

A NUMERICAL METHOD FOR FRACTIONAL DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS

LI Bao-feng

(*Dep. of Math and Info. Sci., Tangshan Normal University, Tangshan 063000, China*)

Abstract: Here is to study the numerical solution of multi-order fractional differential equations (FDEs) with variable coefficients. We derive the operational matrix of fractional integration based on the Chebyshev wavelets. The operational matrix of fractional integration is utilized to reduce the fractional differential equations to a system of algebraic equations. In addition, the convergence of the Chebyshev wavelet bases and the error estimation expression are presented. A numerical example is provided to demonstrate the accuracy and efficiency of the proposed method.

Keywords: fractional integration; the Chebyshev wavelets; operational matrix; fractional differential equations; block pulse function

2010 MR Subject Classification: 41A35

Document code: A

Article ID: 0255-7797(2015)06-1353-10

1 Introduction

During the past decades, the field of fractional differential equations attracted the interest of researchers in several areas including physics, chemistry, engineering and even finance and social sciences (see [1, 2]) and there was significant interest in developing numerical schemes for their solution. These methods include Laplace transforms (see [3]), Fourier transforms (see [4]), eigenvector expansion (see [5]), adomian decomposition method (ADM) (see [6, 7]), variation iteration method (VIM)(see [8, 9]), fractional differential transform method (FDTM) (see [10, 11]), fractional difference method (FDM) (see [12]) and power series method (see [13]). But, few papers were reported to solve the fractional order differential equations by application of wavelets (see [14, 15]).

We intend to use the Chebyshev wavelet method to solve multi-order arbitrary differential equations with variable coefficients. First, we construct the Chebyshev wavelets and derive the operational matrix of fractional integration; then the underlying fractional differential equation is converted to an algebraic equation by introducing the wavelet operational

* **Received date:** 2013-05-03

Accepted date: 2014-05-30

Foundation item: Supported by the Natural Foundation of Hebei Province(A2012203047) and Natural Science foundation of Tangshan Normal University(2014D09).

Biography: Li Baofeng(1971-), male, born at Tangshan, Hebei, associate professor, major in computational mathematics.

matrix of the fractional integration. In this paper, by using the Chebyshev wavelets, we solve the following multi-order fractional differential equations (FDEs) with variable coefficients

$$D_*^v u(t) + \sum_{i=1}^{r-1} \gamma_i(t) D_*^{\beta_i} u(t) + \gamma_r(t) u(t) = g(t) \quad , t \in [0, 1), \quad (1.1)$$

$$u^{(i)}(0) = d_i, \quad i = 0, 1, \dots, n-1, \quad (1.2)$$

where $0 < \beta_1 < \beta_2 < \dots < \beta_{r-1} < v$, $n-1 < v \leq n$ are constants. Moreover, D_*^v denotes the Caputo fractional derivative of order v , the values of $d_i (i = 0, 1, \dots, n-1)$ describe the initial state of $u(t)$ and $g(t)$ is a known function. The existence and uniqueness of solutions of FDEs were studied by [16].

The paper is organized as follows. In Section 2, we introduce some necessary definitions and mathematical preliminaries of fractional calculus. In Section 3, after describing the basic formulation of wavelets and the Chebyshev wavelets, we derive the Chebyshev wavelet operational matrix of the fractional differential equation. In Section 4, the method is defined for approximate solution of the fractional problem (1.1) and (1.2). In Section 5, the error analysis technique based on the residual function is developed for the present method. A numerical example is given to demonstrate the validity of the method in solving fractional differential equation in Section 6. Section 7 comments on the result.

2 Definitions and Notations

We give some necessary definitions and mathematical preliminaries of the fractional calculus theory which are used further in this paper.

