ON FELICITOUS CHARACTER OF GENERALIZED SUN-GRAPHS

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Abstract: Felicitous character of some generalized sun-graphs is investigated in this note, and furthermore the exact felicitous labellings of two classes of generalized sun-graphs are obtained by analyzing the structures of the generalized sun-graphs. And the constructed graph theory models in coding theory, communication networks, logistics and other aspects have important applications.

Keywords: felicitous labelling; sun graph; generalized sun-graph

2010 MR Subject Classification: 05C78


1 Introduction

As known, graph labelling theory has important applications in coding theory, communication networks, logistics and other aspects of science. Some generalized sun-graphs can be regarded as ring-like real-networks. Each node in a network represents a server or client, and every edge joins two nodes if these two nodes are connected in the network. In Operations Research or Systems Engineering Theory and Methods, one very often use graph colorings to divide large systems into subsystems, and one, also, can use graph labellings to distinguish nodes and edges between nodes in order to find some fast algorithms to imitate effective transmission and safe communication in information networks.

On the other hand, Rosa [1] in 1966 proposed a conjecture: Every tree is a graceful tree. There are a lot of results for attacking this conjecture, but it has been known that this conjecture has not been settled down up to now. Lee et al. [2] in 1991 put forward a conjecture: Every tree is a felicitous tree that has the same theoretical value to the graceful tree conjecture and also has the same difficulty to be proved. Both conjectures are NP-hard problems that have attracted many researchers to explore them [3–6]. Attacking mathematical conjecture makes that graph labelling theory rapidly becomes a very active branch in graph theory.

Received date: 2014-01-07 Accepted date: 2014-09-02
Foundation item: Supported by the National Natural Science Foundation of China under Grant (61163054; 61163037; 61363060); Research Projects of Graduate Teacher of Gansu University (1216-01).
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We use standard terminology and notation of graph theory, and graphs mentioned here are finite, simple and undirected. A \((p, q)\)-graph has \(p\) vertices and \(q\) edges. An integer set \(\{m, m+1, m+2, \ldots, n\}\) with integers \(0 \leq m < n\) is denoted as \([m, n]\); similarly, \([k, \ell]^{e}\) denotes an integer set \(\{k, k+2, k+4, \ldots, \ell\}\), where \(k\) and \(\ell\) are even with respect to \(0 \leq k < \ell\); and \([s, t]^{o}\) stands for an integer set \(\{s, s+2, s+4, \ldots, t\}\) with odd integers \(s\) and \(t\) holding \(1 \leq s < t\). A leaf is a vertex of degree one.

**Definition 1.1** [3] A \((p, q)\)-graph \(G\) has a proper mapping \(f : V(G) \to [0, q]\), and the edge label \(f(uv)\) of each edge \(uv \in E(G)\) is defined as \(f(uv) = f(u) + f(v) (\text{mod } q)\). If \(f(u) \neq f(v)\) for distinct \(u, v \in V(G)\), and the edge label set \(\{f(uv) : uv \in E(G)\} = [0, q-1]\), then we say both \(G\) and \(f\) to be felicitous. And write \(f(V(G)) = \{f(u) : u \in V(G)\}\), \(f(E(G)) = \{f(uv) = f(u) + f(v) (\text{mod } q) : uv \in E(G)\}\).

Let \(C_n = w_1w_2 \cdots w_nw_1\) be a cycle of length \(n\), where \(n \geq 3\). Joining each vertex \(w_i\) \((i \in [1, n])\) of \(C_n\) with a new vertex \(y_i\) out of \(C_n\) by an edge obtains a graph, called a sun-graph and denoted as \(SC_n\). Furthermore, for integers \(r_i \geq 1\) and \(k_i \geq 0\) with \(i \in [1, n]\), we join each vertex \(w_i\) of \(C_n\) and the initial vertex \(u^i_{j,m_{i,j}}\) of a path \(P^i_j = u^i_{j,1}u^i_{j,2} \cdots u^i_{j,m_{i,j}}\) for \(m_{i,j} \geq 2\), \(j \in [1, r_i]\), and join each vertex \(w_i\) with \(k_i\) new vertices \(v_{i,t}\) for \(t \in [1, k_i]\) to produce a generalized sun-graph, denoted by \(GSG(m_{i,j}, r_i, k_i : i \in [1, n])\). It is reasonable to allow some \(k_i = 0\), that is, no joining the vertex \(w_i\) of \(C_n\) with new vertex. Clearly, a generalized sun-graph is just a sun-graph when \(m_{i,j} = 1\), \(r_i = 1\) and \(k_i = 0\) for \(i \in [1, n]\).

