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CENTRO-SYMMETRIC MINIMAL RANK SOLUTIONS AND ITS OPTIMAL APPROXIMATION OF THE MATRIX EQUATION $AX = B$

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Abstract: The centro-symmetric solutions of the matrix equation $AX = B$ are considered. By using the generalized singular value decompositions of matrix pairs and generalized inverses of matrices, necessary and sufficient conditions for the existence of such solution and the expression of the maximal and minimal rank solutions are derived. Also, the optimal approximation for the minimal rand solution set to a given matrix is also discussed and the expression of the solution is presented.

Keywords: matrix equation; centro-symmetric matrix; maximal rank; minimal rank; optimal approximate solution

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1 Introduction

Throughout this paper, let $R^{n\times m}$ be the set of all $n \times m$ real matrices, $OR^{n\times n}$ be the set of all $n \times n$ orthogonal matrices. Denote by I_n the identity matrix with order n. For matrix A, A^T, A^+, $\|A\|$ and $r(A)$ represent its transpose, Moore-Penrose inverse, Frobenius norm and rank, respectively.

Definition 1 A matrix $A = (a_{ij}) \in R^{n\times n}$ is said to be a centro-symmetric matrix if $a_{ij} = a_{n+1-i,n+1-j}, i, j = 1, 2, \cdots, n$. the set of all $n \times n$ centro-symmetric matrices is denoted by $CSR^{n\times n}$.

Centro-symmetric matrices have practical applications in information theory, linear system theory, linear estimate theory, and numerical analysis(see, e.g. [1–4]).

We know that investigating minimal ranks of matrix expressions has many immediate motivations in matrix analysis and applications. For example, the classical matrix equation $AX = B$ is consistent if and only if

$$\min_X \operatorname{rank}(B - AX) = 0.$$

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The two consistent matrix equations $A_1X_1B_1 = C_1$, $A_2X_2B_2 = C_2$ where X_1 and X_2 have the same size, have a common solution if and only if

$$\min_{X_1,X_2} \operatorname{rank}(X_1 - X_2) = 0.$$

In 1972, Mitra [5] considered solutions with fixed ranks for the matrix equations $AX = B$ and $AXB = C$. In 1984, Mitra [6] gave common solutions of minimal rank of the pair of complex matrix equations $AX = C, XB = D$. In 1990, Mitra studied the minimal ranks of common solutions to the pair of matrix equations $A_1X_1B_1 = C_1$ and $A_2X_2B_2 = C_2$ over a general field in [7]. In 2003, Tian [8] investigated the extremal rank solutions to the complex matrix equation $AXB = C$ and gave some applications. Xiao et al. [9] in 2009 considered the symmetric minimal rank solution to equation $AX = B$. Recently, the anti-reflexive extremal rank solutions to the matrix equation $AX = B$ was derived by Xiao et al. [10].

In this paper, we consider the centro-symmetric extremal rank solutions of the matrix equation

$$AX = B, \tag{1.1}$$

where A and B are given matrices in $R^{m\times n}$.

We also consider the matrix nearness problem

$$\min_{X\in S_m} \left\|X - \tilde{X}\right\|_F, \tag{1.2}$$

where $\tilde{X}$ is a given matrix in $R^{n\times n}$ and S_m is the minimal rank solution set of eq. (1.1).

We organize this paper as follows. In Section 2, we first establish a representation for the centro-symmetric matrix. Then we give necessary and sufficient conditions for the existence of centro-symmetric solution to (1.1). We also give the expressions of such solutions when the solvability conditions are satisfied. We in Section 3 establish formulas of maximal and minimal ranks of centro-symmetric solutions to (1.1), and present the centro-symmetric extremal rank solutions to (1.1). We in Section 4 present the expression of the optimal approximation solution to the set of the minimal rank solution.

2 Centro-Symmetric Solution to (1.1)

Denote by e_i be the ith column of I_n and set $S_n = (e_n, e_{n-1}, \cdots, e_1)$. It is easy to see that

$$S_n^T = S_n, \quad S_n^T S_n = I.$$

Let $k = [\frac{n}{2}]$, where $[\frac{n}{2}]$ is the maximum integer which is not greater than $\frac{n}{2}$. Define D_n as

$$D_n = \frac{1}{\sqrt{2}}\begin{pmatrix} I_k & I_k \\ S_k & -S_k \end{pmatrix} (n = 2k), \quad D_n = \frac{1}{\sqrt{2}}\begin{pmatrix} I_k & 0 & I_k \\ 0 & \sqrt{2} & 0 \\ S_k & 0 & -S_k \end{pmatrix} (n = 2k + 1), \tag{2.1}$$

then it is easy verified that the above matrices D_n are orthogonal matrices.

