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ORE EXTENSIONS OF NIL-SEMICOMMUTATIVE RINGS

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Abstract: In this paper, we study the properties of Ore extensions of nil-semicommutative rings. Let α be an endomorphism and δ an α-derivation of a ring R. By using the itemized analysis method on polynomials, we prove that if R is (α,δ)-skew Armendariz and (α,δ)-compatible, then $R[x;\alpha,\delta]$ is nil-semicommutative if and only if R is nil-semicommutative; if R is nil-semicommutative and (α,δ)-compatible, then $R[x;\alpha,\delta]$ is weak Armendariz, which generalize some related work on skew polynomial rings.

Keywords: nil-semicommutative ring; Ore extension; (α,δ)-compatible ring; weak (α,δ)-compatible ring; (α,δ)-skew Armendariz ring; weak (α,δ)-skew Armendariz ring

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1 Introduction

Throughout this paper, R denotes an associative ring with identity, α is an endomorphism and δ an α-derivation of R, that is, δ is an additive map such that $\delta(ab) = \delta(a)b + \alpha(a)\delta(b)$ for $a, b \in R$. We denote by $R[x;\alpha,\delta]$ the Ore extension whose elements are the polynomials over R, the addition is defined as usual, and the multiplication subject to the relation $xr = \alpha(r)x + \delta(r)$ for any $r \in R$. When $\delta = 0_R$, we write $R[x;\alpha]$ for $R[x;\alpha,0]$ and call it the skew polynomial ring (also called an Ore extension of endomorphism type); when $\alpha = 1_R$, we write $R[x;\delta]$ for $R[x;1_R,\delta]$ and call it the differential polynomial ring (also called an Ore extension of derivation type). For a ring R, we denote by $nil(R)$ the set of all nilpotent elements of R, and denote by $Nil_*(R)$ its prime radical.

Lambek [1] called a ring R to be symmetric if $abc = 0$ implies $acb = 0$ for all $a, b, c \in R$. Recall that a ring R is called reduced if it has no nonzero nilpotent elements; a ring R is called 2-primal if its prime radical contains every nilpotent element of R; and a ring R is called semicommutative if $ab = 0$ implies $aRb = 0$ for all $a, b \in R$. In [2], semicommutative

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property is called the insertion-of-factors-property, or IFP. There were many papers to study semicommutative rings and their generalization (see [3–5]). Liu and Zhao [6] proved that if R is a semicommutative ring then $nil(R)$ is an ideal of R. Mohammadi et al. [7] called a ring R to be nil-semicommutative if for any $a, b \in nil(R)$, $ab = 0$ implies $aRb = 0$. Obviously, semicommutative rings are nil-semicommutative rings and every subring of a nil-semicommutative ring is nil-semicommutative. [7] proved that if R is nil-semicommutative then $nil(R)$ is an ideal of R and nil-semicommutative rings are 2-primal rings. We have the following implications:

reduced $\Rightarrow$ symmetric $\Rightarrow$ semicommutative $\Rightarrow$ nil-semicommutative $\Rightarrow$ 2-primal.

In general, each of these implications is irreversible (see [7, 8]).

According to Krempa [9], an endomorphism α of a ring R is called rigid if $a\alpha(a) = 0$ implies $a = 0$ for all $a \in R$, and a ring R is called α-rigid if there exists a rigid endomorphism α of R. Notice that every rigid endomorphism of a ring is a monomorphism and α-rigid rings are reduced by Hong et al. [10]. Following Annin [11], for an endomorphism α and an α-derivation δ, a ring R is called α-compatible if for each $a, b \in R$, $ab = 0 \Leftrightarrow a\alpha(b) = 0$. Moreover, R is called δ-compatible if for each $a, b \in R$, $ab = 0 \Rightarrow a\delta(b) = 0$. If R is both α-compatible and δ-compatible, R is called (α, δ)-compatible. In this case, clearly the endomorphism α is injective. Hashemi and Moussavi [12] proved that R is α-rigid if and only if R is α-compatible and reduced.

Rege and Chhawchharia [13] called a ring R to be Armendariz if whenever polynomials $f(x) = \sum_{i=0}^{n} a_i x^i$, $g(x) = \sum_{j=0}^{m} b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_i b_j = 0$ for each i, j. Hong et al. [14] called a ring R with an endomorphism α to be α-skew Armendariz if whenever polynomials $f(x) = \sum_{i=0}^{n} a_i x^i, g(x) = \sum_{j=0}^{m} b_j x^j \in R[x; \alpha]$ satisfy $f(x)g(x) = 0$, then $a_i \alpha^i(b_j) = 0$ for each i, j. Following [15], a ring R with a derivation δ is called δ-skew Armendariz, for each $f(x) = \sum_{i=0}^{n} a_i x^i, g(x) = \sum_{j=0}^{m} b_j x^j \in R[x; \delta]$, if $f(x)g(x) = 0$ implies $a_i \delta^i(b_j) = 0$ (or $a_i b_j = 0$) for each $0 \le i \le n, 0 \le j \le m$. By Moussavi and Hashemi [16], a ring R with an endomorphism α and an α-derivation δ is called (α, δ)-skew Armendariz if whenever polynomials $f(x) = \sum_{i=0}^{n} a_i x^i, g(x) = \sum_{j=0}^{m} b_j x^j \in R[x; \alpha, \delta]$ satisfy $f(x)g(x) = 0$, then $a_i x^i b_j x^j = 0$ for each i, j. Note that each α-skew Armendariz is (α, δ)-skew Armendariz, where δ is the zero mapping. Obviously, every α-rigid ring is (α, δ)-skew Armendariz, but the converse does not hold (see [14], Example 1).

