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WEAK SOLUTIONS TO STATIONARY BIPOLAR QUANTUM DRIFT-DIFFUSION MODEL IN ONE SPACE DIMENSION

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Abstract: In this paper, we study the weak solutions to stationary bipolar quantum drift-diffusion model for semiconductors in one space dimension. The model is reformulated as two coupled fourth-order elliptic equations by using exponential variable transformations. The existence of weak solutions to the reformulated equations is proved by using Schauder fixed-point theorem. Furthermore, the uniqueness of solutions and the semiclassical limit to the equations are obtained.

Keywords: quantum drift-diffusion model; stationary solutions; existence; uniqueness; semiclassical limit

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1 Introduction and Main Results

Recently, the quantum drift-diffusion equations attracted many scientists' interest since they are capable to describe quantum confinement and tunneling effects in metal-oxide-semiconductor structures and to simulate ultra-small semiconductor devices [1, 2]. Quantum drift-diffusion models were derived from a Wigner-Boltzmann equation by a moment method [3]. This paper is concerned with the bipolar quantum drift-diffusion model [4]:

$$n_t = \operatorname{div}\left[-\varepsilon^2 n\nabla\left(\frac{\triangle\sqrt{n}}{\sqrt{n}}\right) + \nabla n - n\nabla V\right], \tag{1.1}$$

$$p_t = \operatorname{div}\left[-\xi\varepsilon^2 p\nabla\left(\frac{\triangle\sqrt{p}}{\sqrt{p}}\right) + \nabla p + p\nabla V\right], \tag{1.2}$$

$$\lambda^2\triangle V = n - p - C(x), \tag{1.3}$$

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where the particle density n, the hole density p and the electrostatic potential V are unknown variables; the scaled Planck constant $\varepsilon > 0$, the scaled Debye length $\lambda > 0$ and the ratio of the effective masses of electrons and holes $\xi > 0$ are physical parameters; the doping profile $C(x)$ representing the distribution of charged background ions. This type of transient model consists of one or two fourth-order parabolic equations (unipolar or bipolar) coupled to a Poisson equation and has been studied in many works [4–14]. Abdallah and Unterreiter [15] showed the existence of solutions to the stationary model of (1.1)–(1.3) over the multi-dimension bounded domain and carried out the semiclassical limit.

The objective of this paper is to analyze the one-dimensional stationary version of (1.1)–(1.3):

$$-\varepsilon^2 n\left(\frac{(\sqrt{n})_{xx}}{\sqrt{n}}\right)_x + n_x - nV_x = J_0, \tag{1.4}$$

$$-\varepsilon^2 p\left(\frac{(\sqrt{p})_{xx}}{\sqrt{p}}\right)_x + p_x + pV_x = J_1, \tag{1.5}$$

$$V_{xx} = n - p - C(x) \quad \text{in } (0,1), \tag{1.6}$$

where we have let $\lambda = \xi = 1$ for convenience as in [4], the electron current density J_0 and the hole current density J_1 are two constants. We choose the physically motivated boundary conditions:

$$n(0) = n(1) = 1, \quad n_x(0) = n_x(1) = 0, \tag{1.7}$$

$$p(0) = p(1) = 1, \quad p_x(0) = p_x(1) = 0, \tag{1.8}$$

$$V(0) = V_0. \tag{1.9}$$

Dividing (1.4) by n and taking the derivative gives

$$-\varepsilon^2\left(\frac{(\sqrt{n})_{xx}}{\sqrt{n}}\right)_{xx} + \left(\frac{n_x}{n}\right)_x - (n - p - C(x)) = \left(\frac{J_0}{n}\right)_x, \tag{1.10}$$

where we have used the Poisson equation (1.6). Similarly, dividing (1.5) by p and taking the derivative leads to

$$-\varepsilon^2\left(\frac{(\sqrt{p})_{xx}}{\sqrt{p}}\right)_{xx} + \left(\frac{p_x}{p}\right)_x + (n - p - C(x)) = \left(\frac{J_1}{p}\right)_x. \tag{1.11}$$

