A NEW FAMILY OF ELEMENTS IN THE STABLE HOMOTOPY GROUPS OF SPHERES

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Abstract: In this paper, we study the non-triviality of the elements in the stable homotopy groups of spheres. Using the May spectral sequence, the authors show that there exists a new product in the $E_2$-term of the Adams spectral sequence, which converges to a family of homotopy elements with order $p$ and higher filtration in the stable homotopy groups of spheres.

Keywords: stable homotopy groups of spheres; Toda-Smith spectrum; sphere spectrum; Adams spectral sequence; May spectral sequence

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1 Introduction

To determine the stable homotopy groups of spheres $\pi_*(S)$ is one of the central problems in homotopy theory. One of the main tools to reach it is the Adams spectral sequence (ASS).

Let $A$ be the mod $p$ Steenrod algebra, and $S$ be the sphere spectrum localized at an odd prime $p$. For connected finite type spectra $X, Y$, there exists the ASS $\{E_{s,t}^r, d_r\}$ such that

1. $d_r : E_{s,t}^r \to E_{s+r,t+r-1}^r$ is the differential;
2. $E_{2}^{s,t} \cong \text{Ext}_A^{s,t}(H^*(X), H^*(Y)) \Rightarrow [\Sigma^{-s}Y, X]_p$, where $E_{2}^{s,t}$ is the cohomology of $A$. When $X$ is sphere spectrum $S$, Toda-Smith spectrum $V(n)(n = 1, 2, 3)$, respectively, $(\pi_{s-r}(X))_p$ is the stable homotopy groups of $S, V(n)$. So, for computing the stable homotopy groups of spheres with the ASS, we must compute the $E_2$-term of the ASS, $\text{Ext}_A^{s,t}(Z_p, Z_p)$.

From [1], $\text{Ext}_A^{s,t}(Z_p, Z_p)$ has the $Z_p$-base consisting of

$$a_0 \in \text{Ext}_A^{1,1}(Z_p, Z_p), h_i \in \text{Ext}_A^{1,p+1}(Z_p, Z_p)$$

for all $i \geq 0$ and $\text{Ext}_A^{2,r}(Z_p, Z_p)$ has the $Z_p$-base consisting of $a_2, a_2^*, a_0 h_i(i > 0), g_i(i \geq 0), k_i(i \geq 0), b_j(i \geq 0)$ and $h_i h_j(j \geq i + 2, i \geq 0)$ whose internal degrees are $2q + 1, 2, p^i q + 1, p^{i+1} q + 2p^i q, 2p^{i+1} q + p^i q, p^{i+1} q$ and $p^i q + p^i q$, respectively. From [2, P.110,
Table 8.1, the \( \mathbb{Z}_p \)-base of \( \text{Ext}^3_A(\mathbb{Z}_p, \mathbb{Z}_p) \) has been completely listed and there is a generator
\( \tilde{\gamma}_t \in \text{Ext}^{t+5, (t+5)p^2+q+(t-1)}(\mathbb{Z}_p, \mathbb{Z}_p) \) which is described in [3].

Our main theorems of this paper are as follows.

**Theorem 1.1** Let \( p \geq 11, 3 \leq t < p - 5 \), then
\[
\tilde{\gamma}_t \, h_0 b_1^5 \in \text{Ext}^{t+11, (t+5)p^2+q+(t-1)+(p+1)q+t-3}(\mathbb{Z}_p, \mathbb{Z}_p)
\]
is a permanent cycle in the ASS and converges to a non-trivial element in \( \pi_*(S) \).

Based on a new homotopy element in \( \pi_*(V(2)) \), the above homotopy element in \( \pi_*(S) \) will be constructed.

For the reader’s convenience, let us firstly give some preliminaries on Toda-Smith spectrum \( V(n) \).

