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ON FELICITOUS CHARACTER OF GENERALIZED SUN-GRAPHS

YANG Si-hua[1] , YAO Bing[1] , YAO Ming[2]

(1. *College of Mathematics and Statistics, Northwest Normal University, Lanzhou 730070, China*)

(2. *Department of Information Process and Control Engineering, Lanzhou Petrochemical College of Vocational Technology, Lanzhou 730060, China*)

Abstract: Felicitous character of some generalized sun-graphs is investigated in this note, and furthermore the exact felicitous labellings of two classes of generalized sun-graphs are obtained by analyzing the structures of the generalized sun-graphs. And the constructed graph theory models in coding theory, communication networks, logistics and other aspects have important applications.

Keywords: felicitous labelling; sun graph; generalized sun-graph

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1 Introduction

As known, graph labelling theory has important applications in coding theory, communication networks, logistics and other aspects of science. Some generalized sun-graphs can be regarded as ring-like real-networks. Each node in a network represents a server or client, and every edge joins two nodes if these two nodes are connected in the network. In Operations Research or Systems Engineering Theory and Methods, one very often use graph colorings to divide large systems into subsystems, and one, also, can use graph labellings to distinguish nodes and edges between nodes in order to find some fast algorithms to imitate effective transmission and safe communication in information networks.

On the other hand, Rosa [1] in 1966 proposed a conjecture: Every tree is a graceful tree. There are a lot of results for attacking this conjecture, but it has been known that this conjecture has not been settled down up to now. Lee et al. [2] in 1991 put forward a conjecture: Every tree is a felicitous tree that has the same theoretical value to the graceful tree conjecture and also has the same difficulty to be proved. Both conjectures are NP-hard problems that have attracted many researchers to explore them [3–6]. Attacking mathematical conjecture makes that graph labelling theory rapidly becomes a very active branch in graph theory.

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Biography: Yang Sihua (1989–), female, born at Songyuan, Jilin, master student, major in graph labellings/colorings.

Corresponding author: Yao Bing.

We use standard terminology and notation of graph theory, and graphs mentioned here are finite, simple and undirected. A (p,q)-graph has p vertices and q edges. An integer set $\{m, m+1, m+2, \cdots, n\}$ with integers $0 \leq m < n$ is denoted as $[m,n]$; similarly, $[k,\ell]^e$ denotes an integer set $\{k, k+2, k+4, \cdots, \ell\}$, where k and ℓ are even with respect to $0 \leq k < \ell$; and $[s,t]^o$ stands for an integer set $\{s, s+2, s+4, \cdots, t\}$ with odd integers s and t holding $1 \leq s < t$. A leaf is a vertex of degree one.

Definition 1.1 [3] A (p,q)-graph G has a proper mapping $f : V(G) \to [0,q]$, and the edge label $f(uv)$ of each edge $uv \in E(G)$ is defined as $f(uv) = f(u) + f(v) (\bmod\ q)$. If $f(u) \neq f(v)$ for distinct $u, v \in V(G)$, and the edge label set $\{f(uv) : uv \in E(G)\} = [0, q-1]$, then we say both G and f to be felicitous. And write $f(V(G)) = \{f(u) : u \in V(G)\}$, $f(E(G)) = \{f(uv) = f(u) + f(v)(\bmod\ q) : uv \in E(G)\}$.

Let $C_n = w_1 w_2 \cdots w_n w_1$ be a cycle of length n, where $n \geq 3$. Joining each vertex w_i $(i \in [1,n])$ of C_n with a new vertex y_i out of C_n by an edge obtains a graph, called a sun-graph and denoted as SC_n. Furthermore, for integers $r_i \geq 1$ and $k_i \geq 0$ with $i \in [1,n]$, we join each vertex w_i of C_n and the initial vertex $u^i_{j,m_{i,j}}$ of a path $P^i_j = u^i_{j,1} u^i_{j,2} \cdots u^i_{j,m_{i,j}}$ for $m_{i,j} \geq 2$, $j \in [1, r_i]$, and join each vertex w_i with k_i new vertices $v_{i,t}$ for $t \in [1, k_i]$ to produce a generalized sun-graph, denoted by $\mathrm{GSG}(m_{i,j}, r_i, k_i : i \in [1,n])$. It is reasonable to allow some $k_i = 0$, that is, no joining the vertex w_i of C_n with new vertex. Clearly, a generalized sun-graph is just a sun-graph when $m_{i,j} = 1$, $r_i = 1$ and $k_i = 0$ for $i \in [1,n]$.