In this article, we focus on finding felicitous labellings of \(GSG(m_{i,j}, r_i, k_i : i \in [1, n])\) for \(r_i = 1\) or \(2\) and \(m_{i,j} = 2\) with \(i \in [1, n]\) and odd \(n\). For the purpose of simplicity, we rewrite \(GSG_{n,1,3} = GSG(m_{i,j} : j \leq r_i, i \in [1, n])\) when \(r_i = 1\), \(m_{i,j} = 2\) and \(k_i \geq 0\). Thereby, we have

\[
V(GSG_{n,1,3}) = \{u^i_{1,1}, u^i_{1,2}, w_i, v_{i,j} \mid j \in [1, k_i], i \in [1, n]\},
\]

where \(u^i_{1,1} = u^i_{1,2}\) and \(u^i_{1,2} = u^i_{1,2}\). Similarly, \(GSG_{n,2,3} = GSG(m_{i,j} : j \leq r_i, i \in [1, n])\) when \(r_i = 2\), \(m_{i,j} = 2\) and \(k_i \geq 0\), and furthermore

\[
V(GSG_{n,2,3}) = \{u^i_{t,1}, u^i_{t,2}, w_i, v_{i,j} \mid t \in [1, 2], j \in [1, k_i], i \in [1, n]\}.
\]

**2 Main Results**

**Theorem 2.1** For odd \(n \geq 3\), every generalized sun-graph \(GSG_{n,1,3}\) admits a felicitous labelling.

**Proof** Apply the notation in (1.1) for \(GSG_{n,1,3}\). Clearly,

\[
|V(GSG_{n,1,3})| = |E(GSG_{n,1,3})| = M,
\]

where \(M = 3n + \sum_{i=1}^{n} k_i\) (it may be some \(k_i = 0\)). Let \(R = \frac{3n+3}{2} + \sum_{s \in [1, o]} k_s\). We define a labelling \(f\) of \(GSG_{n,1,3}\) as follows.

1. \(f(u^i_{1,1}) = 0, f(u^i_{1,2}) = \sum_{s \in [1, o]} (k_s + 3), i \in [3, n]\);
(2) \( f(w_1) = 1, \ f(w_i) = 1 + \sum_{s \in [1,i]} (k_s + 3), \ i \in [3,n]^c; \)
(3) if \( k_2 \neq 0, \ f(v_{2,m}) = m + 1, \ m \in [1, k_2]; \) for \( k_j \neq 0, \ f(v_{j,m}) = 1 + m + \sum_{s \in [1,j-2]} (k_s + 3), \)
\( j \in [3,n]^c, \ m \in [1, k_j]; \)
(4) \( f(u_2^i) = k_2 + 2, \ f(u_2^j) = -1 + \sum_{s \in [1,j]} (k_s + 3), \ j \in [3,n]^c; \)
(5) \( f(u_2^i) = R, \ f(u_2^j) = R + \sum_{t \in [1,j-2]} (k_t + 3), \ i \in [3,n]^o; \)
(6) if \( k_1 \neq 0, \ f(v_{1,m}) = R + m, \ m \in [1, k_1]; \) for \( k_i \neq 0, \ f(v_{i,m}) = R + \sum_{t \in [1,i-2]} (k_t + 3) + m, \)
\( i \in [3,n]^o, \ m \in [1, k_i]; \)
(7) \( f(w_2) = R + k_1 + 1, \ f(w_j) = R + \sum_{t \in [1,j-3]} (k_t + 3) + j, \ j \in [3,n]^c; \)
(8) \( f(u_2^i) = R + k_1 + 2, \ f(u_2^j) = R + \sum_{t \in [1,j-3]} (k_t + 3) + j, \ j \in [3,n]^c. \)