After two exponential transformations $n = e^u,\ p = e^v$, we obtain

$$-\frac{\varepsilon^2}{2}\left(u_{xx} + \frac{u_x^2}{2}\right)_{xx} + u_{xx} - (e^u - e^v - C(x)) = J_0(e^{-u})_x, \tag{1.12}$$

$$-\frac{\varepsilon^2}{2}\left(v_{xx} + \frac{v_x^2}{2}\right)_{xx} + v_{xx} + (e^u - e^v - C(x)) = J_1(e^{-v})_x \tag{1.13}$$

with the boundary conditions

$$u(0) = u(1) = 0, \quad u_x(0) = u_x(1) = 0, \tag{1.14}$$

$$v(0)=v(1)=0, \quad v_x(0)=v_x(1)=0. \tag{1.15}$$

As usual, we call $(u,v)\in H_0^2(0,1)\times H_0^2(0,1)$ a weak solution of the problem (1.12)–(1.15), if for all $\psi\in H_0^2(0,1)$ it holds

$$\frac{\varepsilon^2}{2}\int_0^1\left(u_{xx}+\frac{u_x^2}{2}\right)\psi_{xx}dx+\int_0^1 u_x\psi_x dx+\int_0^1(e^u-e^v-C(x))\psi dx=J_0\int_0^1 e^{-u}\psi_x dx, \tag{1.16}$$

$$\frac{\varepsilon^2}{2}\int_0^1\left(v_{xx}+\frac{v_x^2}{2}\right)\psi_{xx}dx+\int_0^1 v_x\psi_x dx-\int_0^1(e^u-e^v-C(x))\psi dx=J_1\int_0^1 e^{-v}\psi_x dx. \tag{1.17}$$

Our main results are stated as follows:

Theorem 1.1 (Existence) Let $C(x)\in L^2(0,1)$, then there exists a weak solution $(u,v)\in H_0^2(0,1)\times H_0^2(0,1)$ of the problem (1.12)–(1.15) for any $J_0,J_1\in\mathbb{R}$.

Theorem 1.2 (Uniqueness) Let $C(x)\in L^2(0,1)$. If

$$\varepsilon\parallel C(x)\parallel^2_{L^2(0,1)}+(1+\sqrt{2}|J_0|)e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}}\le 2, \tag{1.18}$$

$$\varepsilon\parallel C(x)\parallel^2_{L^2(0,1)}+(1+\sqrt{2}|J_1|)e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}}\le 2, \tag{1.19}$$

then the problem (1.12)–(1.15) has a unique solution.

Theorem 1.3 (Semiclassical limit) Let $(u_\varepsilon,v_\varepsilon)$ be a solution to the problem (1.12)–(1.15) obtained in Theorem 1.1. Then as $\varepsilon\to 0$, maybe for a subsequence,

$$u_\varepsilon\to u, \quad v_\varepsilon\to v \quad \text{weakly in } H^1(0,1) \text{ and strongly in } L^\infty(0,1) \tag{1.20}$$

and (u,v) is a weak solution of

$$u_{xx}-(e^u-e^v-C(x))=J_0(e^{-u})_x, \tag{1.21}$$

$$v_{xx}+(e^u-e^v-C(x))=J_1(e^{-v})_x \tag{1.22}$$

subject to the boundary condition

$$u(0)=u(1)=0, \tag{1.23}$$

$$v(0)=v(1)=0. \tag{1.24}$$

Remark 1.1 Although Abdallah and Unterreiter obtained the stationary bipolar quantum drift-diffusion model in [15], but they did not show the uniqueness. In this paper, we give such a result. In addition, the proof of [15] was based on a Schauder fixed point iteration combined with a minimization procedure, whereas in this paper, we reformulate the model as two coupled fourth-order elliptic equations by using exponential variable transformations and employ the Schauder fixed-point theorem.

This article is organized as follows. In Section 2, we will show the existence of solutions to the problem (1.12)–(1.15) by using the techniques of a priori estimates and Schauder fixed-point theorem. Then we will prove the uniqueness and the semiclassical limit of the solutions in Section 3 and Section 4, respectively.