Due to the fact that many of the quantized algebras and their representations can be expressed in terms of iterated skew polynomial rings, it is interesting to know if the general Ore extension $S = R[x; \alpha]$ of a ring R share the same property with the ring R. In this paper we will show that:

(1) Let R be (α, δ)-skew Armendariz and (α, δ)-compatible. Then R is nil-semicommut ative if and only if $R[x; \alpha, \delta]$ is nil-semicommutative;

(2) Let R be weak (α, δ)-compatible and $nil(R)$ is an ideal of R. Then R is a weak

(α, δ)-skew Armendariz ring;

(3) Let R be weak (α, δ)-compatible and nil-semicommutative. Then $R[x]$ is a weak $(\bar{\alpha}, \bar{\delta})$-skew Armendariz ring.

In the following, for integers i, j with $0 \leqslant i \leqslant j$, $f_i^j \in \mathrm{End}(R, +)$ will denote the map which is the sum of all possible words in α, δ built with i letters α and $i - j$ letters δ. For instance, $f_2^4 = \alpha^2\delta^2 + \delta^2\alpha^2 + \delta\alpha^2\delta + \alpha\delta^2\alpha + \alpha\delta\alpha\delta + \delta\alpha\delta\alpha$. In particular, $f_0^0 = 1$, $f_i^i = \alpha^i$, $f_0^i = \delta^i$, $f_{j-1}^j = \alpha^{j-1}\delta + \alpha^{j-2}\delta\alpha + \cdots + \delta\alpha^{j-1}$. For every $f_i^j \in \mathrm{End}(R, +)$ with $0 \leqslant i \leqslant j$, it has C_j^i monomials in α, δ built with i letters α and $i - j$ letters δ. As is known to all that for any integer n and $r \in R$, we have $x^n r = \sum\limits_{i=0}^{n} f_i^n(r)x^i$ in the ring $R[x; \alpha, \delta]$.

2 Ore Extensions of Nil-Semicommutative Rings

Lemma 2.1 (see [12], Lemma 2.1) Let R be an (α, δ)-compatible ring. Then we have the following:

(1) If $ab = 0$, then $a\alpha^n(b) = \alpha^n(a)b = 0$, for all positive integers n;

(2) If $\alpha^k(a)b = 0$ for some positive integer k, then $ab = 0$;

(3) If $ab = 0$, then $\alpha^n(a)\delta^m(b) = \delta^m(a)\alpha^n(b) = 0$ for all positive integers m, n.

Proposition 2.2 Let R be an (α, δ)-compatible ring. Then we have the following:

(1) If $ab = 0$, then $af_i^j(b) = 0$ for all $0 \leqslant i \leqslant j$ and $a, b \in R$;

(2) For $a, b \in R$ and any positive integer m, $ab \in nil(R)$ if and only if $a\alpha^m(b) \in nil(R)$.

Proof (1) If $ab = 0$, then $a\alpha^i(b) = a\delta^j(b) = 0$ for all $i \geqslant 0$ and $j \geqslant 0$ by Lemma 2.1. Hence $af_i^j(b) = 0$ for all $0 \leqslant i \leqslant j$.

(2) It is an immediate consequence of Lemma 3.1 in [5] and Lemma 2.8 [17].

Proposition 2.3 Let R be an (α, δ)-compatible ring. Then we have the following:

(1) If $abc = 0$, then $a\delta(b)c = 0$ for any $a, b, c \in R$;

(2) If $abc = 0$, then $af_i^j(b)c = 0$ for all $0 \leqslant i \leqslant j$ and $a, b, c \in R$;

(3) If $ab \in nil(R)$, then $a\delta(b) \in nil(R)$ for any $a, b \in R$.

Proof (1) If $abc = 0$, we have $\alpha(ab)\delta(c) = 0$, $\alpha(a)\alpha(b)\delta(c) = 0$ and $a\alpha(b)\delta(c) = 0$. On the other hand, we also have $a\delta(bc) = 0$, $a(\delta(b)c + \alpha(b)\delta(c)) = 0$ and $a\delta(b)c + a\alpha(b)\delta(c) = 0$. So $a\delta(b)c = 0$.

(2) If $abc = 0$, we have $a\alpha(bc) = 0$, $a\alpha(b)\alpha(c) = 0$ and $a\alpha(b)c = 0$. It follows that $a\alpha^m(b)c = 0$ and $a\delta^n\alpha^m(b)c = 0$ for any positive integer m, n. On the other hand, we can obtain that $a\delta(b)c = 0$ by (1). This implies that $a\delta^j(b)c = 0$ and $a\alpha^i\delta^j(b)c = 0$. So $af_i^j(b)c = 0$ for all $0 \leqslant i \leqslant j$.

(3) Since $ab \in nil(R)$, there exists some positive integer k such that $(ab)^k = 0$. In the

following computations, we use freely (1):

$$\begin{aligned}
& (ab)^k = ab(ab\cdots ab) = 0 \\
\Rightarrow\ & a\delta(b)(ab\cdots ab) = (a\delta(b)a)b(ab\cdots ab) = 0 \\
\Rightarrow\ & (a\delta(b)a)\delta(b)(ab\cdots ab) = 0 \\
\Rightarrow\ & \cdots \\
\Rightarrow\ & (a\delta(b))^{k-1}ab1 = 0 \\
\Rightarrow\ & (a\delta(b))^k = 0.
\end{aligned}$$

This implies that $a\delta(b) \in nil(R)$.

Proposition 2.4 If R is an (α, δ)-compatible and nil-semicommutative ring, then $ab \in nil(R)$ implies $af_i^j(b) \in nil(R)$ for all $0 \leqslant i \leqslant j$ and $a, b \in R$.

Proof If $ab \in nil(R)$, we have $a\alpha^i(b) \in nil(R)$ and $a\delta^j(b) \in nil(R)$ for all $i \geqslant 0$ and $j \geqslant 0$ by Proposition 2.2 and Proposition 2.3. This implies that $a\delta^j\alpha^i(b) \in nil(R)$ and $a\alpha^i\delta^j(b) \in nil(R)$. Since $nil(R)$ is a ideal of R, we have $af_i^j(b) \in nil(R)$ for all $0 \leqslant i \leqslant j$.