The \( \mathbb{Z}_p \) cohomology group of Toda-Smith spectrum \( V(n) \) is \( H^* V(n) \cong E[Q_0, Q_1, \ldots, Q_n] \cong Q(2^{n+1}) \), where \( Q_i (i \geq 0) \) is the Milnor’s elements of Steenrod algebra \( A \), and \( E[ ] \) is the exterior algebra. From [4], when \( n = 1, 2, 3 \) and \( p > 2n \), we know that \( V(n) \) is realized, and there exists a cofibre sequence \( V(-1) = S \):
\[
\Sigma^{2(p^n-1)V(n-1)} \xrightarrow{j_n} V(n-1) \xrightarrow{i_n} V(n) \xrightarrow{j_n} \Sigma^{2p^n-1}V(n-1),
\]
where \( \alpha_n (n = 0, 1, 2, 3) \) are \( p, \alpha, \beta, \gamma \), respectively. The cofibre sequence can induce a short exact sequence of \( \mathbb{Z}_p \) cohomology groups. Thus, we get the following long exact sequence of Ext groups:
\[
\cdots \xrightarrow{(j_n \circ i_n)} \text{Ext}^{s, t-(2p^n-1)}(H^* V(n-1), \mathbb{Z}_p) \xrightarrow{(\alpha_n \circ i_n)} \text{Ext}^{s, t}(H^* V(n-1), \mathbb{Z}_p) \xrightarrow{(i_n \circ j_n)} \text{Ext}^{s, t-(2p^n-1)}(H^* V(n-1), \mathbb{Z}_p) \xrightarrow{(\alpha_n \circ j_n)} \cdots
\]

The following theorem is a key step to prove Theorem 1.1.

**Theorem 1.2** Let \( p \geq 11 \), then \( h_0 b_1^5 \in \text{Ext}^{11, 5p^2+q}(H^* V(2), \mathbb{Z}_p) \) is a permanent cycle in the ASS and converges to a non-trivial element in \( \pi_1 V(2) \).

It is very difficult to determine the stable homotopy groups of spheres. So far, not so many nontrivial elements in the stable homotopy groups of spheres were detected. See, for example [1, 5, 6].

The detection of the element \( \tilde{\gamma}_t \, h_0 b_1^5 \) is parallel to that of the element \( \tilde{\gamma}_t \, h_0 b_2^7 \) given in [7]. Actually, our results are more complicated, especially to Proposition 3.3 and Proposition 3.4.

This paper is organized as follows: after giving some preliminaries on the May spectral sequence (MSS) in Section 2, the proofs of the main theorems will be given in Section 3.

### 2 Some Preliminaries on the May Spectral Sequence

The most successful tool for computing \( \text{Ext}_A^*(\mathbb{Z}_p, \mathbb{Z}_p) \) is the MSS. From [8, Theorem 3.2.5], there exists the MSS \( \{ E_r^{s,t,*} \} \) which converges to \( \text{Ext}_A^*(\mathbb{Z}_p, \mathbb{Z}_p) \). The \( E_1 \)-term and
The Convergence of $E_1^{s,t,u}$ in the Adams Spectral Sequence

Let $P$ be the subalgebra of $A$ generated by the reduced power operations $P^i(i > 0)$, then we have the following results.

**Proposition 3.1** (see [9]) $E_1^{s,t}(H^*V(3), \mathbb{Z}_p) \cong \text{Ext}_P^{s,t}(\mathbb{Z}_p, \mathbb{Z}_p)$, $t - s < 2p^4 - 1$.

**Corollary 3.2** Let $s \geq 2$, then

$$\text{Ext}_A^{s+11,5p^2q^2+q+s-1}(H^*V(3), \mathbb{Z}_p) \cong \text{Ext}_P^{s+11,5p^2q^2+q+s-1}(\mathbb{Z}_p, \mathbb{Z}_p).$$

**Proposition 3.3** Let $3 \leq t < p - 5, p \geq 11$, then

$$0 \neq [\gamma_t b_0 b_0^5] \in \text{Ext}_A^{s+11, (t+5)p^2q+(t-1)p+1)(t+2)q+t-3}(\mathbb{Z}_p, \mathbb{Z}_p).$$

The generators of $E_1^{s,t,u}$ and their first, second degrees satisfying $t < p^3q$ are listed in Table 1.

