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EXACT TRAVELING WAVE SOLUTIONS OF THE OSMOSIS $K(2,3)$ EQUATION

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Abstract: In this paper, we study the bifurcation of $K(2,3)$ equation $u_t+(u^2)_x-(u^3)_{xxx}=0$ with osmosis dispersion. Using the qualitative analysis methods of dynamical system and numerical simulation by Maple programs, illustrative phase portraits corresponding to traveling wave system are presented, and expressions of a periodic cusp wave, kink-like and antikink-like wave solutions, and implicit expression of soliton are explicitly given after integrals.

Keywords: periodic cusp wave solution; kink-like wave solution; antikink-like wave solution; soliton; the bifurcation of phase portraits

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1 Introduction

In 1993, Rosenau and Hyman [1] introduced a genuinely nonlinear dispersive equation, namely, a particular generalization of the KdV equation

$$u_t+(u^m)_x+(u^n)_{xxx}=0, m>0, 1<n\le 3,$$

which is called the $K(m,n)$ equation. They studied four cases $m,n=2,3$ with the compacton solutions. Few years later, Ismail and Taha [2] implemented a finite difference method and a finite element method to study equations $K(2,2)$ and $K(3,3)$. Ismail [3] made an extension based on [2] and obtained numerical solutions of the $K(2,3)$ equation. Frutos and Lopez-Marcos [4] presented a finite difference method for the numerical integration of the $K(2,2)$ equation. Zhou and Tian [5] considered soliton solution of the $K(2,2)$ equation. Xu and Tian [6] studied the peaked wave solutions of the $K(2,2)$ equation. Zhou et al. [7] investigated kink-like and antikink-like wave solutions of the $K(2,2)$ equation. He and Meng [8] obtained new exact explicit peakon and smooth periodic wave solutions of the $K(3,2)$ equation. In this paper, we will discuss the osmosis $K(2,3)$ equation

$$u_t+(u^2)_x-(u^3)_{xxx}=0, \tag{1.1}$$

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where the positive convection term denotes the convection moves and the negative dispersive term means the contracting dispersion.

The paper is organized as follows. In Section 2, we discuss the bifurcation of traveling wave system related with equation (1.1) and draw its phase portraits by maple programs in Figure 1. Section 3 is devoted to obtaining exact solutions including soliton, periodic cusp wave and kink-like and antikind-like waves which belong to bounded traveling waves. A short conclusion is given in Section 4.

2 The Bifurcation and Phase Portraits

Consider the osmosis $K(2,3)$ equation (1.1), and change it into the following form

$$u_t + 2uu_x - 6(u_x)^3 - 18uu_xu_{xx} - 3u^2u_{xxx} = 0. \tag{2.1}$$

Let $u = \varphi(\xi) = \varphi(x - ct)$ be the solution for equation (1.1) or (2.1), where c ($\neq 0$) is the wave speed. Substituting $u = \varphi(x - ct)$ into (2.1) gives an ODE

$$-c\varphi' + 2\varphi\varphi' - 6(\varphi')^3 - 18\varphi\varphi'\varphi'' - 3\varphi^2\varphi''' = 0. \tag{2.2}$$

Integrating (2.2) once with respect to ξ, we have

$$-c\varphi + \varphi^2 - 6\varphi(\varphi')^2 - 3\varphi^2\varphi'' = g, \tag{2.3}$$

which can be rewritten as

$$\begin{cases} \frac{d\varphi}{d\xi} = y, \\ \frac{dy}{d\xi} = \frac{g+c\varphi-\varphi^2+6\varphi y^2}{-3\varphi^2}, \end{cases} \tag{2.4}$$

where g is an integral constant. Since (2.4) is a singular traveling wave system with a singular line $\varphi = 0$, we introduce a rescaling of the time $d\xi = -3\varphi^2 d\tau$ to transform it into a regular system. Namely,

$$\begin{cases} \frac{d\varphi}{d\tau} = -3\varphi^2 y, \\ \frac{dy}{d\tau} = g + c\varphi - \varphi^2 + 6\varphi y^2 \end{cases} \tag{2.5}$$

with a first integral

$$H(\varphi, y) = \varphi^3\left(\frac{1}{3}g + \frac{c}{4}\varphi - \frac{1}{5}\varphi^2 + \frac{3}{2}\varphi y^2\right). \tag{2.6}$$