Definition 2.1 The Riemann-Liouville fractional integral operator I^α of order $\alpha (\alpha > 0)$ on usual Lebesgue space $L^2[a, b]$ is given by

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad t > a, \quad (2.1)$$

$$I^0 f(t) = f(t), \quad (2.2)$$

and its fractional derivative of order $\alpha > 0$ is normally used:

$$D^\alpha f(t) = \frac{d^n}{dt^n} (I^{n-\alpha} f(t)), \quad n-1 < \alpha \leq n, \quad (2.3)$$

where n is an integer. For Riemann-Liouville definition, one has

$$I^\alpha t^\nu = \frac{\Gamma(\nu+1)}{\Gamma(\alpha+\nu+1)} t^{\alpha+\nu}. \quad (2.4)$$

The Riemann-Liouville derivative has certain disadvantages when trying to model real-world phenomena with fractional differential equations. Therefore, we shall introduce now a modified fractional differential operator D_*^α proposed by Caputo.

Definition 2.2 The Caputo definition of fractional differential operator is given by

$$D_*^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau, \quad n-1 < \alpha \leq n, \quad (2.5)$$

where n is an integer.

It has the following basic properties for $n-1 < \alpha \leq n$ and $f \in L^2[a, b]$,

$$D_*^\alpha I^\alpha f(t) = f(t), \quad (2.6)$$

$$I^\alpha D_*^\alpha f(t) = f(t) - \sum_{k=0}^{n-1} f^{(k)}(a^+) \frac{t^k}{k!}, \quad t > a, \quad (2.7)$$

$$D_*^\beta f(t) = I^{v-\beta} D_*^v f(t). \quad (2.8)$$

3 The Chebyshev Wavelets and Operational Matrix of the Fractional Integration

In this section, we use the Chebyshev polynomials to construct the Chebyshev wavelets and give some properties of the wavelets.

3.1 The Chebyshev Wavelets and Operational Matrix of the Fractional Integration

Wavelets are a family of functions constructed from dilation and translation of a single function $\psi(t)$ called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously we have the following family of continuous wavelets as [17]:

$$\psi_{ab}(t) = |a|^{-\frac{1}{2}} \psi\left(\frac{t-b}{a}\right), \quad a, b \in R, a \neq 0. \quad (3.1)$$

If we restrict the parameters a and b to discrete values as $a = a_0^{-k}$, $b = nb_0 a_0^{-k}$, $a_0 > 0$, $b_0 > 0$, where n and k are positive integers, the family of discrete wavelets are defined as

$$\psi_{kn}(t) = |a_0|^{\frac{k}{2}} \psi(a_0^k t - nb_0), \quad (3.2)$$

where ψ_{kn} form wavelet bases for $L^2(R)$. In particular, when $a_0 = 2$ and $b_0 = 1$, ψ_{kn} form orthogonal bases.

The Chebyshev wavelets $\psi_{nm}(t) = \psi(k, n, m, t)$, which are defined on the interval $[0, 1]$, involve four arguments. That is, k is assumed any positive integer, $n = 1, 2, \dots, 2^{k-1}$, m is the degree of the Chebyshev polynomials and t is the normalized time. They are defined on the interval $[0, 1]$ as

$$\psi_{nm}(t) = \begin{cases} 2^{\frac{k}{2}} \tilde{T}_m(2^k t - 2n + 1), & \frac{n-1}{2^{k-1}} \leq t < \frac{n}{2^{k-1}}, \\ 0, & \text{otherwise,} \end{cases} \quad (3.3)$$

where

$$\begin{aligned}\tilde{T}_0 &= \frac{1}{\sqrt{\pi}}, \\ \tilde{T}_m(t) &= \sqrt{\frac{2}{\pi}} T_m(t), m = 1, 2, \dots, M-1,\end{aligned}\tag{3.4}$$

here $T_m(t)$ are the Chebyshev polynomials of degree m which respect to the weight function $\omega(t) = 1/\sqrt{1-t^2}$ on interval $[-1, 1]$ and satisfy the following recursive formula

$$T_0(t) = 1, T_1(t) = t, T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t), m = 1, 2, \dots.\tag{3.5}$$