In this article, we focus on finding felicitous labellings of $\mathrm{GSG}(m_{i,j}, r_i, k_i : i \in [1,n])$ for $r_i = 1$ or 2 and $m_{i,j} = 2$ with $i \in [1,n]$ and odd n. For the purpose of simplicity, we rewrite $\mathrm{GSG}_{n,1,3} = \mathrm{GSG}(m_{i,j} : j \leq r_i, i \in [1,n])$ when $r_i = 1$, $m_{i,j} = 2$ and $k_i \geq 0$. Thereby, we have

$$V(\mathrm{GSG}_{n,1,3}) = \{u^i_1, u^i_2, w_i, v_{i,j} \mid j \in [1,k_i], i \in [1,n]\}, \tag{1.1}$$

where $u^i_1 = u^i_{1,1}$ and $u^i_2 = u^i_{1,2}$. Similarly, $\mathrm{GSG}_{n,2,3} = \mathrm{GSG}(m_{i,j} : j \leq r_i, i \in [1,n])$ when $r_i = 2$, $m_{i,j} = 2$ and $k_i \geq 0$, and furthermore

$$V(\mathrm{GSG}_{n,2,3}) = \{u^i_{t,1}, u^i_{t,2}, w_i, v_{i,j} \mid t \in [1,2], j \in [1,k_i], i \in [1,n]\}. \tag{1.2}$$

2 Main Results

Theorem 2.1 For odd $n \geq 3$, every generalized sun-graph $\mathrm{GSG}_{n,1,3}$ admits a felicitous labelling.

Proof Apply the notation in (1.1) for $GSG_{n,1,3}$. Clearly,

$$|V(\mathrm{GSG}_{n,1,3})| = |E(\mathrm{GSG}_{n,1,3})| = M,$$

where $M = 3n + \sum\limits_{i=1}^{n} k_i$ (it may be some $k_i = 0$). Let $R = \frac{3n+3}{2} + \sum\limits_{s \in [1,n]^e} k_s$. We define a labelling f of $\mathrm{GSG}_{n,1,3}$ as follows.

(1) $f(u^1_1) = 0$, $f(u^i_1) = \sum\limits_{s \in [1,i]^e} (k_s + 3)$, $i \in [3,n]^o$;

(2) $f(w_1)=1,\ f(w_i)=1+\sum\limits_{s\in[1,i]^e}(k_s+3),\ i\in[3,n]^o$;

(3) if $k_2\neq 0$, $f(v_{2,m})=m+1,\ m\in[1,k_2]$; for $k_j\neq 0$, $f(v_{j,m})=1+m+\sum\limits_{s\in[1,j-2]^e}(k_s+3)$, $j\in[3,n]^e,\ m\in[1,k_j]$;

(4) $f(u_2^2)=k_2+2,\ f(u_2^j)=-1+\sum\limits_{s\in[1,j]^e}(k_s+3),\ j\in[3,n]^e$;

(5) $f(u_2^1)=R,\ f(u_2^i)=R+\sum\limits_{t\in[1,i-2]^o}(k_t+3),\ i\in[3,n]^o$;

(6) if $k_1\neq 0$, $f(v_{1,m})=R+m,\ m\in[1,k_1]$; for $k_i\neq 0$, $f(v_{i,m})=R+\sum\limits_{t\in[1,i-2]^o}(k_t+3)+m$, $i\in[3,n]^o,\ m\in[1,k_i]$;

(7) $f(w_2)=R+k_1+1,\ f(w_j)=R+\sum\limits_{t\in[1,j-3]^o}(k_t+3)+k_{j-1}+1,\ j\in[3,n]^e$;

(8) $f(u_1^2)=R+k_1+2,\ f(u_1^j)=R+\sum\limits_{t\in[1,j-3]^o}(k_t+3)+k_{j-1}+2,\ j\in[3,n]^e$.