Obviously, the singular traveling wave system (2.4) has the same first integral $H(\varphi, y)$ as (2.5), and accordingly both (2.4) and (2.5) have the same topological phase portraits except for the straight line $\varphi = 0$. Thus, in order to understand the bifurcation sets of system (2.4), we need to study phase portraits of system (2.5). Next, we discuss the isolated and singular points of system (2.5) for their phase portraits. Let $A_\pm = (\frac{c\pm\sqrt{c^2+4g}}{2}, 0)$, then

$$\begin{aligned} H(A_+) &= \tfrac{1}{960}(c + \sqrt{c^2+4g})^3(16g + 3c^2 + 3c\sqrt{c^2+4g}), \\ H(A_-) &= \tfrac{1}{960}(-c + \sqrt{c^2+4g})^3(-16g - 3c^2 + 3c\sqrt{c^2+4g}). \end{aligned}$$

Using the qualitative analysis methods of planar dynamical system, the distribution and property of singular points are given

(1) When $g > 0$, system (2.5) has a saddle point A_+ and a center A_-.

(2) When $g = 0$, system (2.5) has a center $(c, 0)$.

(3) When $-\frac{c^2}{4} < g < 0$, system (2.5) has a saddle point A_+ and a center A_-.

(4) When $g = -\frac{c^2}{4}$, system (2.5) has a cusp $(\frac{c}{2}, 0)$.

(5) When $g < -\frac{c^2}{4}$, system (2.5) has no singular point.

According to the property above, we show phase portraits of system (2.5) in Figure 1. It is easy to see a periodic cusp wave from Figure 1(1-4), a kink-like wave and a antikink-like wave from Figure 1(1-6) and soliton from Figure 1(1-7) and (1-8). Their expressions are presented in next section.