2 Existence of Weak Solutions

In order to use the Schauder fixed-point theorem to prove the existence, we need the following lemma:

Lemma 2.1 Let $C(x) \in L^2(0,1)$ and let $(u,v) \in H_0^2(0,1) \times H_0^2(0,1)$ be a solution of (1.12)–(1.15). Then

$$\varepsilon^2 \parallel u_{xx} \parallel^2_{L^2(0,1)} + \varepsilon^2 \parallel v_{xx} \parallel^2_{L^2(0,1)} + \parallel u_x \parallel^2_{L^2(0,1)} + \parallel v_x \parallel^2_{L^2(0,1)} \leq 2 \parallel C(x) \parallel^2_{L^2(0,1)} . \quad (2.1)$$

Proof We use $\psi = u$ as a test function in the weak formulation of (1.12) to obtain

$$\frac{\varepsilon^2}{2}\int_0^1 \left(u_{xx}^2 + \frac{1}{2}u_x^2 u_{xx}\right) dx + \int_0^1 u_x^2 dx = -\int_0^1 (e^u - e^v - C(x))u dx + J_0 \int_0^1 e^{-u} u_x dx. \quad (2.2)$$

The boundary condition (1.14) gives

$$\int_0^1 u_x^2 u_{xx} dx = \frac{1}{3}[u_x^3(1) - u_x^3(0)] = 0$$

and

$$\int_0^1 e^{-u} u_x dx = -\int_0^1 (e^{-u})_x dx = e^{-u(0)} - e^{-u(1)} = 0.$$

Consequently, equation (2.2) is equivalent to

$$\frac{\varepsilon^2}{2}\int_0^1 u_{xx}^2 dx + \int_0^1 u_x^2 dx = -\int_0^1 (e^u - e^v - C(x))u dx. \quad (2.3)$$

Similarly, using $\psi = v$ as a test function in the weak formulation of (1.13) and using the boundary condition (1.15) we get

$$\frac{\varepsilon^2}{2}\int_0^1 v_{xx}^2 dx + \int_0^1 v_x^2 dx = -\int_0^1 (e^u - e^v - C(x))v dx. \quad (2.4)$$

Summing up (2.3) and (2.4), we have

$$\frac{\varepsilon^2}{2}\int_0^1 u_{xx}^2 dx + \frac{\varepsilon^2}{2}\int_0^1 v_{xx}^2 dx + \int_0^1 u_x^2 dx + \int_0^1 v_x^2 dx = -\int_0^1 (e^u - e^v - C(x))(u-v) dx. \quad (2.5)$$

The monotonicity of $x \mapsto e^x$ implies

$$-\int_0^1 (e^u - e^v)(u-v) dx \leq 0.$$

From the Young inequality and the Poincaré inequality,

$$\begin{aligned}\int_0^1 C(x)(u-v)dx &\leq \frac{1}{2}\int_0^1 u^2 dx + \frac{1}{2}\int_0^1 v^2 dx + \int_0^1 C(x)^2 dx \\ &\leq \frac{1}{2}\int_0^1 u_x^2 dx + \frac{1}{2}\int_0^1 v_x^2 dx + \int_0^1 C(x)^2 dx.\end{aligned}$$

So (2.5) can be estimated as

$$\frac{\varepsilon^2}{2}\int_0^1 u_{xx}^2dx+\frac{\varepsilon^2}{2}\int_0^1 v_{xx}^2dx+\frac{1}{2}\int_0^1 u_x^2dx+\frac{1}{2}\int_0^1 v_x^2dx\le\int_0^1 C(x)^2dx.$$

This proves the lemma.

Proof of Theorem 1.1 Consider the following linear problems for given $(\rho,\eta)\in W_0^{1,4}(0,1)\times W_0^{1,4}(0,1)$ with test functions $\psi\in H_0^2(0,1)$:

$$\frac{\varepsilon^2}{2}\int_0^1 u_{xx}\psi_{xx}dx+\frac{\sigma\varepsilon^2}{4}\int_0^1\rho_x^2\psi_{xx}dx+\int_0^1 u_x\psi_xdx+\sigma\int_0^1(e^\rho-e^\eta-C(x))\psi dx=\sigma J_0\int_0^1 e^{-\rho}\psi_xdx, \tag{2.6}$$

$$\frac{\varepsilon^2}{2}\int_0^1 v_{xx}\psi_{xx}dx+\frac{\sigma\varepsilon^2}{4}\int_0^1\eta_x^2\psi_{xx}dx+\int_0^1 v_x\psi_xdx-\sigma\int_0^1(e^\rho-e^\eta-C(x))\psi dx=\sigma J_1\int_0^1 e^{-\eta}\psi_xdx, \tag{2.7}$$

where $\sigma\in[0,1]$. We define the bilinear form

$$a(u,\psi)=\frac{\varepsilon^2}{2}\int_0^1 u_{xx}\psi_{xx}dx+\int_0^1 u_x\psi_xdx, \tag{2.8}$$

and the linear functional

$$F(\psi)=-\frac{\sigma\varepsilon^2}{4}\int_0^1\rho_x^2\psi_{xx}dx-\sigma\int_0^1(e^\rho-e^\eta-C(x))\psi dx+\sigma J_0\int_0^1 e^{-\rho}\psi_xdx. \tag{2.9}$$