We calculate $f(V(\mathrm{GSG}_{n,1,3}))$. Since

$$\{f(u_1^i)\}\bigcup\{f(w_i)\}\bigcup\{f(v_{j,m})\}\bigcup\{f(u_2^j)\}=[0,\frac{3n-1}{2}+\sum_{s\in[1,n]^e}k_s]=[0,R-2],$$

and

$$\{f(u_2^i)\}\bigcup\{f(v_{i,m})\}\bigcup\{f(w_j)\}\bigcup\{f(u_1^j)\}=[\frac{3n+3}{2}+\sum_{s\in[1,n]^e}k_s,3n+\sum_{i=1}^{n}k_i]=[R,M]$$

for $i\in[1,n]^o$ and $j\in[1,n]^e$. Therefore,

$$f(V(\mathrm{GSG}_{n,1,3}))=[0,R-2]\cup[R,M]\subset[0,M].$$

Next, we compute $f(E(\mathrm{GSG}_{n,1,3}))$ in the following way:

(1) $f(w_1)+f(w_n)=\frac{3n+1}{2}+\sum_{s\in[1,n-1]^e}k_s=R-1$;

(2) for $i\in[1,n]^o$, we have

$$\begin{aligned}f(u_1^i)+f(u_2^i)&=R+\sum_{s\in[1,i]^e}(k_s+3)+\sum_{t\in[1,i-2]^o}(k_t+3)\\&\in\{R,R+k_1+k_2+6,R+k_1+k_2+k_3+k_4+12,\\&\quad R+k_1+k_2+\cdots+k_6+18,\cdots,\\&\quad R+k_1+k_2+\cdots+k_{n-3}+3n-9,\\&\quad R+k_1+k_2+\cdots+k_{n-1}+3n-3\};\end{aligned}$$

(3) for $i\in[1,n]^o$, thus

$$\begin{aligned}f(u_2^i)+f(w_i)&=R+\sum_{s\in[1,i]^e}(k_s+3)+\sum_{t\in[1,i-2]^o}(k_t+3)+1\\&\in\{R+1,R+k_1+k_2+7,R+k_1+k_2+k_3+k_4+13,\\&\quad R+k_1+k_2+\cdots+k_6+19,\cdots,\\&\quad R+k_1+k_2+\cdots+k_{n-3}+3n-8,\\&\quad R+k_1+k_2+\cdots+k_{n-1}+3n-2\};\end{aligned}$$

(4) for $i \in [1,n]^o$, then

$$\begin{aligned} f(w_i)+f(v_{i,m}) &= R+\sum_{s\in[1,i]^e}(k_s+3)+\sum_{t\in[1,i-2]^o}(k_t+3)+1+m_{m\in[1,k_i]} \\ &\in \{R+1+m_{m\in[1,k_1]}, R+k_1+k_2+7+m_{m\in[1,k_3]}, \\ &\quad R+k_1+k_2+k_3+k_4+13+m_{m\in[1,k_5]}, \\ &\quad R+k_1+k_2+\cdots+k_6+19+m_{m\in[1,k_7]},\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-3}+3n-8+m_{m\in[1,k_{n-2}]}, \\ &\quad R+k_1+k_2+\cdots+k_{n-1}+3n-2+m_{m\in[1,k_n]}\}; \end{aligned}$$

(5) when $i \in [1,n]^o$ and $j \in [1,n]^e$, we have

$$\begin{aligned} f(w_i)+f(w_j) &= R+k_{j-1}+2+\sum_{s\in[1,i]^e}(k_s+3)+\sum_{t\in[1,j-3]^o}(k_t+3) \\ &\in \{R+k_1+2, R+k_1+k_2+5, R+k_1+k_2+k_3+8,\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-3}+3n-10, \\ &\quad R+k_1+k_2+\cdots+k_{n-2}+3n-7, \\ &\quad R+k_1+k_2+\cdots+k_{n-1}+3n-4\}; \end{aligned}$$