Lemma 1 [11] Let $X \in R^{n\times n}$ and D_n with the forms of (2.1), then X is the centro-symmetric matrix if and only if there exist $X_1 \in R^{(n-k)\times(n-k)}$ and $X_2 \in R^{k\times k}$, whether n is odd or even, such that

$$X = D_n \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} D_n^T. \tag{2.2}$$

Here, we always assume $k = [\frac{n}{2}]$.

Given matrices $A_1 \in R^{m\times n}$, $B_1 \in R^{m\times p}$, by making generalized singular value decomposition to $[A_1, B_1]$, we have

$$A_1 = M_1\Sigma_{A_1}U_1, \quad B_1 = M_1\Sigma_{B_1}V_1, \tag{2.3}$$

where M_1 is a $m\times m$ nonsingular matrix, $U_1 \in OR^{n\times n}$, $V_1 \in OR^{p\times p}$,

$$\Sigma_{A_1} = \begin{array}{c} \begin{bmatrix} I & 0 & 0 \\ 0 & S_{A_1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} r_1 - s_1 \\ s_1 \\ k_1 - r_1 \\ m - k_1 \end{array}, \qquad \Sigma_{B_1} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 \\ 0 & S_{B_1} & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} r_1 - s_1 \\ s_1 \\ k_1 - r_1 \\ m - k_1 \end{array},$$

$k_1 = r[A_1, B_1]$, $r_1 = r(A_1)$, $s_1 = r(A_1) + r(B_1) - r[A_1, B_1]$, $S_{A_1} = \mathrm{diag}(\alpha_1, \cdots, \alpha_{s_1})$, $S_{B_1} = \mathrm{diag}(\beta_1, \cdots, \beta_{s_1})$, $0 < \alpha_{s_1} \le \cdots \le \alpha_1 < 1$, $0 < \beta_1 \le \cdots \le \beta_{s_1} < 1$, $\alpha_i^2 + \beta_i^2 = 1$, $i = 1, \cdots, s_1$.

Lemma 2 [10] Given matrices $A_1 \in R^{m\times n}$, $B_1 \in R^{m\times p}$, the generalized singular ecomposition of the matrix pair $[A_1, B_1]$ is given by (2.3), then matrix equation $A_1X = B_1$ is consistent, if and only if

$$r[A_1, B_1] = r(A_1), \tag{2.4}$$

and the expression of its general solution is

$$X = U_1^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_1}^{-1}S_{B_1} \\ Y_{31} & Y_{32} \end{bmatrix} V_1, \tag{2.5}$$

where $Y_{31} \in R^{(n-r_1)\times(p-s_1)}$, $Y_{32} \in R^{(n-r_1)\times s_1}$ are arbitrary.

Assume D_n with the form of (2.1), and AD_n and BD_n have the following partition form

$$AD_n = [A_2, A_3], \quad BD_n = [B_2, B_3], \tag{2.6}$$

where $A_2 \in R^{m\times(n-k)}$, $A_3 \in R^{m\times k}$, $B_2 \in R^{m\times(n-k)}$, $B_3 \in R^{m\times k}$, and the generalized singular value decomposition of the matrix pair $[A_2, B_2]$, $[A_3, B_3]$ are, respectively,

$$A_2 = M_2\Sigma_{A_2}U_2, \quad B_2 = M_2\Sigma_{B_2}V_2, \tag{2.7}$$

$$A_3 = M_3\Sigma_{A_3}U_3, \quad B_3 = M_3\Sigma_{B_3}V_3, \tag{2.8}$$

where $U_2 \in OR^{(n-k)\times(n-k)}$, $V_2 \in OR^{(n-k)\times(n-k)}$, $U_3 \in OR^{k\times k}$, $V_3 \in OR^{k\times k}$, nonsingular matrices $M_2, M_3 \in R^{m\times m}$, $k_2 = r[A_2, B_2]$, $r_2 = r(A_2)$, $s_2 = r(A_2) + r(B_2) - r[A_2, B_2]$, and $k_3 = r[A_3, B_3]$, $r_3 = r(A_3)$, $s_3 = r(A_3) + r(B_3) - r[A_3, B_3]$,