Since the bilinear form $a(u,\psi)$ is continuous and coercive on $H_0^2(0,1)\times H_0^2(0,1)$ and the linear functional $F(\psi)$ is continuous on $H_0^2(0,1)$, we can apply the Lax-Milgram theorem to obtain the existence of a solution $u\in H_0^2(0,1)$ of (2.6). Similarly there exists a solution $v\in H_0^2(0,1)$ to (2.7). Thus, the operator

$$S:\ W_0^{1,4}(0,1)\times W_0^{1,4}(0,1)\times[0,1]\to W_0^{1,4}(0,1)\times W_0^{1,4}(0,1),\quad (\rho,\eta,\sigma)\mapsto(u,v)$$

is well defined. Moreover, it is continuous and compact since the embedding $H_0^2(0,1)\hookrightarrow W_0^{1,4}(0,1)$ is compact. Furthermore, $S(\rho,\eta,0)=(0,0)$. Following the steps of the proof of Lemma 2.1, we can show that $\parallel u\parallel_{H_0^2(0,1)}+\parallel v\parallel_{H_0^2(0,1)}\le$ const. for all $(u,v,\sigma)\in W_0^{1,4}(0,1)\times W_0^{1,4}(0,1)\times[0,1]$ satisfying $S(u,v,\sigma)=(u,v)$. Therefore, the existence of a fixed point (u,v) with $S(u,v,1)=(u,v)$ follows from the Schauder fixed-point theorem. This fixed point is a solution of (1.12)–(1.15).

3 Uniqueness of Solutions

To prove the uniqueness, we need the following lemma:

Lemma 3.1 Let (u,v) be a solution of (1.12)–(1.15) obtained in Theorem 1.1. Then

$$\parallel u\parallel_{L^\infty(0,1)},\ \parallel v\parallel_{L^\infty(0,1)}\le\sqrt{2}\parallel C(x)\parallel_{L^2(0,1)}, \tag{3.1}$$

$$\parallel u_x\parallel_{L^\infty(0,1)},\ \parallel v_x\parallel_{L^\infty(0,1)}\le\frac{2\parallel C(x)\parallel_{L^2(0,1)}}{\sqrt{\varepsilon}}. \tag{3.2}$$

Proof For simplicity, we only treat with the case of u. (3.1) can be concluded directly from (2.1) and the Poincaré-Sobolev inequality:

$$\| u \|_{L^\infty(0,1)} \leq \| u_x \|_{L^2(0,1)} \leq \sqrt{2} \| C(x) \|_{L^2(0,1)} .$$

We observe that, due to the boundary conditions for u_x,

$$u_x(x)^2 = 2\int_0^x u_x(s)u_{xx}(s)ds \leq 2 \| u_x \|_{L^2(0,1)} \| u_{xx} \|_{L^2(0,1)}$$

and thus by the Young inequality and (2.1)

$$\begin{aligned} & \| u_x \|_{L^\infty(0,1)} \leq \sqrt{2}\sqrt{\| u_x \|_{L^2(0,1)} \| u_{xx} \|_{L^2(0,1)}} \\ \leq & \frac{\sqrt{2}}{2\sqrt{\varepsilon}} \| u_x \|_{L^2(0,1)} + \frac{\sqrt{2\varepsilon}}{2} \| u_{xx} \|_{L^2(0,1)} \\ \leq & \frac{2 \| C(x) \|_{L^2(0,1)}}{\sqrt{\varepsilon}}. \end{aligned}$$