(6) as $j \in [1,n]^e$, we get

$$\begin{aligned} f(w_j)+f(v_{j,m}) &= R+k_{j-1}+2+\sum_{s\in[1,j-2]^e}(k_s+3)+\sum_{t\in[1,j-3]^o}(k_t+3)+m_{m\in[1,k_j]} \\ &\in \{R+k_1+2+m_{m\in[1,k_2]}, R+k_1+k_2+k_3+8+m_{m\in[1,k_4]}, \\ &\quad R+k_1+k_2+k_3+k_4+k_5+14+m_{m\in[1,k_6]},\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-4}+3n-13+m_{m\in[1,k_{n-3}]}, \\ &\quad R+k_1+k_2+\cdots+k_{n-2}+3n-7+m_{m\in[1,k_{n-1}]}\}; \end{aligned}$$

(7) for $j \in [1,n]^e$,

$$\begin{aligned} f(w_j)+f(u_2^j) &= R+k_{j-1}+\sum_{s\in[1,j]^e}(k_s+3)+\sum_{t\in[1,j-3]^o}(k_t+3) \\ &\in \{R+k_1+k_2+3, R+k_1+k_2+k_3+k_4+9, \\ &\quad R+k_1+k_2+\cdots+k_6+15,\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-3}+3n-12, \\ &\quad R+k_1+k_2+\cdots+k_{n-1}+3n-6\}; \end{aligned}$$

(8) when $j \in [1,n]^e$, we get

$$\begin{aligned}
f(u_1^j)+f(u_2^j) &= R+k_{j-1}+1+\sum_{s\in[1,j]^e}(k_s+3)+\sum_{t\in[1,j-3]^o}(k_t+3)\\
&\in \{R+k_1+k_2+4, R+k_1+k_2+k_3+k_4+10,\\
&\quad R+k_1+k_2+\cdots+k_6+16,\cdots,\\
&\quad R+k_1+k_2+\cdots+k_{n-3}+3n-11,\\
&\quad R+k_1+k_2+\cdots+k_{n-1}+3n-5\}.
\end{aligned}$$

We, finally, obtain

$$\{f(u)+f(v) \mid uv \in E(\mathrm{GSG}_{n,1,3})\} = [R-1, R+\sum_{i=1}^{n} k_i+3n-2]$$

$$= [\frac{3n+1}{2}+\sum_{s\in[1,n-1]^e} k_s, \quad \frac{9n-1}{2}+\sum_{s\in[1,n-1]^e} k_s+\sum_{i=1}^{n} k_i].$$

So

$$f(E(\mathrm{GSG}_{n,1,3})) = \big[0, 3n-1+\sum_{i=1}^{n} k_i\big] = [0, M-1].$$

By Definition 1.1, the generalized sun-graph $\mathrm{GSG}_{n,1,3}$ is felicitous.

Theorem 2.2 For odd $n \geq 3$, every generalized sun-graph $\mathrm{GSG}_{n,2,3}$ admits felicitous labellings.

Proof Use the notation shown in (1.2). Clearly, $|V(\mathrm{GSG}_{n,2,3})| = |E(\mathrm{GSG}_{n,2,3})| = M$, where $M = 5n+\sum\limits_{i=1}^{n} k_i$ (it may be some $k_i = 0$). Let $R = \sum\limits_{s\in[1,n]^e}(k_s+5)+3$. We define a labelling f of the generalized sun-graph $\mathrm{GSG}_{n,2,3}$ by setting

(1) $f(u_{1,1}^1) = 0$, $f(u_{1,1}^i) = \sum\limits_{s\in[1,i]^e}(k_s+5)$, $i \in [3,n]^o$;

(2) $f(w_1) = 1$, $f(w_i) = 1+\sum\limits_{s\in[1,i]^e}(k_s+5)$, $i \in [3,n]^o$;

(3) $f(u_{2,1}^1) = 2$, $f(u_{2,1}^i) = 2+\sum\limits_{s\in[1,i]^e}(k_s+5)$, $i \in [3,n]^o$;

(4) $f(u_{1,2}^2) = 3$, $f(u_{1,2}^j) = 3+\sum\limits_{s\in[1,j-2]^e}(k_s+5)$, $j \in [3,n]^e$;

(5) if $k_2 \neq 0$, $f(v_{2,m}) = m+3$, $m \in [1,k_2]$; for $k_j \neq 0$, $f(v_{j,m}) = 3+m+\sum\limits_{s\in[1,j-2]^e}(k_s+5)$, $j \in [3,n]^e$, $m \in [1,k_j]$;