<table>
<thead>
<tr>
<th>$h_{1,0}$</th>
<th>$h_{1,1}$</th>
<th>$h_{2,0}$</th>
<th>$h_{2,1}$</th>
<th>$h_{1,2}$</th>
<th>$h_{3,0}$</th>
<th>$b_{1,0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, q)</td>
<td>(1, pq)</td>
<td>(1, (p+1)q)</td>
<td>(1, p(p+1)q)</td>
<td>(1, $p^2q$)</td>
<td>(1, $(p^2+p+1)q$)</td>
<td>(2, $pq$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$b_{1,1}$</th>
<th>$b_{2,0}$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, $p^2q$)</td>
<td>(2, $(p^2+p)q$)</td>
<td>(1, 1)</td>
<td>(1, $(q+1)$)</td>
<td>(1, $(p+1)q+1)$</td>
<td>(1, $(p^2+p+1)q+1)$</td>
</tr>
</tbody>
</table>
To compare the degrees, $h_0, b_1 \in \text{Ext}_A^*(\mathbb{Z}_p, \mathbb{Z}_p)$ are represented by $h_{1,0} \in E_1^{t, q, s}$, $b_{1,1} \in E_1^{1, q, s}$ in the MSS. From Corollary 2.3, we conclude that $\gamma_i \in \text{Ext}_A^{t, t, (t+2)q+(t-2)q+t-3} \mathbb{Z}_p$ is represented by $h_{2,1}h_{1,2}h_{3,0}a_{3,3}^{t-3} \in E_1^{1, t, (t+2)q+(t-2)q+t-3} \mathbb{Z}_p$.

Thus, $\gamma_i$ is represented by $h_{1,0}b_1^5 h_{2,1}h_{1,2}h_{3,0}a_{3,3}^{t-3} \in E_1^{1, (t+2)q+(t-2)q+t-3}$ in the MSS. If we want to prove that $0 \neq \gamma_i h_0 b_1^5 \in \text{Ext}_A^{t, (t+2)q+(t-2)q+t-3} \mathbb{Z}_p$, we must prove that $E_1^{t, (t+2)q+(t-2)q+t-3} \mathbb{Z}_p = 0$. For any $\sigma \in E_1^{t, (t+2)q+(t-2)q+t-3}$, we have the following discussions.

**Case 1** When $t \geq 6$, from Lemma 2.5, the number of the factors in $\sigma$ will be $t+9$, $t+8$, $t+7$, $t+6$ or $t+5$.

**Subcase 1.1** If $\sigma$ has $t+9$ factors, there exists a factor $b_{1,0}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, the possible forms will be $\sigma = (1,1) \in \text{Ext}_A^*, \sigma = (1,1,1,1) \in \text{Ext}_A^{t, (t+2)q+(t-2)q+t-3}$.

By Lemma 2.5, the number of the factors in $\sigma_{1,1}$ is $t+7, t+6$ or $t+5$, thus the number of the factors in $\sigma$ will be $t+8, t+7$ or $t+6$. It is in contradiction with that $\sigma$ has $t+9$ factors, so $\sigma_{1,1} = 0$. Similarly, we conclude that $\sigma_{1,2} = 0, \sigma_{1,3} = 0$, so $\sigma = 0$.

**Subcase 1.2** If $\sigma$ has $t+8$ factors, there exist two factors $b_{1,0}, b_{1,1}, b_{2,0}$. Due to the commutativity, the possible forms will be $\sigma = (1,1,1) \in \text{Ext}_A^*, \sigma = (1,1,1,1,1) \in \text{Ext}_A^{t, (t+2)q+(t-2)q+t-3}$.

By the similar argument in Subcase 1.1, we can get that $\sigma_{2,0} = 0(i = 1, 2, \cdots, 6)$, thus $\sigma = 0$.