A function $f(t)$ defined on the interval $[0, 1]$ may be expanded as

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{nm} \psi_{nm}(t),\tag{3.6}$$

where

$$c_{nm} = (f(t), \psi_{nm}(t))_{\omega_n} = \int_0^1 \omega_n(t) \psi_{nm}(t) f(t) dt.\tag{3.7}$$

If the infinite series in eq. (3.6) is truncated, then eq. (3.6) can be written as

$$f(t) \approx \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{nm} \psi_{nm}(t) = C^T \Psi(t),\tag{3.8}$$

where T indicates transposition, C and $\Psi(t)$ are $2^{k-1}M$ dimensional column vectors given by

$$C = [c_{10}, c_{11}, \dots, c_{1(M-1)}, c_{20}, \dots, c_{2(M-1)}, \dots, c_{2^{k-1}0}, \dots, c_{2^{k-1}(M-1)}]^T,\tag{3.9}$$

$$\Psi(t) = [\psi_{10}(t), \dots, \psi_{1(M-1)}(t), \psi_{20}(t), \dots, \psi_{2(M-1)}(t), \dots, \psi_{2^{k-1}0}(t), \dots, \psi_{2^{k-1}(M-1)}(t)]^T.\tag{3.10}$$

Taking the collocation points as following

$$t_i = \frac{2i-1}{2^k M}, i = 1, 2, \dots, 2^{k-1}M.\tag{3.11}$$

We define the Chebyshev wavelet matrix $\Phi_{m' \times m'}$ as

$$\Phi_{m' \times m'} = [\Psi(t_1), \Psi(t_2), \dots, \Psi(t_{m'})],\tag{3.12}$$

where $m' = 2^{k-1}M$.

3.2 Operational Matrix of the Fractional Integration

The integration of the vector $\Psi(t)$ defined in eq. (3.10) can be obtained as

$$\int_0^1 \Psi(t) dt \approx P \Psi(t),\tag{3.13}$$

where P is the $2^{k-1}M \times 2^{k-1}M$ operational matrix for integration [17].

Our purpose is to derive the Chebyshev wavelet operational matrix of the fractional integration. For this purpose, we rewrite Riemann-Liouville fractional integration, as following

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau = \frac{1}{\Gamma(\alpha)} t^{\alpha-1} * f(t), t > 0. \quad (3.14)$$

Now, if $f(t)$ is expanded in the Chebyshev wavelets, as showed in eq. (3.8), the Riemann-Liouville fractional integration becomes

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} t^{\alpha-1} * f(t) \approx C^T \frac{1}{\Gamma(\alpha)} \{t^{\alpha-1} * \Psi(t)\}. \quad (3.15)$$

Thus if $t^{\alpha-1} * f(t)$ can be integrated, then expanded in the Chebyshev wavelets, the Riemann-Liouville fractional integration is solved.

Also, we define a m' -set of Block Pulse function (BPF) as

$$b_i(t) = \begin{cases} 1, & \frac{i}{m'} \leq t < \frac{i+1}{m'}, \\ 0, & \text{otherwise,} \end{cases} \quad (3.16)$$

where $i = 0, 1, 2, \dots, m' - 1$.

The functions $b_i(t)$ are disjoint and orthogonal. That is

$$b_i(t)b_l(t) = \begin{cases} b_i(t), & i = l, \\ 0, & i \neq l, \end{cases} \quad (3.17)$$

$$\int_0^1 b_i(t)b_l(t) dt = \begin{cases} \frac{1}{m'}, & i = l, \\ 0, & i \neq l. \end{cases} \quad (3.18)$$

From the orthogonality property of BPF, it is possible to expand functions into their Block Pulse series. This means that for every $f(t) \in [0, 1)$ we can write

$$f(t) \approx \sum_{i=0}^{m'-1} f_i b_i(t) = f^T B_{m'}(t), \quad (3.19)$$

where

$$f^T = [f_0, f_1, \dots, f_{m'-1}], \quad B_{m'}^T(t) = [b_0(t), b_1(t), \dots, b_{m'-1}(t)], \quad (3.20)$$

such that f_i for $i = 0, 1, 2, \dots, m' - 1$ are obtained by $f_i = m' \int_0^1 f(t)b_i(t)dt$.