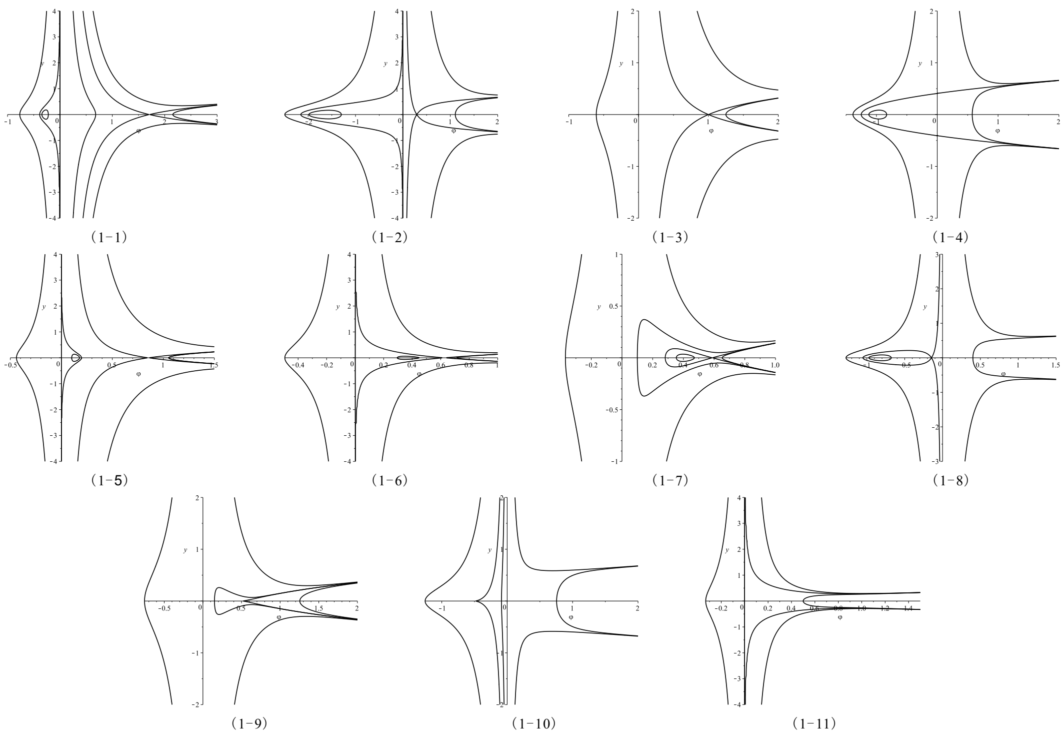

Figure 1: The phase portraits of system (2.5): (1-1) $g > 0, c > 0$; (1-2) $g > 0, c < 0$; (1-3) $g = 0, c > 0$; (1-4) $g = 0, c < 0$; (1-5) $-\frac{15c^2}{64} < g < 0, c > 0$; (1-6) $g = -\frac{15c^2}{64}, c > 0$; (1-7) $-\frac{c^2}{4} < g < -\frac{15c^2}{64}, c > 0$; (1-8) $-\frac{c^2}{4} < g < 0, c < 0$; (1-9) $g = -\frac{c^2}{4}, c > 0$; (1-10) $g = -\frac{c^2}{4}, c < 0$; (1-11) $g < -\frac{c^2}{4}$.

3 Main Results and Their Proofs

3.1 Periodic Cusp Wave Solution

Theorem 3.1 When $g = 0$ and $c < 0$, equation (1.1) has a periodic cusp wave solution of peak type $u = \varphi_1(\xi + 2nT), n = 0, \pm 1, \pm 2 \cdots$ and $\xi \in [(2n-1)T, (2n+1)T]$, where

$\varphi_1(\xi) = \frac{1}{30}\xi^2 + \frac{5c}{4}$, $\xi \in [-T, T]$ with $T = \frac{5}{2}\sqrt{-6c}$.

Proof In Figure 1 (1-4), the periodic orbit of system (2.4) intersects y-axis, which can be expressed as $H(\varphi, y) = \varphi^4(\frac{c}{4} - \frac{1}{5}\varphi + \frac{3}{2}y^2) = 0$. Namely,

$$y = \pm\sqrt{\frac{2\varphi}{15} - \frac{c}{6}}, \quad \frac{5}{4}c \le \varphi \le 0,\ c < 0, \tag{3.1}$$

which intersects y-axis at $(0, \sqrt{-\frac{c}{6}})$ and $(0, -\sqrt{-\frac{c}{6}})$, and x-axis at $(\frac{5}{4}c, 0)$. Substituting (3.1) into system (2.4), we get

$$\begin{cases} \frac{d\varphi}{d\xi} = y, \\ \frac{dy}{d\xi} = \frac{1}{15}, \end{cases}$$

which gives $u = \varphi_1(\xi + 2nT), n = 0, \pm1, \pm2\cdots$ and $\varphi_1(\xi) = \frac{1}{30}\xi^2 + \frac{5c}{4}$, $\xi \in [-T, T]$. In addition, $\varphi_1(T) = 0$ reveals $T = \frac{5}{2}\sqrt{-6c}$. The proof is completed.

The graph of relevant periodic cusp wave of peak type for equation (1.1) is shown in Figure 2.

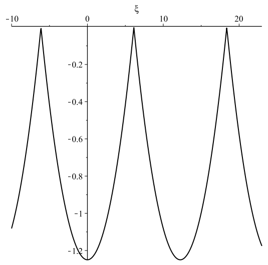

Figure 2: The periodic cusp wave of peak type for equation (1.1) : $g = 0, c = -1$.