We calculate \( f(V(GSG_{n,1,3})). \) Since
\[
\{f(u_1^i)\} \bigcup \{f(v_i)\} \bigcup \{f(v_{j,m})\} \bigcup \{f(u_2^i)\} = [0, \frac{3n - 1}{2} + \sum_{s \in [1,n]} k_s] = [0, R - 2],
\]
and
\[
\{f(u_2^i)\} \bigcup \{f(v_{i,m})\} \bigcup \{f(v_j)\} \bigcup \{f(u_2^j)\} = [\frac{3n + 3}{2} + \sum_{s \in [1,n]} k_s, 3n + \sum_{i=1}^{n} k_i] = [R, M]
\]
for \( i \in [1,n]^o \) and \( j \in [1,n]^c. \) Therefore,
\[
f(V(GSG_{n,1,3})) = [0, R - 2] \cup [R, M] \subset [0, M].
\]

Next, we compute \( f(E(GSG_{n,1,3})) \) in the following way:
(1) \( f(w_1) + f(w_n) = \frac{3n+1}{2} + \sum_{s \in [1,n-1]} k_s = R - 1; \)
(2) for \( i \in [1,n]^o, \) we have
\[
f(u_1^i) + f(u_2^i) = R + \sum_{s \in [1,i]} (k_s + 3) + \sum_{t \in [1,i-2]} (k_t + 3)
\in \{ R, R + k_1 + k_2 + 6, R + k_1 + k_2 + k_3 + k_4 + 12, \\
R + k_1 + k_2 + \cdots + k_6 + 18, \cdots, \\
R + k_1 + k_2 + \cdots + k_{n-3} + 3n - 9, \\
R + k_1 + k_2 + \cdots + k_{n-1} + 3n - 3 \};
\]
(3) for \( i \in [1,n]^o, \) thus
\[
f(u_1^i) + f(w_i) = R + \sum_{s \in [1,i]} (k_s + 3) + \sum_{t \in [1,i-2]} (k_t + 3) + 1
\in \{ R + 1, R + k_1 + k_2 + 7, R + k_1 + k_2 + k_3 + k_4 + 13, \\
R + k_1 + k_2 + \cdots + k_6 + 19, \cdots, \\
R + k_1 + k_2 + \cdots + k_{n-3} + 3n - 8, \\
R + k_1 + k_2 + \cdots + k_{n-1} + 3n - 2 \};
(4) for \( i \in [1, n]^{\circ} \), then

\[
\begin{align*}
    f(w_i) + f(v_{i,m}) & = R + \sum_{s \in [1,i]} (k_s + 3) + \sum_{t \in [1,i-2]} (k_t + 3) + 1 + m_{m \in [1,k_i]} \\
    & \in \{ R + 1 + m_{m \in [1,k_i]}, R + k_1 + k_2 + 7 + m_{m \in [1,k_3]}, R + k_1 + k_2 + k_3 + k_4 + 13 + m_{m \in [1,k_5]}, R + k_1 + k_2 + \cdots + k_6 + 19 + m_{m \in [1,k_7]}, \cdots, R + k_1 + k_2 + \cdots + k_{n-3} + 3n - 8 + m_{m \in [1,k_{n-2}]}, R + k_1 + k_2 + \cdots + k_{n-1} + 3n - 2 + m_{m \in [1,k_n]} \};
\end{align*}
\]

(5) when \( i \in [1, n]^{\circ} \) and \( j \in [1, n]^{\circ} \), we have

\[
\begin{align*}
    f(w_i) + f(w_j) & = R + k_{j-1} + 2 + \sum_{s \in [1,j]} (k_s + 3) + \sum_{t \in [1,j-3]} (k_t + 3) \\
    & \in \{ R + k_1 + 2, R + k_1 + k_2 + 5, R + k_1 + k_2 + k_3 + 8, \cdots, R + k_1 + k_2 + \cdots + k_{n-3} + 3n - 10, R + k_1 + k_2 + \cdots + k_{n-2} + 3n - 7, R + k_1 + k_2 + \cdots + k_{n-1} + 3n - 4 \};
\end{align*}
\]