Similarly, the Chebyshev wavelets may be expanded into an m' -term block pulse functions (BPF) as

$$\Psi(t) \approx \Phi_{m' \times m'} B_{m'}(t). \quad (3.21)$$

We derive the Block Pulse operational matrix of the fractional integration F^α as following

$$I^\alpha B_{m'}(t) = F^\alpha B_{m'}(t), \quad (3.22)$$

where

$$F^\alpha = \frac{1}{2^\alpha m'^\alpha \Gamma(\alpha + 1)} \begin{bmatrix} 1 & \xi_1 & \xi_2 & \xi_3 & \cdots & \xi_{m'-1} \\ 0 & 1 & \xi_1 & \xi_2 & \cdots & \xi_{m'-2} \\ 0 & 0 & 1 & \xi_1 & \cdots & \xi_{m'-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \xi_1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.23)$$

with $\xi_k = (2k + 1)^\alpha - (2k - 1)^\alpha$, $k = 1, 2, \dots, m' - 1$.

Next, we derive the Chebyshev wavelet operational matrix of the fractional integration. Let

$$I^\alpha \Psi(t) \approx P_{m' \times m'}^\alpha \Psi(t). \quad (3.24)$$

Using eqs. (3.21), (3.22), we have

$$I^\alpha \Psi(t) \approx I^\alpha \Phi_{m' \times m'} B_{m'}(t) = \Phi_{m' \times m'} I^\alpha B_{m'}(t) \approx \Phi_{m' \times m'} F^\alpha B_{m'}(t). \quad (3.25)$$

From eqs. (3.24), (3.25), we get

$$P_{m' \times m'}^\alpha \Psi(t) \approx \Phi_{m' \times m'} F^\alpha B_{m'}(t). \quad (3.26)$$

Then from eqs. (3.21), (3.26), the Chebyshev wavelet operational matrix of the fractional integration $P_{m' \times m'}^\alpha$ is given by

$$P_{m' \times m'}^\alpha = \Phi_{m' \times m'} F^\alpha \Phi_{m' \times m'}^{-1}. \quad (3.27)$$

It should be noted that the operational matrix $P_{m' \times m'}^\alpha$ contains many zero entries. This phenomenon makes calculations fast. The calculation for the matrix $P_{m' \times m'}^\alpha$ is carried out once and is used to solve fractional order as well as integer order differential equations.

4 Solution of Eqs. (1.1) and (1.2)

By approximating the function $D_*^v u(t)$, we have

$$D_*^v u(t) u(t) \approx C^T \Psi(t) \quad (4.1)$$

together with the initial states, we get

$$D_*^{\beta_i} u(t) \approx C^T P_{m' \times m'}^{v-\beta_i} \Psi(t), i = 1, 2, \dots, v-1, \quad (4.2)$$

$$u(t) \approx C^T P_{m' \times m'}^v \Psi(t) + \sum_{k=0}^{n-1} u^{(k)}(0) \frac{t^k}{k!}. \quad (4.3)$$

Substituting eqs. (4.1), (4.2) and (4.3) into eq. (1.1), we have

$$C^T \Psi(t) + \sum_{i=1}^{r-1} \gamma_i(t) C^T P_{m' \times m'}^{v-\beta_i} \Psi(t) + \gamma_r(t) (C^T P_{m' \times m'}^v \Psi(t) + \sum_{k=0}^{n-1} u^{(k)}(0) \frac{t^k}{k!}) = g(t). \quad (4.4)$$