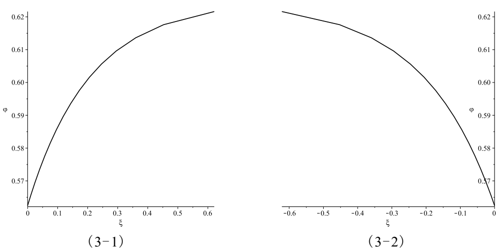

Figure 3: $g = 0, c = -1, a = \frac{16}{9}$. (3-1) The kink-like wave for equation (1.1); (3-2) The antikink-like wave for equation (1.1).

3.2 Kink-Like and Antikink-Like Wave Solutions

Theorem 3.2 When $g = -\frac{15}{64}c^2$ and $c > 0$, equation (1.1) has a kink-like wave solution $u = \varphi_2(\xi)$ and a antikink-like wave solution $u = \varphi_3(\xi)$,

$$\beta(\varphi_2) = \beta(a) + \xi, \quad \xi \in (-\xi_0, +\infty), \tag{3.2}$$

$$\beta(\varphi_3) = \beta(a) - \xi, \quad \xi \in (-\infty, \xi_0), \tag{3.3}$$

where $\frac{c}{2} < \varphi < \frac{5}{8}c$ and $\beta(\varphi) = \frac{\sqrt{30}}{2}[-2\sqrt{\varphi} + \frac{\sqrt{10c}}{2}\text{arctanh}\frac{2\sqrt{10\varphi}}{5\sqrt{c}}]$, $\xi_0 = \beta(a) - \beta(\frac{c}{2})$ with a is a constant satisfying $\varphi_2(0) = \varphi_3(0) = a$.

Proof In Figure 1 (1-6), system (2.4) has four orbits connecting with a saddle point A_+, L_2 denotes a orbit lying on the upper-left side of A_+, and L_3 on the lower-left. Note that $H(A_+) = 0$, then L_2 and L_3 can be respectively described as the following forms for

$0 \le \varphi \le \frac{5}{8}c$:

$$L_2 : y = \frac{\sqrt{2}(5c - 8\varphi)}{8\sqrt{15\varphi}}, \tag{3.4}$$

$$L_3 : y = -\frac{\sqrt{2}(5c - 8\varphi)}{8\sqrt{15\varphi}}. \tag{3.5}$$

Let $\varphi_2(\xi)$ and $\varphi_3(\xi)$ be respective solutions of system (2.4) on L_2 and L_3. Substituting (3.4) and (3.5) into the first equation of system (2.4) separately and integrating along orbits L_2, L_3, respectively, we get

$$\int_{\varphi_2}^{a} \frac{8\sqrt{15s}}{\sqrt{2}(5c - 8s)} ds = \int_{\xi}^{0} ds, \tag{3.6}$$

$$\int_{a}^{\varphi_3} \frac{-8\sqrt{15s}}{\sqrt{2}(5c - 8s)} ds = \int_{0}^{\xi} ds, \tag{3.7}$$

which imply implicit functions in (3.2) and (3.3) by maple programs.

Besides, assume that $\varphi_2(\xi) \to \frac{c}{2}$ as $\xi \to -\xi_0$, and $\varphi_3(\xi) \to \frac{c}{2}$ as $\xi \to \xi_1$, then we have from (3.6)–(3.7) that

$$\xi_0 = \xi_1 = \int_{a}^{\frac{c}{2}} \frac{-8\sqrt{15s}}{\sqrt{2}(5c - 8s)} ds = \beta(a) - \beta(\frac{c}{2}).$$

The proof is completed.

The graph of relevant kink-like and antikink-like waves for equation (1.1) is shown in Figure 3.