(6) as \( j \in [1, n]^{\circ} \), we get

\[
\begin{align*}
    f(w_j) + f(v_{j,m}) & = R + k_{j-1} + 2 + \sum_{s \in [1,j-2]} (k_s + 3) + \sum_{t \in [1,j-3]} (k_t + 3) + m_{m \in [1,k_i]} \\
    & \in \{ R + k_1 + 2 + m_{m \in [1,k_2]}, R + k_1 + k_2 + 3 + k_3 + 8 + m_{m \in [1,k_4]}, R + k_1 + k_2 + k_3 + k_4 + 14 + m_{m \in [1,k_6]}, \cdots, R + k_1 + k_2 + \cdots + k_{n-4} + 3n - 13 + m_{m \in [1,k_{n-3}]}, R + k_1 + k_2 + \cdots + k_{n-2} + 3n - 7 + m_{m \in [1,k_{n-1}]} \};
\end{align*}
\]

(7) for \( j \in [1, n]^{\circ} \),

\[
\begin{align*}
    f(w_j) + f(u_j^l) & = R + k_{j-1} + \sum_{s \in [1,j]} (k_s + 3) + \sum_{t \in [1,j-3]} (k_t + 3) \\
    & \in \{ R + k_1 + k_2 + 3, R + k_1 + k_2 + k_3 + k_4 + 9, R + k_1 + k_2 + \cdots + k_6 + 15, \cdots, R + k_1 + k_2 + \cdots + k_{n-3} + 3n - 12, R + k_1 + k_2 + \cdots + k_{n-1} + 3n - 6 \};
\end{align*}
\]
(8) when \( j \in [1, n]^c \), we get
\[
f(u'_1) + f(u'_2) = R + k_{j-1} + 1 + \sum_{s \in [1, j]^e} (k_s + 3) + \sum_{t \in [1, j-3]^o} (k_t + 3)
\]
\[
\in \{ R + k_1 + k_2 + 4, R + k_1 + k_2 + k_3 + k_4 + 10, R + k_1 + k_2 + \cdots + k_6 + 16, \cdots, R + k_1 + k_2 + \cdots + k_{n-3} + 3n - 11, R + k_1 + k_2 + \cdots + k_{n-1} + 3n - 5 \}.
\]

We, finally, obtain
\[
\{ f(u) + f(v) \mid uv \in E(GSG_{n,1,3}) \} = [R - 1, R + \sum_{i=1}^{n} k_i + 3n - 2]
\]
\[
= \left[ \frac{3n+1}{2} + \sum_{s \in [1, n-1]^e} k_s, \frac{9n-1}{2} + \sum_{s \in [1, n-1]^e} k_s + \sum_{i=1}^{n} k_i \right].
\]

So
\[
f(E(GSG_{n,1,3})) = [0, 3n - 1 + \sum_{i=1}^{n} k_i] = [0, M - 1].
\]

By Definition 1.1, the generalized sun-graph \( GSG_{n,1,3} \) is felicitous.

**Theorem 2.2** For odd \( n \geq 3 \), every generalized sun-graph \( GSG_{n,2,3} \) admits felicitous labellings.

**Proof** Use the notation shown in (1.2). Clearly, \( |V(GSG_{n,2,3})| = |E(GSG_{n,2,3})| = M \), where \( M = 5n + \sum_{i=1}^{n} k_i \) (it may be some \( k_i = 0 \)). Let \( R = \sum_{s \in [1, n]^e} (k_s + 5) + 3 \). We define a labelling \( f \) of the generalized sun-graph \( GSG_{n,2,3} \) by setting