Theorem 2.5 Let R be an (α, δ)-compatible and nil-semicommutative ring, and $f(x) = \sum\limits_{i=0}^{n} a_i x^i \in R[x; \alpha, \delta]$. Then $f(x) \in nil(R[x; \alpha, \delta])$ if and only if $a_i \in nil(R)$ for each i. That is, we have

$$nil(R[x; \alpha, \delta]) = nil(R)[x; \alpha, \delta].$$

Proof $(\Rightarrow)$ Suppose that $f(x) = \sum\limits_{i=0}^{n} a_i x^i \in R[x; \alpha, \delta]$. Then there exists a positive integer k such that $f(x)^k = (a_0 + a_1 x + \cdots + a_n x^n)^k = 0$. It follows that

$$f(x)^k = \text{“lower terms”} + a_n\alpha^n(a_n)\alpha^{2n}(a_n)\cdots\alpha^{(k-1)n}(a_n)x^{nk} = 0.$$

Hence

$$\begin{aligned}
& a_n\alpha^n(a_n)\alpha^{2n}(a_n)\cdots\alpha^{(k-1)n}(a_n) = 0 \\
\Rightarrow\ & a_n\alpha^n(a_n\alpha^n(a_n)\cdots\alpha^{(k-2)n}(a_n)) = 0 \\
\Rightarrow\ & a_n a_n\alpha^n(a_n)\cdots\alpha^{(k-2)n}(a_n) = 0 \\
\Rightarrow\ & a_n^2\alpha^n(a_n\alpha^n(a_n)\cdots\alpha^{(k-3)n}(a_n)) = 0 \\
\Rightarrow\ & \cdots \Rightarrow a_n^k = 0 \Rightarrow a_n \in nil(R).
\end{aligned}$$

Therefore, by Proposition 2.4, $a_n = 1 \cdot a_n \in nil(R)$ implies $1 \cdot f_s^t(a_n) = f_s^t(a_n) \in nil(R)$ for all $0 \leqslant s \leqslant t$. Let $Q = a_0 + a_1 x + \cdots + a_{n-1}x^{n-1}$. Then we have

$$\begin{aligned}
0 &= (Q + a_n x^n)^k \\
&= (Q + a_n x^n)(Q + a_n x^n)\cdots(Q + a_n x^n) \\
&= (Q^2 + Q\cdot a_n x^n + a_n x^n \cdot Q + a_n x^n \cdot a_n x^n)(Q + a_n x^n)\cdots(Q + a_n x^n) \\
&= \cdots = Q^k + \Delta,
\end{aligned}$$

where $\Delta \in R[x;\alpha,\delta]$. Notice that the coefficients of Δ can be written as sums of monomials in a_i and $f_u^v(a_j)$ where $a_i, a_j \in \{a_0, a_1, \cdots, a_n\}$ and $0 \leqslant u \leqslant v$ are positive integers, and each monomial has a_n or $f_s^t(a_n)$. Since $nil(R)$ is an ideal of R, we obtain that each monomial is in $nil(R)$, and then $\Delta \in nil(R)[x;\alpha,\delta]$. Thus we obtain $(a_0 + a_1x + \cdots + a_{n-1}x^{n-1})^k =$ "lower terms" $+ a_{n-1}\alpha^{n-1}(a_{n-1})\cdots\alpha^{(n-1)(k-1)}(a_{n-1})x^{(n-1)k} \in nil(R)[x;\alpha,\delta]$. Hence by Proposition 2.3 we have

$$\begin{aligned}
& a_{n-1}\alpha^{n-1}(a_{n-1})\cdots\alpha^{(n-1)(k-1)}(a_{n-1}) \in nil(R) \\
\Rightarrow\quad & a_{n-1}\alpha^{n-1}(a_{n-1}\alpha^{n-1}(a_{n-1})\cdots\alpha^{(n-1)(k-2)}(a_{n-1})) \in nil(R) \\
\Rightarrow\quad & a_{n-1}^2\alpha^{n-1}(a_{n-1})\cdots\alpha^{(n-1)(k-2)}(a_{n-1}) \in nil(R) \\
\Rightarrow\quad & a_{n-1}^3\alpha^{n-1}(a_{n-1})\cdots\alpha^{(n-1)(k-3)}(a_{n-1}) \in nil(R) \\
\Rightarrow\quad & \cdots \Rightarrow a_{n-1}^{k-1} \in nil(R) \Rightarrow a_{n-1} \in nil(R).
\end{aligned}$$

Using induction on n we have $a_i \in nil(R)$ for all $0 \leqslant i \leqslant n$.

($\Leftarrow$) Let $f(x) = \sum\limits_{i=0}^{n} a_ix^i \in nil(R)[x;\alpha,\delta]$, where $a_i \in nil(R)$ for all $0 \leqslant i \leqslant n$. Suppose that $a_i^{m_i} = 0$ for $i = 0, 1, 2, \cdots, n$. Putting $k = m_0 + m_1 + \cdots + m_n + 1$, we claim that

$$f(x)^k = (a_0 + a_1x + \cdots + a_nx^n)^k = 0.$$

From

$$\begin{aligned}
(\sum_i^n a_ix^i)^2 =& (\sum_i^n a_ix^i)(\sum_i^n a_ix^i) \\
=& (\sum_i^n a_ix^i)a_0 + (\sum_i^n a_ix^i)a_1x + \cdots + (\sum_i^n a_ix^i)a_sx^s + \cdots + (\sum_i^n a_ix^i)a_nx^n \\
=& \sum_{i=0}^n a_if_0^i(a_0) + (\sum_{i=1}^n a_if_1^i(a_0))x + \cdots + (\sum_{i=s}^n a_if_s^i(a_0))x^s + \cdots + a_n\alpha^n(a_0)x^n \\
& + (\sum_{i=0}^n a_if_0^i(a_1) + (\sum_{i=1}^n a_if_1^i(a_1))x + \cdots + a_n\alpha^n(a_1)x^n)x \\
& + \cdots + (\sum_{i=0}^n a_if_0^i(a_s) + (\sum_{i=1}^n a_if_1^i(a_s))x + \cdots + a_n\alpha^n(a_s)x^n)x^s \\
& + \cdots + (\sum_{i=0}^n a_if_0^i(a_n) + (\sum_{i=1}^n a_if_1^i(a_n))x + \cdots + a_n\alpha^n(a_n)x^n)x^n \\
=& \sum_{i=0}^n a_if_0^i(a_0) + (\sum_{i=1}^n a_if_1^i(a_0) + \sum_{i=0}^n a_if_0^i(a_1))x \\
& + \cdots + (\sum_{s+t=k}(\sum_{i=s}^n a_if_s^i(a_t)))x^k + \cdots + a_n\alpha^n(a_n)x^{2n},
\end{aligned}$$