3.3 Soliton Solutions

Theorem 3.3 When $-\frac{c^2}{4} < g < -\frac{15}{64}c^2$ and $c > 0$, equation (1.1) has a soliton solution

$$\begin{cases} u(x,t) = \varphi(x - ct), \\ \phi = \arccos(\frac{A+l_1-\varphi}{A-l_1+\varphi}), \\ 2hAE(\phi,k) - h(l + l_1 - A - \frac{l^2}{l-l_1+A})F(\phi,k) - \frac{2Al^2h}{(\alpha^2-1)(l-l_1-A)^2}\Pi(\phi, \frac{\alpha^2}{\alpha^2-1}, k) - 2hA\frac{\sqrt{1-k^2\phi^2}\phi}{1+\sqrt{1-\phi^2}} \\ +\frac{Al^2h\alpha\sqrt{\alpha^2-1}}{(l-l_1-A)^2(\alpha^2-1)\sqrt{k^2+\alpha^2-k^2\alpha^2}} \ln \frac{\sqrt{k^2+\alpha^2-k^2\alpha^2}\phi+\sqrt{(\alpha^2-1)(1-k^2\phi^2)}}{\sqrt{k^2+\alpha^2-k^2\alpha^2}\phi-\sqrt{(\alpha^2-1)(1-k^2\phi^2)}} = \sqrt{\frac{2}{15}}|\xi|, \end{cases} \tag{3.8}$$

where $l = \frac{1}{2}(c+\sqrt{c^2+4g})$, $b_1 = -\frac{1}{2}l_2$, $a_1^2 = l_3 - \frac{1}{4}l_2^2$, $A = \sqrt{(b_1 - l_1)^2 + a_1^2}$, $k = \sqrt{\frac{A+b_1-l_1}{2A}}$, $h = \frac{1}{\sqrt{A}}$, $\alpha = \frac{l-l_1+A}{l-l_1-A}$ with l_1, l_2, l_3 given in (3.12), here $F(\cdot,\cdot)$ is the normal elliptic integral of the first kind, $E(\cdot,\cdot)$ is the normal elliptic integral of the second kind and $\Pi(\cdot,\cdot,\cdot)$ is the Legendre's incomplete elliptic integral of the third kind (see [9]).

Proof In Figure 1 (1-7), system (2.4) has a homoclinic loop consisting of a saddle point A_+, which can be expressed as $H(\varphi, y) = \varphi^3(\frac{1}{3}g + \frac{c}{4}\varphi - \frac{1}{5}\varphi^2 + \frac{3}{2}\varphi y^2) = H(A_+)$. Namely,

$$\frac{3}{2}\varphi^4 y^2 = \frac{1}{5}(\varphi - \frac{c + \sqrt{c^2 + 4g}}{2})^2(\varphi^3 + m_1\varphi^2 + m_2\varphi + m_3), \tag{3.9}$$

where $m_1=-\frac{c}{4}+\sqrt{c^2+4g}$, $m_2=\frac{c^2}{4}+\frac{c\sqrt{c^2+4g}}{4}+\frac{4g}{3}$, $m_3=\frac{c^3}{8}+\frac{c^2\sqrt{c^2+4g}}{8}+\frac{7cg}{12}+\frac{g\sqrt{c^2+4g}}{3}$. Let

$$f(\varphi)=\varphi^3+m_1\varphi^2+m_2\varphi+m_3. \tag{3.10}$$

By introducing a new variable, $x=\varphi+\frac{m_1}{3}$, (3.10) becomes

$$f(\varphi)=x^3+px+q,$$

where $p=-\frac{5c^2}{48}+\frac{5c\sqrt{c^2+4g}}{12}$, $q=\frac{5c^3}{864}+\frac{65c^2\sqrt{c^2+4g}}{432}+\frac{5cg}{36}+\frac{5g\sqrt{c^2+4g}}{27}$. Besides, $(\frac{q}{2})^2+(\frac{p}{3})^3>0$ under $-\frac{c^2}{4}<g<-\frac{15}{64}c^2$ and $c>0$, then (3.10) has a sole real root. Thus, (3.9) becomes