1. \( f(u'_{1,1}) = 0 \), \( f(u'_{1,2}) = \sum_{s \in [1, j]^o} (k_s + 5), i \in [3, n]^o; \)
2. \( f(w_1) = 1 \), \( f(w_i) = 1 + \sum_{s \in [1, j]^o} (k_s + 5), i \in [3, n]^o; \)
3. \( f(u'_{2,1}) = 2 \), \( f(u'_{2,2}) = 2 + \sum_{s \in [1, j]^o} (k_s + 5), i \in [3, n]^o; \)
4. \( f(u'_{2,1}) = 3 \), \( f(u'_{2,2}) = 3 + \sum_{s \in [1, j-2]^o} (k_s + 5), j \in [3, n]^c; \)
5. if \( k_2 \neq 0 \), \( f(v_{2,m}) = m+3, m \in [1, k_2]; \) for \( k_j \neq 0 \), \( f(v_{j,m}) = 3 + m + \sum_{s \in [1, j-2]^e} (k_s+5), j \in [3, n]^c, m \in [1, k_j]; \)
6. \( f(u'_{2,2}) = k_2 + 4, f(u'_{2,2}) = \sum_{s \in [1, j-2]^e} (k_s + 5) + k_j + 4, j \in [3, n]^c; \)
7. \( f(u'_{1,2}) = R \), \( f(u'_{1,2}) = R + \sum_{t \in [1, j-2]^o} (k_t + 5), i \in [3, n]^o; \)
8. if \( k_1 \neq 0 \), \( f(v_{1,m}) = R + m, m \in [1, k_1]; \) for \( k_i \neq 0 \), \( f(v_{i,m}) = R + \sum_{t \in [1, j-2]^o} (k_t+5) + m, i \in [3, n]^o, m \in [1, k_i]; \)
9. \( f(u'_{2,2}) = R + k_1 + 1 \), \( f(u'_2) = R + \sum_{t \in [1, j-2]^o} (k_t + 5) + k_i + 1, i \in [3, n]^o; \)
(10) \( f(u_{1,1}^2) = R + k_1 + 2 \), \( f(u_{1,4}) = R + \sum_{t \in [1,j-3]^o} (k_t + 5) + k_{j-1} + 2, j \in [3,n]^e \);  

(11) \( f(w_2) = R + k_1 + 3 \), \( f(w_j) = R + \sum_{t \in [1,j-3]^o} (k_t + 5) + k_{j-1} + 3, j \in [3,n]^e \);  

(12) \( f(u_{2,1}^2) = R + k_1 + 4 \), \( f(u_{2,4}) = R + \sum_{t \in [1,j-3]^o} (k_t + 5) + k_{j-1} + 4, j \in [3,n]^e \).

The subsets of \( f(V(GSG_{n,2,3})) \) are: for \( i \in [1,n]^o \) and \( j \in [1,n]^e \), we have

\[
\{f(u_{1,1}^i)\} \cup \{f(w_i)\} \cup \{f(u_{2,1}^i)\} \cup \{f(u_{1,2}^i)\} \cup \{f(v_i, m)\} \cup \{f(u_{2,2}^i)\} = [0, R - 1],
\]

and

\[
\{f(u_{1,2}^i)\} \cup \{f(v_i, m)\} \cup \{f(u_{2,2}^i)\} \cup \{f(u_{1,1}^i)\} \cup \{f(w_i)\} \cup \{f(u_{2,1}^i)\} = [R, M - 1].
\]

Hence,

\[
f(V(GSG_{n,2,3})) = [0, M - 1] \subset [0, M].
\]

We come to compute \( f(E(GSG_{n,2,3})) \) as follows.

(1) \( f(w_1) + f(w_n) = 2 + \sum_{s \in [1,n]^o} (k_s + 5) = R - 1 \);  

(2) for \( i \in [1,n]^o \), we have

\[
f(u_{1,1}^i) + f(u_{1,2}^i) = R + \sum_{s \in [1,i]^o} (k_s + 5) + \sum_{t \in [1,i-2]^o} (k_t + 5) \\
\in \{R, R + k_1 + k_2 + 10, R + k_1 + k_2 + k_3 + k_4 + 20, \cdots , R + k_1 + k_2 + \cdots + k_{n-3} + 5n - 15, R + k_1 + k_2 + \cdots + k_{n-1} + 5n - 5\};
\]