it is easy to check that the coefficients of $(\sum\limits_{i=0}^n a_ix^i)^k$ can be written as sums of monomials of length k in a_i and $f_u^v(a_j)$, where $a_i, a_j \in \{a_0, a_1, \cdots, a_n\}$ and $0 \leqslant u \leqslant v$ are positive integers. Consider each monomial $a_{i_1}f_{s_2}^{t_2}(a_{i_2})\cdots f_{s_p}^{t_p}(a_{i_p})$ where $a_{i_1},a_{i_2},\cdots,a_{i_p}\in\{a_0,a_1,\cdots,a_n\}$, and $t_j,s_j(t_j\geqslant s_j,2\leqslant j\leqslant p)$ are nonnegative integers. We will show that $a_{i_1}f_{s_2}^{t_2}(a_{i_2})\cdots f_{s_p}^{t_p}(a_{i_p})=0$. If the number of a_0 in $a_{i_1}f_{s_2}^{t_2}(a_{i_2})\cdots f_{s_p}^{t_p}(a_{i_p})$ is greater than m_0, then we write $a_{i_1}f_{s_2}^{t_2}(a_{i_2})\cdots f_{s_p}^{t_p}(a_{i_p})$ as

$$b_1(f_{s_{01}}^{t_{01}}(a_0))^{j_1}b_2(f_{s_{02}}^{t_{02}}(a_0))^{j_2}\cdots b_v(f_{s_{0v}}^{t_{0v}}(a_0))^{j_v}b_{v+1},$$

where $j_1+j_2+\cdots+j_v>m_0$, $1\leqslant j_1,j_2,\cdots,j_v$, and $b_q(q=1,2,\cdots,v+1)$ is a product of some elements chosen from $\{a_{i_1},f_{s_2}^{t_2}(a_{i_2}),\cdots,f_{s_p}^{t_p}(a_{i_p})\}$ or is equal to 1. Since $a_0^{j_1+j_2+\cdots+j_v}=0$, by Proposition 2.4 we have

$$\begin{aligned}
&0=a_0^{j_1+j_2+\cdots+j_v}=a_0a_0\cdots a_0\\
\Rightarrow\ &1\cdot f_{s_{02}}^{t_{01}}(a_0)a_0\cdots a_0=0\\
\Rightarrow\ &(f_{s_{01}}^{t_{01}}(a_0))^{j_1}a_0\cdots a_0=0\\
\Rightarrow\ &(f_{s_{01}}^{t_{01}}(a_0))^{j_1}f_{s_{02}}^{t_{02}}(a_0)a_0\cdots a_0=0\\
\Rightarrow\ &(f_{s_{01}}^{t_{01}}(a_0))^{j_1}(f_{s_{02}}^{t_{02}}(a_0))^{j_2}a_0\cdots a_0=0\\
\Rightarrow\ &\cdots\\
\Rightarrow\ &(f_{s_{01}}^{t_{01}}(a_0))^{j_1}(f_{s_{02}}^{t_{02}}(a_0))^{j_2}\cdots(f_{s_{0v}}^{t_{0v}}(a_0))^{j_v}=0.
\end{aligned}$$

By Proposition 2.4, $a_0=1\cdot a_0\in nil(R)$ implies $1\cdot f_s^t(a_0)=f_s^t(a_0)\in nil(R)$ for $0\leqslant s\leqslant t$. Since R is nil-semicommutative, we have

$$\begin{aligned}
&(f_{s_{01}}^{t_{01}}(a_0))^{j_1}(f_{s_{02}}^{t_{02}}(a_0))^{j_2}\cdots(f_{s_{0v}}^{t_{0v}}(a_0))^{j_v}=0\\
\Rightarrow\ &b_1(f_{s_{01}}^{t_{01}}(a_0))^{j_1}(f_{s_{02}}^{t_{02}}(a_0))^{j_2}\cdots(f_{s_{0v}}^{t_{0v}}(a_0))^{j_v}=0\\
\Rightarrow\ &b_1(f_{s_{01}}^{t_{01}}(a_0))^{j_1}b_2(f_{s_{02}}^{t_{02}}(a_0))^{j_2}\cdots(f_{s_{0v}}^{t_{0v}}(a_0))^{j_v}=0\\
\Rightarrow\ &\cdots\\
\Rightarrow\ &b_1(f_{s_{01}}^{t_{01}}(a_0))^{j_1}b_2(f_{s_{02}}^{t_{02}}(a_0))^{j_2}\cdots b_v(f_{s_{0v}}^{t_{0v}}(a_0))^{j_v}b_{v+1}=0.
\end{aligned}$$

Thus $a_{i_1}f_{s_2}^{t_2}(a_{i_2})\cdots f_{s_p}^{t_p}(a_{i_p})=0$. If the number of a_i in $a_{i_1}f_{s_2}^{t_2}(a_{i_2})\cdots f_{s_p}^{t_p}(a_{i_p})$ is greater than m_i, then similar discussion yields that $a_{i_1}f_{s_2}^{t_2}(a_{i_2})\cdots f_{s_p}^{t_p}(a_{i_p})=0$. So each term appeared in $(\sum\limits_{i=0}^{n}a_ix^i)^k$ equal 0. Therefore, $\sum\limits_{i=0}^{n}a_ix^i\in R[x;\alpha,\delta]$ is a nilpotent element.