$$y=\sqrt{\frac{2}{15}}\frac{(l-\varphi)\sqrt{(\varphi-l_1)(\varphi^2+l_2\varphi+l_3)}}{\varphi^2},\ l_1<\varphi<l, \tag{3.11}$$

where

$$\begin{aligned}
l_1 \;=\; & -\frac{1}{12}\Big\{-\Big[-5c^3-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g\\
&+20\sqrt{(3c^2+2g)(9c^4+9c^3\sqrt{c^2+4g}+66c^2g+48c\sqrt{c^2+4g}g+128g^2)}\,\Big]^{\frac{2}{3}}-5c^2\\
&+20c\sqrt{c^2+4g}-c\Big[-5c^3-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g\\
&+20\sqrt{(3c^2+2g)(9c^4+9c^3\sqrt{c^2+4g}+66c^2g+48c\sqrt{c^2+4g}g+128g^2)}\,\Big]^{\frac{1}{3}}\\
&+4\sqrt{c^2+4g}\Big[-5c^3-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g\\
&+20\sqrt{(3c^2+2g)(9c^4+9c^3\sqrt{c^2+4g}+66c^2g+48c\sqrt{c^2+4g}g+128g^2)}\Big]^{\frac{1}{3}}\Big\}/\Big[-5c^3\\
&-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g\\
&+20\sqrt{(3c^2+2g)(9c^4+9c^3\sqrt{c^2+4g}+66c^2g+48c\sqrt{c^2+4g}g+128g^2)}\,\Big]^{\frac{1}{3}},\\
l_2 \;=\; & -\frac{1}{60c^2}\Big\{2250c^7-9000c^6\sqrt{c^2+4g}+19200c^5g-76800c^4g\sqrt{c^2+4g}+40960c^3g^2\\
&-163840c^2g^2\sqrt{c^2+4g}-(1125c^6+9600c^4g+20480c^2g^2)\\
&\Big[-5c^3-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g+20\\
&\sqrt{162c^3\sqrt{c^2+4g}g+216c^4g+516c^2g^2+96cg^2\sqrt{c^2+4g}+27c^5\sqrt{c^2+4g}+27c^6+256g^3}\Big]^{\frac{1}{3}}\\
&+\Big[-5c^3-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g+20\\
&\sqrt{162c^3\sqrt{c^2+4g}g+216c^4g+516c^2g^2+96cg^2\sqrt{c^2+4g}+27c^5\sqrt{c^2+4g}+27c^6+256g^3}\Big]^{\frac{2}{3}}\\
&\Big[+225c^5+450c^4\sqrt{c^2+4g}+1560c^3g+2400c^2\sqrt{c^2+4g}g+2560cg^2+2048\sqrt{c^2+4g}g^2\\
&+(68c^2+32c\sqrt{c^2+4g}+256g)\\
&\sqrt{162c^3\sqrt{c^2+4g}g+216c^4g+516c^2g^2+96cg^2\sqrt{c^2+4g}+27c^5\sqrt{c^2+4g}+27c^6+256g^3}\Big]\Big\}\\
&/(15c^2+64g)^2,
\end{aligned}$$