(3) for \( i \in [1,n]^o \),

\[
f(u_{1,2}^i) + f(w_i) = R + 1 + \sum_{s \in [1,i]^o} (k_s + 5) + \sum_{t \in [1,i-2]^o} (k_t + 5) \\
\in \{R + 1, R + k_1 + k_2 + 11, R + k_1 + k_2 + k_3 + k_4 + 21, \cdots , R + k_1 + k_2 + \cdots + k_{n-3} + 5n - 14, R + k_1 + k_2 + \cdots + k_{n-1} + 5n - 4\};
\]

(4) for \( i \in [1,n]^o \),

\[
f(w_i) + f(v_{i,m}) = R + \sum_{s \in [1,i]^o} (k_s + 5) + \sum_{t \in [1,i-2]^o} (k_t + 5) + 1 + m_{m \in [1,k_i]} \\
\in \{R + 1 + m_{m \in [1,k_i]}, R + k_1 + k_2 + 11 + m_{m \in [1,k_i]}, R + k_1 + k_2 + k_3 + k_4 + 21 + m_{m \in [1,k_i]}, \cdots , R + k_1 + k_2 + \cdots + k_{n-3} + 5n - 14 + m_{m \in [1,k_{n-2}]}, R + k_1 + k_2 + \cdots + k_{n-1} + 5n - 4 + m_{m \in [1,k_{n-1}]})};
\]
(5) for $i \in [1, n]^o$,

$$f(w_i) + f(u_{2,2}^i) = R + k_i + 2 + \sum_{s \in [1, i]^o} (k_s + 5) + \sum_{t \in [1, i-2]^o} (k_t + 5)$$

$$\in \{ R + k_1 + 2, R + k_1 + k_2 + k_3 + 12, R + k_1 + k_2 + \cdots + k_5 + 22, \cdots, R + k_1 + k_2 + \cdots + k_{n-2} + 5n - 13, R + k_1 + k_2 + \cdots + k_n + 5n - 3 \};$$

(6) for $i \in [1, n]^o$,

$$f(u_{2,1}^i) + f(u_{2,2}^i) = R + k_i + 3 + \sum_{s \in [1, i]^o} (k_s + 5) + \sum_{t \in [1, i-2]^o} (k_t + 5)$$

$$\in \{ R + k_1 + 3, R + k_1 + k_2 + k_3 + 13, R + k_1 + k_2 + \cdots + k_5 + 23, \cdots, R + k_1 + k_2 + \cdots + k_{n-2} + 5n - 12, R + k_1 + k_2 + \cdots + k_n + 5n - 2 \};$$

(7) when $i \in [1, n]^o$ and $j \in [1, n]^e$,

$$f(w_i) + f(w_j) = R + k_{j-1} + 4 + \sum_{s \in [1, i]^o} (k_s + 5) + \sum_{t \in [1, j-3]^o} (k_t + 5)$$

$$\in \{ R + k_1 + 4, R + k_1 + k_2 + 9, R + k_1 + k_2 + k_3 + 14, \cdots, R + k_1 + k_2 + \cdots + k_{n-3} + 5n - 16, R + k_1 + k_2 + \cdots + k_{n-2} + 5n - 11, R + k_1 + k_2 + \cdots + k_{n-1} + 5n - 6 \};$$

(8) for $j \in [1, n]^e$,

$$f(u_{1,1}^j) + f(u_{1,2}^j) = R + k_{j-1} + 5 + \sum_{s \in [1, j-2]^o} (k_s + 5) + \sum_{t \in [1, j-3]^o} (k_t + 5)$$

$$\in \{ R + k_1 + 5, R + k_1 + k_2 + k_3 + 15, R + k_1 + k_2 + \cdots + k_5 + 25, \cdots, R + k_1 + k_2 + \cdots + k_{n-4} + 5n - 20, R + k_1 + k_2 + \cdots + k_{n-2} + 5n - 10 \};$$
(9) for $j \in [1, n]^c$,
\[
    f(w_j) + f(u_{1,j}^1) = R + k_{j-1} + 6 + \sum_{s \in [1,j-2]^c} (k_s + 5) + \sum_{t \in [1,j-3]^c} (k_t + 5)
\]
\[
\in \{ R + k_1 + 6, R + k_1 + k_2 + k_3 + 16, \\
R + k_1 + k_2 + \cdots + k_5 + 26, \cdots, \\
R + k_1 + k_2 + \cdots + k_{n-4} + 5n - 19, \\
R + k_1 + k_2 + \cdots + k_{n-2} + 5n - 9 \};
\]