Coefficient $\gamma_i(t) (i = 1, 2, \dots, r)$ can be dispersed into $\gamma_i(t_l)$ and $g(t)$ may be dispersed into $g(t_l) (l = 1, 2, \dots, m')$. Let

$$R_i = \begin{bmatrix} \gamma_i(t_1) & 0 & \cdots & 0 \\ 0 & \gamma_i(t_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_i(t_{m'}) \end{bmatrix} \quad (i = 1, 2, \dots, r), \quad (4.5)$$

$$g = \left[g(t_1) - \gamma_r(t_1) \sum_{k=0}^{n-1} u^{(k)}(0) \frac{t_1^k}{k!} \quad \cdots \quad g(t_{m'}) - \gamma_r(t_{m'}) \sum_{k=0}^{n-1} u^{(k)}(0) \frac{t_{m'}^k}{k!} \right]^T. \quad (4.6)$$

Therefore, eq. (4.4) can be written as

$$C^T \left(\Phi_{m' \times m'} + \sum_{i=1}^r P_{m' \times m'}^{v-\beta_i} \Phi_{m' \times m'} R_i \right) = g^T. \quad (4.7)$$

Eq. (4.7) is a linear system of algebraic equations.

5 Convergence of the Chebyshev Wavelet Bases

Theorem 5.1 (see [18]) Let A function $f(t)$ define on the interval $[0, 1]$ may be expanded as

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{nm} \psi_{nm}(t), \quad \tilde{f}(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{nm} \psi_{nm}(t), \quad (5.1)$$

then

$$\sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{nm}^2 \leq \int_0^1 f^2(t) dt, \quad (5.2)$$

where

$$c_{nm} = (f(t), \psi_{nm}(t))_{\omega_n} = \int_0^1 \omega_n(t) \psi_{nm}(t) f(t) dt. \quad (5.3)$$

Theorem 5.2 (see [18]) Let A function $f(t)$ be $L^2[0, 1]$, and

$$R_{K,M} = f(t) - \tilde{f}(t), \quad (5.4)$$

then

$$\lim_{K, M \rightarrow \infty} \|R_{K,M}\| = 0, \quad (5.5)$$

where $f(t), \tilde{f}(t)$ be defined as above and $K = 2^{k-1}$.

Also, we present an error estimation for eq. (5.4) with the residual error function. Let $u(t), \tilde{u}(t)$ be the exact solution and the Chebyshev wavelet solution of the problem (1.1) and (1.2), $R(t) = u(t) - \tilde{u}(t)$. Therefore, $R(t)$ satisfies the following problem

$$D_*^v R(t) + \sum_{i=1}^{r-1} \gamma_i(t) D_*^{\beta_i} R(t) + \gamma_r(t) R(t) = g(t) - h(t), \quad (5.6)$$

$$R^{(i)}(0) = 0, i = 0, 1, \dots, n-1, \quad (5.7)$$

where $h(t) = C^T \Psi(t) + \sum_{i=1}^{r-1} \gamma_i(t) C^T P_{m' \times m'}^{v-\beta_i} \Psi(t) + \gamma_r(t) C^T P_{m' \times m'}^{\alpha} \Psi(t)$.

By solving the error problem (5.6)–(5.7) in the same way as Section 4, we get the approximation

$$\tilde{R}(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{nm}^* \psi_{nm}(t) \quad (5.8)$$

to $R(t)$. Here, the coefficients $c_{nm}^*, n = 1, 2, \dots, 2^{k-1}, m = 0, 1, 2, \dots, M-1$ are determined by solving the error problem (5.6)–(5.7). Hence, the maximum absolute error can be estimated approximately by using $\max |\tilde{R}(t)|$. If the exact solution of the problem is not known, this error estimation can be used to test the reliability of the results.