$$\Sigma_{A_2} = \begin{array}{c} \begin{bmatrix} I & 0 & 0 \\ 0 & S_{A_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} r_2 - s_2 \\ s_2 \\ k_2 - r_2 \\ m - k_2 \end{array}, \quad \Sigma_{B_2} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 \\ 0 & S_{B_2} & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} r_2 - s_2 \\ s_2 \\ k_2 - r_2 \\ m - k_2 \end{array},$$

$$\Sigma_{A_3} = \begin{array}{c} \begin{bmatrix} I & 0 & 0 \\ 0 & S_{A_3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} r_3 - s_3 \\ s_3 \\ k_3 - r_3 \\ m - k_3 \end{array}, \quad \Sigma_{B_3} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 \\ 0 & S_{B_3} & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} r_3 - s_3 \\ s_3 \\ k_3 - r_3 \\ m - k_3 \end{array}.$$

Then we can establish the existence theorems as follows.

Theorem 1 Let $A, B \in R^{m\times n}$ and D_n with the form of (2.1), AD_n, BD_n have the partition forms of (2.6), and the generalized singular value decompositions of the matrix pair $[A_2, B_2]$ and $[A_3, B_3]$ are given by (2.7) and (2.8). Then equation (1.1) has a centro-symmetric solution X if and only if

$$r[A_2, B_2] = r(A_2), \quad r[A_3, B_3] = r(A_3), \tag{2.9}$$

and its general solution can be expressed as

$$X = D_n \begin{bmatrix} U_2^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1} S_{B_2} \\ Z_{31} & Z_{32} \end{bmatrix} V_2 & 0 \\ 0 & U_3^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1} S_{B_3} \\ W_{31} & W_{32} \end{bmatrix} V_3 \end{bmatrix} D_n^T, \tag{2.10}$$

where $Z_{31} \in R^{(n-k-r_2)\times(n-k-s_2)}$, $Z_{32} \in R^{(n-k-r_2)\times s_2}$, $W_{31} \in R^{(k-r_3)\times(k-s_3)}$, $W_{32} \in R^{(k-r_3)\times s_3}$ are arbitrary.

Proof Suppose the matrix equation (1.1) has a solution X is centro-symmetric, then it follows from Lemma 1 that there exist $X_1 \in R^{(n-k)\times(n-k)}$, $X_2 \in R^{k\times k}$ satisfying

$$X = D_n \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} D_n^T \quad \text{and} \quad AX = B. \tag{2.11}$$

By (2.6), that is

$$[A_2 \ \ A_3] \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} = [B_2 \ \ B_3], \tag{2.12}$$

i.e.,

$$A_2 X_1 = B_2, \quad A_3 X_2 = B_3. \tag{2.13}$$

Therefore by Lemma 2, (2.9) hold, and

$$X_1 = U_2^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1} S_{B_2} \\ Z_{31} & Z_{32} \end{bmatrix} V_2, \quad X_2 = U_3^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1} S_{B_3} \\ W_{31} & W_{32} \end{bmatrix} V_3, \tag{2.14}$$

where $Z_{31} \in R^{(n-k-r_2)\times(n-k-s_2)}$, $Z_{32} \in R^{(n-k-r_2)\times s_2}$, $W_{31} \in R^{(k-r_3)\times(k-s_3)}$, $W_{32} \in R^{(k-r_3)\times s_3}$ are arbitrary. Substituting (2.14) into (2.11) yields that the centro-symmetric solution X of the matrix equation (1.1) can be represented by (2.10). The proof is completed.

3 Centro-Symmetric Extremal Rank Solutions to (1.1)

Theorem 2 Suppose that the matrix equation (1.1) has a centro-symmetric solution X and Ω is the set of all centro-symmetric solutions of (1.1). Then the extreme ranks of X are as follows:

(1) The maximal rank of X is

$$\max_{X\in\Omega} r(X) = \min\{n-k, n-k-r(A_2)+r(B_2)\} + \min\{k, k-r(A_3)+r(B_3)\}. \tag{3.1}$$