Corollary 2.6 (see [7], Theorem 3.3) If R is a nil-semicommutative ring, then $nil(R[x])=nil(R)[x]$.

Corollary 2.7 If R is a semicommutative ring, then $nil(R[x])=nil(R)[x]$.

Theorem 2.8 Let R be an (α,δ)-skew Armendariz and (α,δ)-compatible ring. Then R is nil-semicommutative if and only if $R[x;\alpha,\delta]$ is nil-semicommutative.

Proof $(\Rightarrow)$ Suppose that $R[x;\alpha,\delta]$ is nil-semicommutative. Since any subring of nil-semicommutative rings is also nil-semicommutative, thus it is easy to see that R is a nil-semicommutative ring.

($\Leftarrow$) Let $f(x) = \sum\limits_{i=0}^{m} a_i x^i, g(x) = \sum\limits_{j=0}^{n} b_j x^j \in nil(R[x;\alpha,\delta])$. Since R is nil-semicommutative and (α,δ)-compatible, by Theorem 2.5, we have $nil(R[x;\alpha,\delta]) = nil(R)[x;\alpha,\delta]$. So $a_i, b_j \in nil(R)$ for $0 \leqslant i \leqslant m$, $0 \leqslant j \leqslant n$. Suppose $f(x)g(x) = 0$. Since R is (α,δ)-skew Armendariz, we have $a_i x^i b_j x^j = 0$ for $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$. Thus we can obtain $\sum\limits_{k=0}^{i} a_i f_k^i(b_j) x^{k+j} = 0$ and $a_i f_k^i(b_j) = 0$ for $k = 0, 1, \cdots, i$, $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$. Particularly, we have $a_i f_i^i(b_j) = 0$, and hence $a_i b_j = 0$ and $a_i R b_j = 0$ for $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$ since R is (α,δ)-compatible and nil-semicommutative. Thus, for any $h(x) = \sum\limits_{k=0}^{p} c_k x^k \in R[x;\alpha,\delta]$, we have $a_i c_k b_j = 0$ for all $0 \leqslant i \leqslant m$, $0 \leqslant j \leqslant n$ and $0 \leqslant k \leqslant p$. According to the proof of Theorem 2.5, it is easy to check that the coefficients of $f(x)h(x)g(x)$ can be written as sums of monomials $a_i f_{s2}^{t2}(c_k) f_{s3}^{t3}(b_j)$, where $a_i \in \{a_0, a_1, \cdots, a_m\}$, $b_j \in \{b_0, b_1, \cdots, b_n\}$ and $c_k \in \{c_0, c_1, \cdots, c_p\}$, and $t_2 \geqslant s_2$, $t_3 \geqslant s_3$ are nonnegative integers. Then from $a_i c_k b_j = 0$ we obtain that $a_i c_k f_{s3}^{t3}(b_j) = 0$, and hence $a_i f_{s2}^{t2}(c_k) f_{s3}^{t3}(b_j) = 0$ by Proposition 2.2 and Proposition 2.3. Thus, each term appears in $f(x)h(x)g(x)$ is equal 0. So we have $f(x)h(x)g(x) = 0$. Therefore, $R[x;\alpha,\delta]$ is nil-semicommutative.

Corollary 2.9 Let α be an endomorphism of R and δ an α-derivation of R. If R is α-rigid, then R is nil-semicommutative if and only if $R[x;\alpha,\delta]$ is nil-semicommutative.

Corollary 2.10 Let α be an endomorphism of R. If R is α-skew Armendariz and α-compatible, then R is nil-semicommutative if and only if $R[x;\alpha]$ is nil-semicommutative.

Corollary 2.11 Let δ be a derivation of R. If R is δ-skew Armendariz and δ-compatible, then R is nil-semicommutative if and only if $R[x;\delta]$ is nil-semicommutative.

Corollary 2.12 (see [7], Theorem 3.5) If R is skew Armendariz, then R is nil-semicommutative if and only if $R[x]$ is nil-semicommutative.

3 Nil-Semicommutative Rings and Weak (α,δ)-Skew Armendariz Rings

Ouyang and Liu [18] introduced the notion of weak (α,δ)-compatible rings, that is a generalization of α-rigid rings and (α,δ)-compatible rings. For an endomorphism α and α-derivation δ, we say that R is weak α-compatible if for each $a, b \in R$, $ab \in nil(R) \Leftrightarrow a\alpha(b) \in nil(R)$. Moreover, R is weak δ-compatible if for each $a, b \in nil(R)$, $ab \in nil(R) \Rightarrow a\delta(b) \in nil(R)$. If R is both weak α-compatible and weak δ-compatible, we say that R is weak (α,δ)-compatible. Ouyang [18] proved that all (α,δ)-compatible rings are weak (α,δ)-compatible, but the converse does not hold (see [18], Example 2.5). Ouyang [19] called a ring R with an endomorphism α to be weak α-rigid if $a\alpha(a) \in nil(R) \Leftrightarrow a \in nil(R)$ for $a \in R$. Obviously, all weak α-compatible rings are weak α-rigid.

Lemma 3.1 (see [18], Lemma 2.2) Let R be a weak (α,δ)-compatible ring. Then we have the following:

(1) If $ab \in nil(R)$, then $a\alpha^n(b) \in nil(R)$, $\alpha^m(a)b \in nil(R)$ for all positive integers m, n;

(2) If $\alpha^k(a)b \in nil(R)$ for some positive integer k, then $ab \in nil(R)$;

(3) If $a\alpha^s(b) \in nil(R)$ for some positive integer s, then $ab \in nil(R)$;

(4) If $ab \in nil(R)$, then $\alpha^n(a)\delta^m(b) \in nil(R)$, and $\delta^s(a)\alpha^t(b) \in nil(R)$ for all positive integers m, n, s, t.

Lemma 3.2 (see [19], Proposition 2.3) Let R be a weak α-rigid ring and $nil(R)$ be an ideal of R. Then we have the following:

(1) If $ab \in nil(R)$, then $a\alpha^m(b) \in nil(R)$, $\alpha^n(a)b \in nil(R)$ for all positive integers m, n;

(2) If $\alpha^k(a)b \in nil(R)$ for some positive integer k, then ab, $ba \in nil(R)$;

(3) If $a\alpha^t(b) \in nil(R)$ for some positive integer t, then ab, $ba \in nil(R)$.