(6) $f(u_{2,2}^2) = k_2+4$, $f(u_{2,2}^j) = \sum\limits_{s\in[1,j-2]^e}(k_s+5)+k_j+4$, $j \in [3,n]^e$;

(7) $f(u_{1,2}^1) = R$, $f(u_{1,2}^i) = R+\sum\limits_{t\in[1,i-2]^o}(k_t+5)$, $i \in [3,n]^o$;

(8) if $k_1 \neq 0$, $f(v_{1,m}) = R+m$, $m \in [1,k_1]$; for $k_i \neq 0$, $f(v_{i,m}) = R+\sum\limits_{t\in[1,i-2]^o}(k_t+5)+m$, $i \in [3,n]^o$, $m \in [1,k_i]$;

(9) $f(u_{2,2}^1) = R+k_1+1$, $f(u_{2,2}^i) = R+\sum\limits_{t\in[1,i-2]^o}(k_t+5)+k_i+1$, $i \in [3,n]^o$;

(10) $f(u^2_{1,1}) = R + k_1 + 2$, $f(u^j_{1,1}) = R + \sum_{t\in[1,j-3]^o}(k_t+5) + k_{j-1} + 2$, $j \in [3,n]^e$;

(11) $f(w_2) = R + k_1 + 3$, $f(w_j) = R + \sum_{t\in[1,j-3]^o}(k_t+5) + k_{j-1} + 3$, $j \in [3,n]^e$;

(12) $f(u^2_{2,1}) = R + k_1 + 4$, $f(u^j_{2,1}) = R + \sum_{t\in[1,j-3]^o}(k_t+5) + k_{j-1} + 4$, $j \in [3,n]^e$.

The subsets of $f(V(\mathrm{GSG}_{n,2,3}))$ are: for $i \in [1,n]^o$ and $j \in [1,n]^e$, we have

$$\{f(u^i_{1,1})\}\bigcup\{f(w_i)\}\bigcup\{f(u^i_{2,1})\}\bigcup\{f(u^j_{1,2})\}\bigcup\{f(v_{j,m})\}\bigcup\{f(u^j_{2,2})\} = [0, R-1],$$

and

$$\{f(u^i_{1,2})\}\bigcup\{f(v_{i,m})\}\bigcup\{f(u^i_{2,2})\}\bigcup\{f(u^j_{1,1})\}\bigcup\{f(w_j)\}\bigcup\{f(u^j_{2,1})\} = [R, M-1].$$

Hence,

$$f(V(\mathrm{GSG}_{n,2,3})) = [0, M-1] \subset [0, M].$$

We come to compute $f(E(\mathrm{GSG}_{n,2,3}))$ as follows.

(1) $f(w_1) + f(w_n) = 2 + \sum_{s\in[1,n]^e}(k_s+5) = R - 1$;

(2) for $i \in [1,n]^o$, we have

$$\begin{aligned} f(u^i_{1,1}) + f(u^i_{1,2}) &= R + \sum_{s\in[1,i]^e}(k_s+5) + \sum_{t\in[1,i-2]^o}(k_t+5) \\ &\in \{R, R+k_1+k_2+10, R+k_1+k_2+k_3+k_4+20, \cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-3}+5n-15, \\ &\quad R+k_1+k_2+\cdots+k_{n-1}+5n-5\}; \end{aligned}$$

(3) for $i \in [1,n]^o$,

$$\begin{aligned} f(u^i_{1,2}) + f(w_i) &= R + 1 + \sum_{s\in[1,i]^e}(k_s+5) + \sum_{t\in[1,i-2]^o}(k_t+5) \\ &\in \{R+1, R+k_1+k_2+11, R+k_1+k_2+k_3+k_4+21, \cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-3}+5n-14, \\ &\quad R+k_1+k_2+\cdots+k_{n-1}+5n-4\}; \end{aligned}$$