$$
\begin{aligned}
l_3 \;=\; & \frac{1}{360c^2}\Bigg\{450c^6+450c^5\sqrt{c^2+4g}+4320c^4g+1920c^3\sqrt{c^2+4g}g+10240c^2g^2+ \\
& \Big[-5c^3-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g+20 \\
& \sqrt{162c^3\sqrt{c^2+4g}g+216c^4g+516c^2g^2+96cg^2\sqrt{c^2+4g}+27c^5\sqrt{c^2+4g}+27c^6+256g^3}\Big]^{\frac{1}{3}} \\
& \cdot\Big[225c^5+225c^4+1260c^3g+960c^2g\sqrt{c^2+4g}+1280cg^2 \\
& +(10c^2+40c\sqrt{c^2+4g}) \\
& \sqrt{162c^3\sqrt{c^2+4g}g+216c^4g+516c^2g^2+96cg^2\sqrt{c^2+4g}+27c^5\sqrt{c^2+4g}+27c^6+256g^3}\Big] \\
& +\Big[-5c^3-130c^2\sqrt{c^2+4g}-120cg-160\sqrt{c^2+4g}g+20 \\
& \sqrt{162c^3\sqrt{c^2+4g}g+216c^4g+516c^2g^2+96cg^2\sqrt{c^2+4g}+27c^5\sqrt{c^2+4g}+27c^6+256g^3}\Big]^{\frac{2}{3}} \\
& \cdot\Big[-15c^4+225c^4-15c^3\sqrt{c^2+4g}-124c^2g-64cg\sqrt{c^2+4g} \\
& -(2c+8\sqrt{c^2+4g}) \\
& \sqrt{162c^3\sqrt{c^2+4g}g+216c^4g+516c^2g^2+96cg^2\sqrt{c^2+4g}+27c^5\sqrt{c^2+4g}+27c^6+256g^3} \\
& -256g^2\Big]\Bigg\}/(15c^2+64g)
\end{aligned}
\tag{3.12}
$$

with $l_2^2-4l_3>0$. Substituting (3.11) into the first equation of system (2.4) and integrating along the homoclinic orbit, we get

$$
\begin{aligned}
& -lhF(\phi,k)-h(l_1-A)F(\phi,k)-2hA\int_0^{u_1}\frac{du}{1+cn\,u}+l^2h\int_0^{u_1}\frac{1+cn\,udu}{(l-l_1+A)cn\,u+l-l_1-A]} \\
= \; & \sqrt{\tfrac{2}{15}}|\xi|,
\end{aligned}
\tag{3.13}
$$

where $cn\,u_1=\cos\phi, cn\,u=cn(u,k)$ is Jacobian elliptic function (see [9]). Therefore, (3.8) holds by computation.

The graph of relevant soliton for equation (1.1) is shown in Figure 4 (4-1).

Remark When $-\frac{c^2}{4}<g<0$ and $c<0$ (Figure 1 (1-8)), equation (1.1) also has a soliton solution and (3.8) holds. Its graph is shown in Figure 4 (4-2).

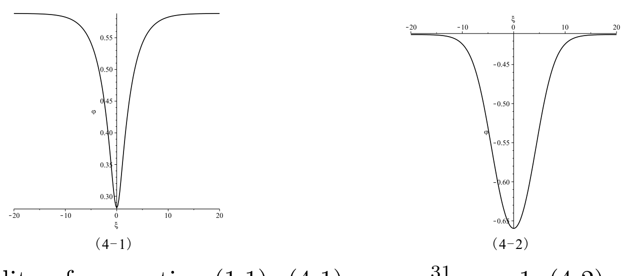

Figure 4: The soliton for equation (1.1). (4-1) $g=-\frac{31}{128}, c=1$; (4-2) $g=-\frac{31}{128}, c=-1$.

4 Conclusion

In this paper, using the qualitative analysis methods of planar dynamical system, we studied the bifurcation of the osmosis $K(2,3)$ equation and obtained exact periodic cusp wave solution, kink-like and antikink-like wave solutions, and soliton solution.

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含频散项的 $K(2,3)$ 方程的精确行波解

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摘 要: 本文研究了包含频散项的 $K(2,3)$ 方程 $u_t+(u^2)_x-(u^3)_{xxx}=0$ 的分支问题. 利用动力系统的定性分析, 并且借助 Maple 软件进行数值模拟得到行波解系统相应的相图, 然后通过积分计算得到周期尖波解、类扭波和类反扭波的精确解的函数表达式, 以及孤立波精确解的隐函数表达式.

关键词: 周期尖波解; 类扭波解; 类反扭波解; 孤立波; 相图分支

MR(2010)**主题分类号**: 34K13; 37K10 **中图分类号**: O175.1