Proposition 3.3 If $nil(R)$ is an ideal of a ring R, then R is a weak α-compatible ring if and only if R is a weak α-rigid ring.

Proof Obviously, weak α-compatible rings are weak α-rigid. Conversely, if R is a weak α-rigid ring, then from $ab \in nil(R)$ we have $a\alpha(b) \in nil(R)$ and $a\alpha(b) \in nil(R)$, and hence $ab \in nil(R)$ by Lemma 3.2 for all $a, b \in R$. Therefore, R is a weak α-compatible ring.

Proposition 3.4 Let δ be an α-derivation of R. If R is a weak (α, δ)-compatible ring and $nil(R)$ is an ideal of R, then $ab \in nil(R)$ implies $af_i^j(b) \in nil(R)$ for all $0 \leqslant i \leqslant j$ and $a, b \in nil(R)$.

Proof If $ab \in nil(R)$, then $a\alpha^i(b) \in nil(R)$ and $a\delta^j(b) \in nil(R)$ for all $i \geqslant 0$ and $j \geqslant 0$ since R is weak (α, δ)-compatible. It follows that $af_i^j(b) \in nil(R)$ for all $0 \leqslant i \leqslant j$ since $nil(R)$ is an ideal of R.

Alhevaz et al. [20] generalized (α, δ)-skew Armendariz rings by introducing the notion of weak (α, δ)-skew Armendariz rings. Let α be an endomorphism and δ an α-derivation of a ring R. A ring R is called weak (α, δ)-skew Armendariz ring, if for polynomials $f(x) = \sum\limits_{i=0}^{m} a_i x^i$, $g(x) = \sum\limits_{j=0}^{n} b_j x^j \in R[x; \alpha, \delta]$, $f(x)g(x) = 0$ implies $a_i x^i b_j x^j \in nil(R)[x; \alpha, \delta]$ for all i, j. Obviously, all (α, δ)-skew Armendariz rings are weak (α, δ)-skew Armendariz.

Theorem 3.5 Let R be a weak (α, δ)-compatible ring and $nil(R)$ is an ideal of R, then R is a weak (α, δ)-skew Armendariz ring.

Proof Suppose that $f(x) = \sum\limits_{i=0}^{m} a_i x^i$, $g(x) = \sum\limits_{j=0}^{n} b_j x^j \in R[x; \alpha, \delta]$ such that $f(x)g(x) = 0$. From

$$\begin{aligned}
f(x)g(x) &= (\sum_{i=0}^{m} a_i x^i)(\sum_{j=0}^{n} b_j x^j) \\
&= (\sum_{i=0}^{m} a_i x^i) b_0 + (\sum_{i=0}^{m} a_i x^i) b_1 x + \cdots + (\sum_{i=0}^{m} a_i x^i) b_n x^n \\
&= \sum_{i=0}^{m} a_i f_0^i(b_0) + (\sum_{i=1}^{m} a_i f_1^i(b_0))x + \cdots + (\sum_{i=s}^{m} a_i f_s^i(b_0))x^s + \cdots + a_m \alpha^m(b_0) x^m \\
&\quad + (\sum_{i=0}^{m} a_i f_0^i(b_1) + \sum_{i=1}^{m} a_i f_1^i(b_1)x + \cdots + \sum_{i=s}^{m} a_i f_s^i(b_1)x^s + \cdots + a_m \alpha^m(b_1) x^m)x \\
&\quad + \cdots + (\sum_{i=0}^{m} a_i f_0^i(b_n) + \cdots + (\sum_{i=s}^{m} a_i f_s^i(b_1))x^s + \cdots + a_m \alpha^m(b_n) x^m)x^n
\end{aligned}$$

$$= \sum_{i=0}^{m} a_i f_0^i(b_0) + (\sum_{i=1}^{m} a_i f_1^i(b_0) + \sum_{i=0}^{m} a_i f_0^i(b_1))x + \cdots + (\sum_{s+t=k} (\sum_{i=s}^{m} a_i f_s^i(b_t)))x^k$$
$$+ \cdots + a_m \alpha^m(b_n)x^{m+n} = 0.$$

We have the following system of equations

$$\Delta_{m+n} = a_m \alpha^m(b_n) = 0, \quad (1)$$

$$\Delta_{m+n-1} = a_m \alpha^m(b_{n-1}) + a_{m-1}\alpha^{m-1}(b_n) + a_m f_{m-1}^m(b_n) = 0, \quad (2)$$

$$\Delta_{m+n-2} = a_m \alpha^m(b_{n-2}) + \sum_{i=m-1}^{m} f_{m-1}^i(b_{n-1}) + \sum_{i=m-2}^{m} f_{m-2}^i(b_n) = 0, \quad (3)$$

$$\vdots$$

$$\Delta_k = \sum_{s+t=k} (\sum_{i=s}^{m} a_i f_s^i(b_t)) = 0. \quad (4)$$

By eq. (1), we have $a_m \alpha^m(b_n) = 0 \in nil(R)$, it implies $a_m b_n \in nil(R)$ since R is weak α-compatible. By Proposition 3.4, we obtain $a_m f_s^t(b_n) = 0$ for all $0 \leqslant s \leqslant t$. For eq. (2), we have

$$\Delta'_{m+n-1} = a_m \alpha^m(b_{n-1}) + a_{m-1}\alpha^{m-1}(b_n) \in nil(R). \quad (5)$$

If we multiply eq. (5) on the left side by b_n, then we obtain

$$b_n a_{m-1}\alpha^{m-1}(b_n) = b_n \Delta'_{m+n-1} - b_n a_m \alpha^m(b_{n-1}) \in nil(R).$$