6 Numerical Examples

Consider the equation

$$D^2 u(t) + \sin(t) D_*^{\frac{1}{2}} u(t) + t u(t) = f(t), u(0) = u'(0) = 0, \quad (6.1)$$

where $f(t) = t^9 - t^8 + 56t^6 - 42t^5 + \sin(t) \left(\frac{32768}{6435} t^{\frac{15}{2}} - \frac{2048}{429} t^{\frac{13}{2}} \right)$.

One can easily check that $u(t) = t^8 - t^7$ is the unique analytical solution. The comparison between approximate and exact solution for $k = 3, M = 5$ is presented in Fig.1 and we list the absolute errors for $M = 3$ and different values of k and Ref. [Li and Zhao (2010)] (see[19]). From Table 1, we can achieve a better approximation by the Chebyshev wavelet method than Ref. [Li and Zhao (2010)]. We may also see that the error is smaller and smaller when k increases. Therefore for better results, using a larger k is recommended. The computational results show that the method in this article can be effectively used in numerical calculus for fractional differential equation with variable coefficient, and the method is also feasibility to the realistic fractional differential equation.

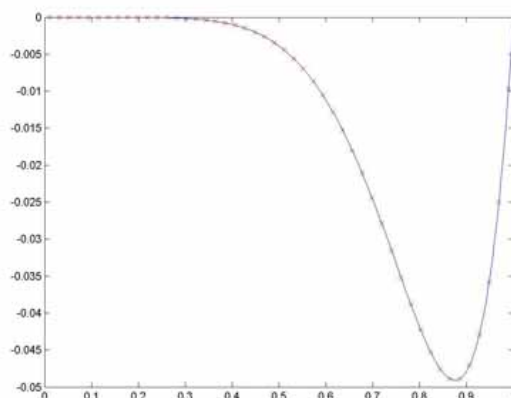


Figure 1: Comparison of Num. Sol. and Exam. Sol. of $k = 3, M = 3$

Table 1: The absolution errors for $M = 3$ and different values of k

t	$k = 3,$	$M=3$	$k = 4,$	$M=3$	$k = 5,$	$M = 3$
	ours	Ref.[Liand Zhao(2010)]	ours	Ref.[Liand Zhao(2010)]	ours	Ref.[Li and Zhao(2010)]
0.0625	6.816e-08	9.204e-08	5.575e-10	7.447e-10	4.452e-12	5.985e-12
0.1875	6.202e-06	1.242e-05	6.428e-07	8.363e-07	8.317e-08	5.903e-07
0.3125	5.788e-05	4.618e-07	8.146e-06	3.935e-07	1.555e-06	1.875e-07
0.4375	1.979e-04	8.814e-04	3.840e-05	8.146e-04	9.692e-06	8.026e-04
0.5625	4.184e-04	5.755e-03	1.164e-04	5.164e-03	4.490e-05	4.570e-03
0.6875	5.208e-04	2.000e-02	2.905e-04	1.736e-02	1.849e-04	1.571e-02
0.8125	1.136e-04	4.515e-02	6.403e-04	3.876e-02	5.273e-04	3.606e-02
0.9375	3.138e-03	5.972e-02	1.608e-03	5.010e-02	8.347e-04	4.936e-02

7 Conclusion

This article adopts the Chebyshev wavelet method to solve a class of multi-order fractional differential equations with variable coefficients. We derive the Chebyshev wavelet operational matrix of fractional order integration and use the wavelet basis together with operational matrix to reduce the fractional differential equation to a system of algebraic equations. Furthermore, we present an error estimation for eq. (5.4) with the residual error function. The matrix elements of the discrete operators are provided explicitly and this in turn greatly simplifies the steps for obtaining solutions. An example is given to demonstrate that the method is effective and accurate for solving multi-order FDEs. It is obvious that the accuracy improves when we take a relatively small fixed M and only increase k . Usually, it can reach the higher precision, even though M, k are small.