The general expression of X satisfying (3.1) is

$$X = D_n \begin{bmatrix} U_2^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1} S_{B_2} \\ Z_{31} & Z_{32} \end{bmatrix} V_2 & 0 \\ 0 & U_3^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1} S_{B_3} \\ W_{31} & W_{32} \end{bmatrix} V_3 \end{bmatrix} D_n^T, \tag{3.2}$$

where $Z_{31} \in R^{(n-k-r_2)\times(n-k-s_2)}$, $W_{31} \in R^{(k-r_3)\times(k-s_3)}$ are chosen such that $r(Z_{31}) = \min(n-k-r_2, n-k-s_2)$, $r(W_{31}) = \min(k-r_3, k-s_3)$, $Z_{32} \in R^{(n-k-r_2)\times s_2}$, $W_{32} \in R^{(k-r_3)\times s_3}$ are arbitrary.

(2) The minimal rank of X is

$$\min_{X\in\Omega} r(X) = r(B_2) + r(B_3). \tag{3.3}$$

The general expression of X satisfying (3.3) is

$$X = D_n \begin{bmatrix} U_2^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1} S_{B_2} \\ 0 & Z_{32} \end{bmatrix} V_2 & 0 \\ 0 & U_3^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1} S_{B_3} \\ 0 & W_{32} \end{bmatrix} V_3 \end{bmatrix} D_n^T, \tag{3.4}$$

where $Z_{32} \in R^{(n-k-r_2)\times s_2}$, $W_{32} \in R^{(k-r_3)\times s_3}$ are arbitrary.

Proof (1) By (2.10),

$$\max_{X\in\Omega} r(X)=\max_{Z_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1}S_{B_2} \\ Z_{31} & Z_{32} \end{bmatrix}+\max_{W_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1}S_{B_3} \\ W_{31} & W_{32} \end{bmatrix}, \tag{3.5}$$

$$\begin{aligned}&\max_{Z_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1}S_{B_2} \\ Z_{31} & Z_{32} \end{bmatrix}\\ =\;& s_2+\min\{n-k-r_2, n-k-s_2\}\\ =\;& \min\{n-k, n-k-r_2+s_2\}=\min\{n-k, n-k-r(A_2)+r(B_2)\},\end{aligned} \tag{3.6}$$

and

$$\begin{aligned}&\max_{W_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1}S_{B_3} \\ W_{31} & W_{32} \end{bmatrix}=s_3+\min\{k-r_3, k-s_3\}\\ =\;& \min\{k, k-r_3+s_3\}=\min\{k, k-r(A_3)+r(B_3)\}.\end{aligned} \tag{3.7}$$

Taking (3.6) and (3.7) into (3.5) yields (3.1).

According to the general expression of the solution in Theorem 1, it is easy to verify the rest of part in (1).

(2) By (2.10),

$$\min_{X\in\Omega} r(X)=\min_{Z_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1}S_{B_2} \\ Z_{31} & Z_{32} \end{bmatrix}+\min_{W_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1}S_{B_3} \\ W_{31} & W_{32} \end{bmatrix}, \tag{3.8}$$

$$\min_{Z_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1}S_{B_2} \\ Z_{31} & Z_{32} \end{bmatrix}=s_2=r(B_2),\quad \min_{W_{31}} r\begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1}S_{B_3} \\ W_{31} & W_{32} \end{bmatrix}=s_3=r(B_3). \tag{3.9}$$

Taking (3.9) into (3.8) yields (3.3).

According to the general expression of the solution in theorem 1, it is easy to verify the rest of part in (2). The proof is completed.

4 The Expression of the Optimal Approximation Solution to the Set of the Minimal Rank Solution

From (3.4), when $S_m=\{X \mid AX=B, X\in CSR^{n\times n}, r(X)=\min\limits_{Y\in\Omega} r(Y)\}$ is nonempty, it is easy to verify that S_m is a closed convex set, therefore there exists a unique solution $\hat{X}$ to the matrix nearness problem (1.2).

