4): 247–308.
- [11] Duvillard T C, Guionnet A. Large deviations upper bounds for the laws of matrix-valued processes and non-communicative entropies[J]. Ann. Prob., 2001, 1205–1261.
- [12] Guionnet A, Zeitouni O. Large deviations asymptotics for spherical integrals[J]. J. Funct. Anal., 2002, 188(2): 461–515.
- [13] Dembo A, Zeitouni O. Large deviations techniques and applications[M]. Berlin: Springer-Verlag, 1998.

矩阵值 Ornstein-Uhlenbeck 过程的大偏差上界

陈 磊

(江苏师范大学数学与统计学院, 江苏 徐州 221116)

摘要: 本文研究了矩阵值Ornstein-Uhlenbeck 过程的大偏差问题. 通过构造指数鞅, 得到了矩阵值Ornstein-Uhlenbeck过程的经验谱过程的大偏差上界, 推广了厄米特布朗运动相应的结果.

关键词: 大偏差; Ornstein-Uhlenbeck 过程; 经验过程

MR (2010) 主题分类号: 60J60; 60F10

中图分类号: O211.62