Proof of Theorem 1.2 Let $(u_1, v_1),\ (u_2, v_2) \in H_0^2(0,1)\times H_0^2(0,1)$ be two weak solutions of (1.12)–(1.15). The weak formulations of the difference of the equations satisfied by (u_1, v_1) and (u_2, v_2), with the test functions $u_1 - u_2$ and $v_1 - v_2$, respectively, read as follows:

$$\begin{aligned} & \frac{\varepsilon^2}{2}\int_0^1 (u_1-u_2)_{xx}^2 dx + \frac{\varepsilon^2}{4}\int_0^1 (u_{1x}^2 - u_{2x}^2)(u_1-u_2)_{xx}dx + \int_0^1 (u_1-u_2)_x^2 dx \\ = & -\int_0^1 (e^{u_1} - e^{v_1} - e^{u_2} + e^{v_2})(u_1-u_2)dx - J_0\int_0^1 (e^{-u_1} - e^{-u_2})(u_1-u_2)_x dx, \quad (3.3) \\ & \frac{\varepsilon^2}{2}\int_0^1 (v_1-v_2)_{xx}^2 dx + \frac{\varepsilon^2}{4}\int_0^1 (v_{1x}^2 - v_{2x}^2)(v_1-v_2)_{xx}dx + \int_0^1 (v_1-v_2)_x^2 dx \\ = & \int_0^1 (e^{u_1} - e^{v_1} - e^{u_2} + e^{v_2})(v_1-v_2)dx - J_1\int_0^1 (e^{-v_1} - e^{-v_2})(v_1-v_2)_x dx. \quad (3.4) \end{aligned}$$

Using (3.2) and the Young inequality, we can estimate the second integral on the left-hand side of (3.3) as

$$\begin{aligned} & \frac{\varepsilon^2}{4}\int_0^1 (u_{1x}^2 - u_{2x}^2)(u_1-u_2)_{xx}dx \\ = & \frac{\varepsilon^2}{4}\int_0^1 (u_{1x} + u_{2x})(u_1-u_2)_x(u_1-u_2)_{xx}dx \\ \geq & -\varepsilon^{\frac{3}{2}} \| C(x) \|_{L^2(0,1)} \int_0^1 |(u_1-u_2)_{xx}| \cdot |(u_1-u_2)_x| dx \\ \geq & -\frac{\varepsilon^2}{2}\int_0^1 (u_1-u_2)_{xx}^2 dx - \frac{\varepsilon}{2} \| C(x) \|_{L^2(0,1)}^2 \int_0^1 (u_1-u_2)_x^2 dx. \quad (3.5) \end{aligned}$$

The mean value theorem and estimate (3.1) for v yields

$$|e^{v_1} - e^{v_2}| \leq e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}}|v_1 - v_2|. \quad (3.6)$$

The monotonicity of $x \mapsto e^x$, inequality (3.6), the Young inequality and the Poincaré inequality leads to

$$\begin{aligned}
& -\int_0^1 (e^{u_1} - e^{v_1} - e^{u_2} + e^{v_2})(u_1 - u_2)dx \leq \int_0^1 (e^{v_1} - e^{v_2})(u_1 - u_2)dx \\
\leq\ & e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \int_0^1 |v_1 - v_2| \cdot |u_1 - u_2| dx \\
\leq\ & \frac{1}{2} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \left[\int_0^1 (u_1 - u_2)^2 dx + \int_0^1 (v_1 - v_2)^2 dx \right] \\
\leq\ & \frac{1}{4} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \left[\int_0^1 (u_1 - u_2)_x^2 dx + \int_0^1 (v_1 - v_2)_x^2 dx \right]. \qquad (3.7)
\end{aligned}$$

For the estimate of the second integral on the right-hand side of (3.3), we obtain similarly as above

$$\begin{aligned}
& -J_0 \int_0^1 (e^{-u_1} - e^{-u_2})(u_1 - u_2)_x dx \\
\leq\ & |J_0| e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \int_0^1 |u_1 - u_2| \cdot |(u_1 - u_2)_x| dx \\
\leq\ & |J_0| e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \left[\int_0^1 (u_1 - u_2)^2 dx \right]^{\frac{1}{2}} \left[\int_0^1 (u_1 - u_2)_x^2 dx \right]^{\frac{1}{2}} \\
\leq\ & \frac{|J_0|}{\sqrt{2}} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \int_0^1 (u_1 - u_2)_x^2 dx, \qquad (3.8)
\end{aligned}$$

where we have used the Hölder inequality in the second inequality of (3.8). By (3.3), (3.5), (3.7) and (3.8), we get

$$\begin{aligned}
& \left[1 - \frac{\varepsilon}{2} \parallel C(x) \parallel_{L^2(0,1)}^2 - \frac{1 + 2\sqrt{2}|J_0|}{4} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \right] \int_0^1 (u_1 - u_2)_x^2 dx \\
\leq\ & \frac{1}{4} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \int_0^1 (v_1 - v_2)_x^2 dx. \qquad (3.9)
\end{aligned}$$