By $b_n a_m \in nil(R)$, we have $b_n a_{m-1}\alpha^{m-1}(b_n) = -b_n a_m \alpha^m(b_{n-1}) \in nil(R)$ because the $nil(R)$ is an ideal of R, and hence $b_n a_{m-1} b_n \in nil(R)$ since R is weak α-compatible. So $b_n a_{m-1} \in nil(R)$, $a_{m-1} b_n \in nil(R)$ and $a_{m-1}\alpha^{m-1}(b_n) \in nil(R)$. Thus, we have $a_m \alpha^m(b_{n-1}) \in nil(R)$ and $a_m b_{n-1} \in nil(R)$. Therefore, we have obtained $a_m b_{n-1}$, $a_{m-1} b_n \in nil(R)$. By Proposition 3.4 and eq. (3), we have

$$\begin{aligned} \Delta_{m+n-2} &= a_m \alpha^m(b_{n-2}) + a_{m-1}\alpha^{m-1}(b_{n-1}) + a_m f_{m-1}^m(b_{n-1}) \\ &\quad + a_{m-2}\alpha^{m-2}(b_n) + a_{m-1} f_{m-2}^{m-1}(b_n) + a_m f_{m-2}^m(b_n) = 0. \end{aligned}$$

It follows that

$$\Delta'_{m+n-2} = a_m \alpha^m(b_{n-2}) + a_{m-1}\alpha^{m-1}(b_{n-1}) + a_{m-2}\alpha^{m-2}(b_n) \in nil(R). \quad (6)$$

If we multiply eq. (6) on the left side by b_n, b_{n-1}, b_{n-2}, respectively, then we obtain $a_{m-2} b_n \in nil(R)$, $a_{m-1} b_{n-1} \in nil(R)$ and $a_m b_{n-2} \in nil(R)$ in turn. Continuing this procedure yields that $a_i b_j \in nil(R)$ for all i, j. Next we consider

$$a_i x^i b_j x^j = a_i(\sum_{k=0}^{i} f_k^i(b_j)x^k)x^j = \sum_{k=0}^{i} a_i f_k^i(b_j)x^{k+j}.$$

Since R is a weak (α,δ)-compatible ring and $nil(R)$ is an ideal of R, by Proposition 3.4, $a_ib_j \in nil(R)$ implies $a_if_k^i(b_j) \in nil(R)$ for all $0 \leqslant k \leqslant i$. Thus, we have

$$a_ix^ib_jx^j = \sum_{k=0}^{i} a_if_k^i(b_j)x^{k+j} \in nil(R)[x;\alpha,\delta].$$

Therefore, R is weak (α,δ)-skew Armendariz.

Corollary 3.6 (see [20], Theorem 3.6) Every (α,δ)-compatible semicommutative ring is weak (α,δ)-skew Armendariz.

Corollary 3.7 (see [19], Theorem 3.3) Let R be a weak α-rigid ring and $nil(R)$ is an ideal of R. Then R is a weak α-skew Armendariz ring.

Corollary 3.8 Let R be a weak δ-compatible ring and $nil(R)$ is an ideal of R. Then R is a weak δ-skew Armendariz ring.

Corollary 3.9 If $nil(R)$ is an ideal of a ring R, then R is a weak Armendariz ring.

Let α be an endomorphism and δ an α-derivation of a ring R. Then the map $\bar{\alpha}: R[x] \to R[x]$ defined by $\bar{\alpha}(\sum\limits_{i=0}^{m} a_ix^i) = \sum\limits_{i=0}^{m} \alpha(a_i)x^i$ is an endomorphism of the polynomial ring $R[x]$, and the α-derivation δ of R is extended to $\bar{\delta}: R[x] \to R[x]$, defined by

$$\bar{\delta}(\sum_{i=0}^{m} a_ix^i) = \sum_{i=0}^{m} \delta(a_i)x^i.$$

Then $\bar{\delta}$ is an $\bar{\alpha}$-derivation of $R[x]$.

Antoine [21] called a ring R to be nil-Armendariz if whenever two polynomials

$$f(x) = \sum_{i=0}^{m} a_ix^i, \quad g(x) = \sum_{j=0}^{n} b_jx^j \in R[x]$$

satisfy $f(x)g(x) \in nil(R)[x]$ then $a_ib_j \in nil(R)$ for all $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$. Mohammadi et al. [7] proved that nil-semicommutative rings are nil-Armendariz.

Theorem 3.10 If R is a weak (α,δ)-compatible and nil-semicommutative ring, then $R[x]$ is weak $(\bar{\alpha},\bar{\delta})$-skew Armendariz.

Proof Let $F(y) = \sum\limits_{i=0}^{m} f_iy^i$, $G(y) = \sum\limits_{j=0}^{n} g_jy^j \in R[x][y;\bar{\alpha},\bar{\delta}]$ such that $F(y)G(y) = 0$, where

$$f_i = \sum_{s=0}^{p_i} a_{is}x^s, \quad g_j = \sum_{t=0}^{q_j} b_{jt}x^t \in R[x].$$

Put $k = \sum\limits_{i=0}^{m} deg(f_i) + \sum\limits_{j=0}^{n} deg(g_j)$, where $deg(h_j)$ is the degree of an polynomial $h(x)$ in x and the degree of zero polynomial is taken to be 0. Then $F(x^k) = \sum\limits_{i=0}^{m} f_ix^{ik}$ and $F(x^k)G(x^k) = 0$ in $R[x]$. Because R is nil-semicommutative, R is nil-Armendariz by Corollary 2.9 of [7]. Thus $F(x^k)G(x^k) = 0 \in nil(R)[x]$ can imply $a_{is}b_{jt} \in nil(R)$ for all $0 \leqslant i \leqslant m$, $0 \leqslant j \leqslant n$,