(4) for $i \in [1,n]^o$,

$$\begin{aligned} f(w_i) + f(v_{i,m}) &= R + \sum_{s\in[1,i]^e}(k_s+5) + \sum_{t\in[1,i-2]^o}(k_t+5) + 1 + m_{m\in[1,k_i]} \\ &\in \{R+1+m_{m\in[1,k_1]}, R+k_1+k_2+11+m_{m\in[1,k_3]}, \\ &\quad R+k_1+k_2+k_3+k_4+21+m_{m\in[1,k_5]}, \cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-3}+5n-14+m_{m\in[1,k_{n-2}]}, \\ &\quad R+k_1+k_2+\cdots+k_{n-1}+5n-4+m_{m\in[1,k_n]}\}; \end{aligned}$$

(5) for $i \in [1,n]^o$,

$$\begin{aligned} f(w_i)+f(u^i_{2,2}) &= R+k_i+2+\sum_{s\in[1,i]^e}(k_s+5)+\sum_{t\in[1,i-2]^o}(k_t+5) \\ &\in \{R+k_1+2, R+k_1+k_2+k_3+12, \\ &\quad R+k_1+k_2+\cdots+k_5+22,\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-2}+5n-13, \\ &\quad R+k_1+k_2+\cdots+k_n+5n-3\}; \end{aligned}$$

(6) for $i \in [1,n]^o$,

$$\begin{aligned} f(u^i_{2,1})+f(u^i_{2,2}) &= R+k_i+3+\sum_{s\in[1,i]^e}(k_s+5)+\sum_{t\in[1,i-2]^o}(k_t+5) \\ &\in \{R+k_1+3, R+k_1+k_2+k_3+13, \\ &\quad R+k_1+k_2+\cdots+k_5+23,\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-2}+5n-12, \\ &\quad R+k_1+k_2+\cdots+k_n+5n-2\}; \end{aligned}$$

(7) when $i \in [1,n]^o$ and $j \in [1,n]^e$,

$$\begin{aligned} f(w_i)+f(w_j) &= R+k_{j-1}+4+\sum_{s\in[1,i]^e}(k_s+5)+\sum_{t\in[1,j-3]^o}(k_t+5) \\ &\in \{R+k_1+4, R+k_1+k_2+9, \\ &\quad R+k_1+k_2+k_3+14,\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-3}+5n-16, \\ &\quad R+k_1+k_2+\cdots+k_{n-2}+5n-11, \\ &\quad R+k_1+k_2+\cdots+k_{n-1}+5n-6\}; \end{aligned}$$

(8) for $j \in [1,n]^e$,

$$\begin{aligned} f(u^j_{1,1})+f(u^j_{1,2}) &= R+k_{j-1}+5+\sum_{s\in[1,j-2]^e}(k_s+5)+\sum_{t\in[1,j-3]^o}(k_t+5) \\ &\in \{R+k_1+5, R+k_1+k_2+k_3+15, \\ &\quad R+k_1+k_2+\cdots+k_5+25,\cdots, \\ &\quad R+k_1+k_2+\cdots+k_{n-4}+5n-20, \\ &\quad R+k_1+k_2+\cdots+k_{n-2}+5n-10\}; \end{aligned}$$

(9) for $j \in [1,n]^e$,

$$\begin{aligned} f(w_j)+f(u^j_{1,2}) &= R+k_{j-1}+6+\sum_{s\in[1,j-2]^e}(k_s+5)+\sum_{t\in[1,j-3]^o}(k_t+5)\\ &\in \{R+k_1+6, R+k_1+k_2+k_3+16,\\ &\quad R+k_1+k_2+\cdots+k_5+26,\cdots,\\ &\quad R+k_1+k_2+\cdots+k_{n-4}+5n-19,\\ &\quad R+k_1+k_2+\cdots+k_{n-2}+5n-9\}; \end{aligned}$$

(10) $j \in [1,n]^e$,

$$\begin{aligned} f(w_j)+f(v_{j,m}) &= R+k_{j-1}+6+\sum_{s\in[1,j-2]^e}(k_s+5)+\sum_{t\in[1,j-3]^o}(k_t+5)+m_{m\in[1,k_j]}\\ &\in \{R+k_1+6+m_{m\in[1,k_2]}, R+k_1+k_2+k_3+16+m_{m\in[1,k_4]},\\ &\quad R+k_1+k_2+k_3+k_4+k_5+26+m_{m\in[1,k_6]},\cdots,\\ &\quad R+k_1+k_2+\cdots+k_{n-4}+5n-19+m_{m\in[1,k_{n-3}]},\\ &\quad R+k_1+k_2+\cdots+k_{n-2}+5n-9+m_{m\in[1,k_{n-1}]}\}; \end{aligned}$$