References

- [1] Garrappa R, Popolizio M. On the use of matrix functions for fractional partial differential equations[J]. Math. Comput. Simulation, 2011, 81(5): 1045–1056.
- [2] Podlubny I. Fractional differential equations: mathematics in science and engineering, vol. 198[M]. San Diego: Academic Press, 1999.
- [3] Podlubny I. Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications[M]. New York: Academic Press, 1999.
- [4] Gaul L, Klein P, Kemple S. Damping description involving fractional operators[J]. Mech Syst Signal Pr, 1991, 5(2), 81–88.
- [5] Suarez L, Shokooh A. An eigenvector expansion method for the solution of motion containing fractional derivatives[J]. J. Appl. Mech., 1997, 64(3): 629–635.
- [6] Momani S. An algorithm for solving the fractional convection-diffusion equation with nonlinear source term[J]. Commun. Nonlinear Sci. Numer. Simul., 2007, 12(7): 1283–1290.
- [7] Jafari H, Sei S. Solving a system of nonlinear fractional partial differential equations using homotopy analysis method[J]. Commun. Nonlinear Sci. Numer. Simul., 2009, 14(5): 1962–1969.

- [8] Sweilam N H, Khader M M, Al-Bar R F. Numerical studies for a multi-order fractional differential equation[J]. Phys. Lett. A, 2007, 371(1-2): 26–33.
- [9] Das S. Analytical solution of a fractional diffusion equation by variational iteration method[J]. Comput. Math. Appl., 2009, 57(3): 483–487.
- [10] Arikoglu A, Ozkol I. Solution of fractional integro-differential equations by using fractional differential transform method[J]. Chaos Solitons Fract., 2009, 40(2): 521–529.
- [11] Erturk V S, Momani S, Odibat Z. Application of generalized differential transform method to multi-order fractional differential equations[J]. Commun. Nonlinear Sci. Numer. Simul., 2008, 13(8), 1642–1654.
- [12] Meerschaert M, Tadjeran C. Finite difference approximations for two-sided space-fractional partial differential equations[J]. Appl. Numer. Math., 2006, 56(1): 80–90.
- [13] Odibat Z, Shawagfeh N. Generalized Taylor's formula[J]. Appl. Math. Comput., 2007, 186(1), 286–293.
- [14] Wu J L. A wavelet operational method for solving fractional partial differential equations numerically[J]. Appl. Math. Comput., 2009, 214(1): 31–40.
- [15] Lepik. Solving fractional integral equations by the Haar wavelet method[J]. Appl. Math. Comput., 2009, 214(2), 468–478.
- [16] Deng Jiqin, Ma Lifeng. Existence and uniqueness of solutions of initial value problems for nonlinear fractional differential equations[J]. Appl. Math. Lett., 2010, 23(6): 676–680.
- [17] Kajani M, Vencheh A. The Chebyshev wavelets operational matrix of integration and product operation matrix[J]. Int. J. Comput. Math., 2008, 86(7): 1118–1125.
- [18] Li B F. The second kind Chebyshev Wavelet method for fractional differential equations with variable coefficients[J]. Computer Modeling in Engineering & Sciences, 2013, 93(3): 187–202.
- [19] Li Yuanlu, Zhao Weiwei. Haar wavelet operational matrix of fractional order integration and its applications in solving the fractional order differential equations[J]. Appl. Math. Comput., 2010, 216(8): 2276–2285.

一类变系数分数阶微分方程的数值解法

李宝凤

(唐山师范学院数学与信息科学系, 河北 唐山 063000)

摘要: 本文研究了一类变系数分数阶微分方程的数值解法问题. 利用Cheyshev小波推导出的分数阶微分方程的算子矩阵把分数阶微分方程转换为代数方程组. 同时给出了Cheyshev小波基的收敛性和误差估计表达式, 并给出数值算例说明所提方法的精确性和有效性.

关键词: 分数阶积分; Chebvshev小波; 算子矩阵; 分数阶微分方程; block pulse函数

MR(2010)主题分类号: 41A35 中图分类号: O175.8