$0 \leqslant s \leqslant p_i$ and $0 \leqslant t \leqslant q_j$. By Proposition 2.4 of [19], $\alpha(1) = 1$ and $\delta(1) = 0$ since R is weak α-compatible nil-semicommutative. So we have $xy = yx$. Next we consider

$$\begin{aligned} f_i y^i g_j y^j &= (\sum_{s=0}^{p_i} a_{is} x^s) y^i (\sum_{t=0}^{q_j} b_{jt} x^t) y^j \\ &= (\sum_{s=0}^{p_i} a_{is} y^i x^s)(\sum_{t=0}^{q_j} b_{jt} y^j x^t) \\ &= \sum_{l=0}^{p_i+q_j} (\sum_{s_1+s_2=l} a_{is_1} y^i b_{js_2} y^j) x^l \\ &= \sum_{l=0}^{p_i+q_j} (\sum_{s_1+s_2=l} a_{is_1} (\sum_{t_1=0}^{i} f_{t_1}^i (b_{js_2}) y^{t_1}) y^j) x^l \\ &= \sum_{l=0}^{p_i+q_j} \sum_{s_1+s_2=l} \sum_{t_1=0}^{i} a_{is_1} f_{t_1}^i (b_{js_2}) y^{t_1+j} x^l \\ &= \sum_{t_1=0}^{i} (\sum_{l=0}^{p_i+q_j} (\sum_{s_1+s_2=l} a_{is_1} f_{t_1}^i (b_{js_2})) x^l) y^{t_1+j}. \end{aligned}$$

Since R is a weak (α, δ)-compatible nil-semicommutative ring and $a_{is_1} b_{js_2} \in nil(R)$ implies $a_{is_1} f_{t_1}^i (b_{js_2}) \in nil(R)$, we have $\sum_{s_1+s_2=l} a_{is_1} f_{t_1}^i (b_{js_2}) \in nil(R)$ for all $0 \leqslant l \leqslant p_i + q_j$ by Proposition 3.4. Thus, $\sum_{l=0}^{p_i+q_j} (\sum_{s_1+s_2=l} a_{is_1} f_{t_1}^i (b_{js_2})) x^l \in nil(R)[x]$. Furthermore, we have

$$\sum_{l=0}^{p_i+q_j} (\sum_{s_1+s_2=l} a_{is_1} f_{t_1}^i (b_{js_2})) x^l \in nil(R[x])$$

by Theorem 3.3 of [7]. This implies that

$$f_i y^i g_j y^j = \sum_{t_1=0}^{i} (\sum_{l=0}^{p_i+q_j} (\sum_{s_1+s_2=l} a_{is_1} f_{t_1}^i (b_{js_2})) x^l) y^{t_1+j} \in nil(R[x])[y; \bar{\alpha}, \bar{\delta}]$$

for each $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$. Therefore, $R[x]$ is weak $(\bar{\alpha}, \bar{\delta})$-skew Armendariz.

Corollary 3.11 (see [20], Theorem 3.11 (ii)) Let R be a semicommutative (α, δ)-compatible ring. Then $R[x]$ is weak $(\bar{\alpha}, \bar{\delta})$-skew Armendariz.

Corollary 3.12 (see [7], Theorem 3.13) If R is a weak α-rigid and nil-semicommutative ring, then $R[x]$ is a weak $\bar{\alpha}$-skew Armendariz ring.

Corollary 3.13 If R is a weak δ-compatible and nil-semicommutative ring, then $R[x]$ is a weak $\bar{\delta}$-skew Armendariz ring.

Corollary 3.14 (see [7], Theorem 3.7) If R is a nil-semicommutative ring, then $R[x]$ is a weak Armendariz ring.

Theorem 3.15 Let R be a nil-semicommutative (α, δ)-compatible ring. Then $R[x; \alpha, \delta]$ is weak Armendariz.

Proof Let

$$F(y)=\sum_{i=0}^{m} f_i y^i, \quad G(y)=\sum_{j=0}^{n} g_j y^j \in R[x;\alpha,\delta][y]$$

such that $F(y)G(y)=0$, where

$$f_i=\sum_{s=0}^{p_i} a_{is}x^s, \quad g_j=\sum_{t=0}^{q_j} b_{jt}x^t \in R[x;\alpha,\delta].$$

Put $k=\sum\limits_{i=0}^{m}\deg(f_i)+\sum\limits_{j=0}^{n}\deg(g_j)$, where $\deg(g_j)$ is the degree of an polynomial $h(x)$ in x and the degree of zero polynomial is taken to be 0. Then

$$F(x^k)=\sum_{i=0}^{m} f_i x^{ik}, \quad G(x^k)=\sum_{j=0}^{n} g_j x^{jk} \in R[x;\alpha,\delta]$$

and $F(x^k)G(x^k)=0\in R[x;\alpha,\delta]$. By Theorem 3.5, we have $a_{is}x^i b_{jt}x^j \in nil(R)[x;\alpha,\delta]$. Since R is nil-semicommutative (α,δ)-compatible, we obtain that $a_{is}x^i b_{jt}x^j \in nil(R[x;\alpha,\delta])$ and then $f_ig_j\in nil(R[x;\alpha,\delta])$ by Theorem 2.5. Therefore, $R[x;\alpha,\delta]$ is weak Armendariz.

Corollary 3.16 (see [20], Theorem 3.11(i)) Let R be a semicommutative (α,δ)-compatible ring. Then $R[x;\alpha,\delta]$ is weak Armendariz.

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诣零半交换环上的Ore 扩张

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摘要: 本文研究诣零半交换环上的 Ore 扩张环的性质. 利用对多项式的逐项分析方法, 我们证明了: 设 α 是环 R 上的一个自同态, δ 是环 R 上的一个 α -导子. 如果 R 是 (α,δ) -斜 Armendariz 的 (α,δ)-compatible 环, 则 $R[x;\alpha,\delta]$ 是诣零半交换环当且仅当环 R 是诣零半交换环; 如果 R 是诣零半交换的 (α,δ)-compatible 环, 则 $R[x;\alpha,\delta]$ 是斜Armendariz 环. 所得结果推广了近期关于斜多项式环的相关结论.

关键词: 诣零半交换环; Ore 扩张; (α,δ)-compatible 环; 弱(α,δ)-compatible 环; (α,δ) -斜Armendari 环; 弱(α,δ) -斜Armendari 环

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