(11) $j \in [1,n]^e$,

$$\begin{aligned} f(w_j)+f(u^j_{2,2}) &= R+k_{j-1}+k_j+7+\sum_{s\in[1,j-2]^e}(k_s+5)+\sum_{t\in[1,j-3]^o}(k_t+5)\\ &\in \{R+k_1+k_2+7, R+k_1+k_2+k_3+k_4+17,\\ &\quad R+k_1+k_2+\cdots+k_6+27,\cdots,\\ &\quad R+k_1+k_2+\cdots+k_{n-3}+5n-18,\\ &\quad R+k_1+k_2+\cdots+k_{n-1}+5n-8\}; \end{aligned}$$

(12) $j \in [1,n]^e$,

$$\begin{aligned} f(u^j_{2,1})+f(u^j_{2,2}) &= R+k_{j-1}+k_j+8+\sum_{s\in[1,j-2]^e}(k_s+5)+\sum_{t\in[1,j-3]^o}(k_t+5)\\ &\in \{R+k_1+k_2+8, R+k_1+k_2+k_3+k_4+18,\\ &\quad R+k_1+k_2+\cdots+k_6+28,\cdots,\\ &\quad R+k_1+k_2+\cdots+k_{n-3}+5n-17,\\ &\quad R+k_1+k_2+\cdots+k_{n-1}+5n-7\}. \end{aligned}$$

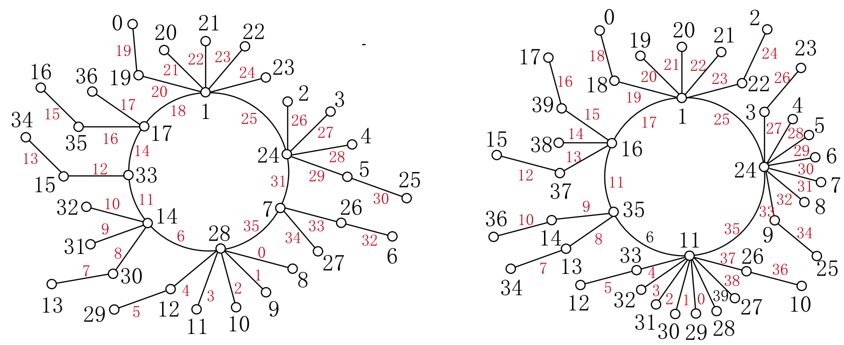
Thereby, we get

$$\{f(u)+f(v) \mid uv\in E(\mathrm{GSG}_{n,2,3})\} = \big[R-1, R+\sum_{i=1}^{n}k_i+5n-2\big]$$

$$= \big[\sum_{s\in[1,n]^e}(k_s+5)+2, \sum_{s\in[1,n]^e}(k_s+5)+\sum_{i=1}^{n}k_i+5n+1\big],$$

and furthermore $f(E(\mathrm{GSG}_{n,2,3})) = \left[0, \sum\limits_{i=1}^{n} k_i + 5n - 1\right] = [0, M-1]$. We claim that $\mathrm{GSG}_{n,2,3}$ is felicitous.

We propose a conjecture: every unicyclic graph is felicitous.



(a) (b)

Figure 1: (a) is for illustrating Theorem 2.1; (b) is for illustrating Theorem 2.2.

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广义太阳图的 Felicitous 性质

杨思华[1], 姚 兵[1], 姚 明[2]

(1. 西北师范大学数学与统计学院, 甘肃 兰州 730070)

(2. 兰州石化职业技术学院信息处理与控制工程系, 甘肃 兰州 730060)

摘 要: 本文研究了广义太阳图的 felicitous 标号. 利用广义太阳图的结构特征, 获得了 2 类特殊广义太阳图的精确 felicitous 标号. 并且, 这些类图论模型在编码理论、通讯网络、物流等方面均有重要的应用.

关键词: felicitous 标号; 太阳图; 广义太阳图

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