(10) $j \in [1, n]^c$,
\[
f(w_j) + f(v_{j,m}) = R + k_{j-1} + 6 + \sum_{s \in [1,j-2]^c} (k_s + 5) + \sum_{t \in [1,j-3]^c} (k_t + 5) + m_{m \in [1,k_j]}
\]
\[
\in \{ R + k_1 + 6 + m_{m \in [1,k_2]}, R + k_1 + k_2 + k_3 + 16 + m_{m \in [1,k_3]}, \\
R + k_1 + k_2 + k_3 + k_4 + k_5 + 26 + m_{m \in [1,k_4]}, \cdots, \\
R + k_1 + k_2 + \cdots + k_{n-4} + 5n - 19 + m_{m \in [1,k_{n-3}]}, \\
R + k_1 + k_2 + \cdots + k_{n-2} + 5n - 9 + m_{m \in [1,k_{n-1}]} \};
\]

(11) $j \in [1, n]^c$,
\[
f(w_j) + f(u_{2,j}^1) = R + k_{j-1} + k_j + 7 + \sum_{s \in [1,j-2]^c} (k_s + 5) + \sum_{t \in [1,j-3]^c} (k_t + 5)
\]
\[
\in \{ R + k_1 + k_2 + 7, R + k_1 + k_2 + k_3 + k_4 + 17, \\
R + k_1 + k_2 + \cdots + k_6 + 27, \cdots, \\
R + k_1 + k_2 + \cdots + k_{n-3} + 5n - 18, \\
R + k_1 + k_2 + \cdots + k_{n-1} + 5n - 8 \};
\]

(12) $j \in [1, n]^c$,
\[
f(u_{2,1}^1) + f(u_{2,j}^2) = R + k_{j-1} + k_j + 8 + \sum_{s \in [1,j-2]^c} (k_s + 5) + \sum_{t \in [1,j-3]^c} (k_t + 5)
\]
\[
\in \{ R + k_1 + k_2 + 8, R + k_1 + k_2 + k_3 + k_4 + 18, \\
R + k_1 + k_2 + \cdots + k_6 + 28, \cdots, \\
R + k_1 + k_2 + \cdots + k_{n-3} + 5n - 17, \\
R + k_1 + k_2 + \cdots + k_{n-1} + 5n - 7 \}.
\]

Thereby, we get
\[
\{ f(u) + f(v) \mid uv \in E(GSG_{n,2,3}) \} = [ R - 1, R + \sum_{i=1}^{n} k_i + 5n - 2 ]
\]
\[
= \left[ \sum_{s \in [1,n]^c} (k_s + 5) + 2, \sum_{s \in [1,n]^c} (k_s + 5) + \sum_{i=1}^{n} k_i + 5n + 1 \right],
\]
and furthermore \( f(E(GSG_{n,2,3})) = \sum_{i=1}^{n} k_i + 5n - 1 = [0, M - 1] \). We claim that \( GSG_{n,2,3} \) is felicitous.

We propose a conjecture: every unicyclic graph is felicitous.

![Figure 1: (a) is for illustrating Theorem 2.1; (b) is for illustrating Theorem 2.2.](image-url)

References


广义太阳图的 Felicitous 性质

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摘要: 本文研究了广义太阳图的 felicitous 标号, 利用广义太阳图的结构特征, 获得了 2 类特殊广义太阳图的精确定位 felicitous 标号, 并且, 这些类图论模型在编码理论、通讯网络、物流等方面均有的重要的应用。关键词: felicitous 标号; 太阳图; 广义太阳图