Theorem 3 Given matrix $\tilde{X}$, and the other given notations and conditions are the same as in Theorem 1. Let

$$D_n^T \tilde{X} D_n = \begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} \\ \tilde{X}_{21} & \tilde{X}_{22} \end{bmatrix}, \quad \tilde{X}_{11} \in C^{(n-k)\times(n-k)}, \quad \tilde{X}_{22} \in C^{k\times k}, \tag{4.1}$$

and we denote

$$U_2 \tilde{X}_{11} V_2^T = \begin{bmatrix} \tilde{Z}_{11} & \tilde{Z}_{12} \\ \tilde{Z}_{21} & \tilde{Z}_{22} \\ \tilde{Z}_{31} & \tilde{Z}_{32} \end{bmatrix}, \quad U_3 \tilde{X}_{22} V_3^T = \begin{bmatrix} \tilde{W}_{11} & \tilde{W}_{12} \\ \tilde{W}_{21} & \tilde{W}_{22} \\ \tilde{W}_{31} & \tilde{W}_{32} \end{bmatrix}. \tag{4.2}$$

If S_m is nonempty, then problem (1.2) has a unique $\hat{X}$ which can be represented as

$$\hat{X} = D_n \begin{bmatrix} U_2^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1} S_{B_2} \\ 0 & \tilde{Z}_{32} \end{bmatrix} V_2 & 0 \\ 0 & U_3^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1} S_{B_3} \\ 0 & \tilde{W}_{32} \end{bmatrix} V_3 \end{bmatrix} D_n^T, \tag{4.3}$$

where $\tilde{Z}_{32}, \tilde{W}_{32}$ are the same as in (4.2).

Proof When S_m is nonempty, it is easy to verify from (3.4) that S_m is a closed convex set. Since $R^{n\times n}$ is a uniformly convex banach space under Frobenius norm, there exists a unique solution for problem (1.2). By theorem 2, for any $X \in S_m$, X can be expressed as

$$X = D_n \begin{bmatrix} U_2^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1} S_{B_2} \\ 0 & Z_{32} \end{bmatrix} V_2 & 0 \\ 0 & U_3^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1} S_{B_3} \\ 0 & W_{32} \end{bmatrix} V_3 \end{bmatrix} D_n^T, \tag{4.4}$$

where $Z_{32} \in R^{(n-k-r_2)\times s_2}$, $W_{32} \in R^{(k-r_3)\times s_3}$ are arbitrary.

Using the invariance of the Frobenius norm under unitary transformations, we have

$$\begin{aligned} \|X - \tilde{X}\|^2 &= \left\| \begin{bmatrix} U_2^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_2}^{-1} S_{B_2} \\ 0 & Z_{32} \end{bmatrix} V_2 & 0 \\ 0 & U_3^T \begin{bmatrix} 0 & 0 \\ 0 & S_{A_3}^{-1} S_{B_3} \\ 0 & W_{32} \end{bmatrix} V_3 \end{bmatrix} - D_n^T \tilde{X} D_n \right\|^2 \\ &= \left\|Z_{32} - \tilde{Z}_{32}\right\|^2 + \left\|W_{32} - \tilde{W}_{32}\right\|^2 + \left\|S_{A_2}^{-1} S_{B_2} - \tilde{Z}_{22}\right\|^2 + \left\|S_{A_3}^{-1} S_{B_3} - \tilde{W}_{22}\right\|^2 \\ &\quad + \left\|\tilde{X}_{12}\right\|^2 + \left\|\tilde{X}_{21}\right\|^2 + \left\|\tilde{Z}_{11}\right\|^2 + \left\|\tilde{Z}_{12}\right\|^2 + \left\|\tilde{Z}_{21}\right\|^2 + \left\|\tilde{Z}_{31}\right\|^2 \\ &\quad + \left\|\tilde{W}_{11}\right\|^2 + \left\|\tilde{W}_{12}\right\|^2 + \left\|\tilde{W}_{21}\right\|^2 + \left\|\tilde{W}_{31}\right\|^2. \end{aligned}$$

Therefore, $\|X - \tilde{X}\|$ reaches its minimum if and only if

$$Z_{32} = \tilde{Z}_{32}, \quad W_{32} = \tilde{W}_{32}. \tag{4.5}$$

Substituting (4.5) into (4.4) yields (4.3). The proof is completed.

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矩阵方程$AX = B$的中心对称定秩解及其最佳逼近

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摘 要: 本文研究了矩阵方程$AX = B$ 的中心对称解. 利用矩阵对的广义奇异值分解和广义逆矩阵, 获得了该方程有中心对称解的充要条件以及有解时, 最大秩解、最小秩解的一般表达式, 并讨论了中心对称最小秩解集合中与给定矩阵的最佳逼近解.

关键词: 矩阵方程; 中心对称矩阵; 最大秩解; 最小秩解; 最佳逼近解

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