Employing the same techniques as above, we can estimate (3.4) as

$$\begin{aligned}
& \left[1 - \frac{\varepsilon}{2} \parallel C(x) \parallel_{L^2(0,1)}^2 - \frac{1 + 2\sqrt{2}|J_1|}{4} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \right] \int_0^1 (v_1 - v_2)_x^2 dx \\
\leq\ & \frac{1}{4} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \int_0^1 (u_1 - u_2)_x^2 dx. \qquad (3.10)
\end{aligned}$$

It follows from (3.9) and (3.10) that

$$\begin{aligned}
& \left[1 - \frac{\varepsilon}{2} \parallel C(x) \parallel_{L^2(0,1)}^2 - \frac{1 + \sqrt{2}|J_0|}{2} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \right] \int_0^1 (u_1 - u_2)_x^2 dx \\
& + \left[1 - \frac{\varepsilon}{2} \parallel C(x) \parallel_{L^2(0,1)}^2 - \frac{1 + \sqrt{2}|J_1|}{2} e^{\sqrt{2}\|C(x)\|_{L^2(0,1)}} \right] \int_0^1 (v_1 - v_2)_x^2 dx \\
\leq\ & 0. \qquad (3.11)
\end{aligned}$$

This inequality and (1.18), (1.19) implies $u_1 = u_2,\ v_1 = v_2$ in $(0,1)$.

4 Semiclassical Limit

Proof of Theorem 1.3 From Lemma 2.1 and the Poincaré inequality we obtain a uniform $H^1(0,1)$ bound for u_ε and v_ε. Then there exists a subsequence of $(u_\varepsilon, v_\varepsilon)$ (not relabeled) such that (1.20) holds. The weak formulations of (1.12) and (1.13) read, for any $\psi \in C_0^\infty(0,1)$, after integration by parts,

$$\begin{aligned} & -\frac{\varepsilon^2}{2}\int_0^1 u_\varepsilon \psi_{xxxx}dx - \frac{\varepsilon^2}{4}\int_0^1 u_{\varepsilon,x}^2\psi_{xx}dx \\ = & \int_0^1 u_{\varepsilon,x}\psi_x dx + \int_0^1 (e^{u_\varepsilon} - e^{v_\varepsilon} - C(x))\psi dx - J_0\int_0^1 e^{-u_\varepsilon}\psi_x dx, \end{aligned} \tag{4.1}$$

$$\begin{aligned} & -\frac{\varepsilon^2}{2}\int_0^1 v_\varepsilon \psi_{xxxx}dx - \frac{\varepsilon^2}{4}\int_0^1 v_{\varepsilon,x}^2\psi_{xx}dx \\ = & \int_0^1 v_{\varepsilon,x}\psi_x dx - \int_0^1 (e^{u_\varepsilon} - e^{v_\varepsilon} - C(x))\psi dx - J_1\int_0^1 e^{-v_\varepsilon}\psi_x dx. \end{aligned} \tag{4.2}$$

Convergences (1.20) allow us to pass to the limit $\varepsilon \to 0$ in the above equations, observing that the left-hand sides of (4.1) and (4.2) vanish in the limit:

$$0 = \int_0^1 u_x\psi_x dx + \int_0^1 (e^u - e^v - C(x))\psi dx - J_0\int_0^1 e^{-u}\psi_x dx, \tag{4.3}$$

$$0 = \int_0^1 v_x\psi_x dx - \int_0^1 (e^u - e^v - C(x))\psi dx - J_1\int_0^1 e^{-v}\psi_x dx. \tag{4.4}$$

This shows the weak forms of (1.21) and (1.22) hold.

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一维双极量子漂移-扩散稳态模型的弱解

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摘 要: 本文研究了半导体中一维双极量子漂移–扩散稳态模型的弱解. 利用指数变换法把此模型转化成两个四阶椭圆方程, 然后利用Schauder不动点定理证明了转化后的方程组弱解的存在性. 另外得到了方程组解的唯一性和半古典极限.

关键词: 量子漂移-扩散模型; 稳态解; 存在性; 唯一性; 半古典极限

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