**Subcase 1.3** If $\sigma$ has $t+7$ factors, there exist three factors $b_{1,0}, b_{1,1}, b_{2,0}$. Due to the commutativity, the possible forms will be $\sigma = (1,1,1,1) \in \text{Ext}_A^*, \sigma = (1,1,1,1,1,1) \in \text{Ext}_A^{t, (t+2)q+(t-2)q+t-3}$.

It is obvious that $\sigma_{3,1} = 0$. By the similar argument in Subcase 1.1, we can get that $\sigma_{3,1} = 0 (i = 4, 5, \cdots, 10)$. From Lemma 2.4, note that $t-3 < t-1, t-4 < t-1$, thus the remainder are all zero. Therefore, we can get $\sigma = 0$. 
**Subcase 1.4** If $\sigma$ has $t + 6$ factors, there exist four factors $b_{i,j}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, $\sigma = \sigma' b_{i,j}^t b_{i,1}^q b_{2,0}$, where $x + y + z = 4, x, y, z \geq 0$ and $\sigma' \in E_{t+2}^1 T$, $T = (t + 5 - y - z)p^2 q + (t - 1 - x - z)p q + (t - 1)q + (t - 3)$. If $x \geq 2$, we have that $y + z < 3$ and $t + 5 - y - z > t + 2$. It is obvious that $\sigma' = 0$. Thus, the possible nontrivial forms will be $\sigma = \sigma_{4,1} b_{1,1}, \sigma = \sigma_{4,2} b_{2,0}, \sigma = \sigma_{4,4} b_{1,1} b_{1,0}, \sigma = \sigma_{4,6} b_{1,0} b_{1,0}, \sigma = \sigma_{4,8} b_{1,0} b_{1,0}$, where the first degrees of $\sigma_{4,i}(i = 1, 2, 3, \cdots, 9)$ are all $t + 2$ and the second degrees of them are listed in Table 2 ($M = t - 1$, and $N = t - 3$).

<table>
<thead>
<tr>
<th>$\sigma_{4,i}$</th>
<th>the second degree $\sigma_{4,i}$</th>
<th>the second degree $\sigma_{4,i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{4,1}$</td>
<td>$(t + 1)p^2 q + (t - 1)p q + M q + N$</td>
<td>$\sigma_{4,2}$</td>
</tr>
<tr>
<td>$\sigma_{4,3}$</td>
<td>$(t + 2)p^2 q + (t - 2)p q + M q + N$</td>
<td>$\sigma_{4,4}$</td>
</tr>
<tr>
<td>$\sigma_{4,5}$</td>
<td>$(t + 2)p^2 q + (t - 5)p q + M q + N$</td>
<td>$\sigma_{4,6}$</td>
</tr>
<tr>
<td>$\sigma_{4,7}$</td>
<td>$(t + 1)p^2 q + (t - 3)p q + M q + N$</td>
<td>$\sigma_{4,8}$</td>
</tr>
<tr>
<td>$\sigma_{4,9}$</td>
<td>$(t + 2)p^2 q + (t - 4)p q + M q + N$</td>
<td></td>
</tr>
</tbody>
</table>

By the argument similar to Subcase 1.3, we get that $\sigma_{4,i} = 0(i = 1, 2 \cdots 9)$, thus $\sigma = 0$.

**Subcase 1.5** If $\sigma$ has $t + 5$ factors, there exist five factors $b_{i,j}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{5,1} b_{1,1}, \sigma = \sigma_{5,3} b_{1,1} b_{2,0}, \sigma = \sigma_{5,4} b_{1,1} b_{2,0}, \sigma = \sigma_{5,5} b_{1,1} b_{2,0}, \sigma = \sigma_{5,6} b_{1,1}$, where the first degrees of $\sigma_{5,i}(i = 1, 2, 3, \cdots, 6)$ are all $t$ and the second degrees of them are listed in Table 3 ($M = t - 1$, and $N = t - 3$).

<table>
<thead>
<tr>
<th>$\sigma_{5,i}$</th>
<th>the second degree $\sigma_{5,i}$</th>
<th>the second degree $\sigma_{5,i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{5,1}$</td>
<td>$t p^2 q + (t - 3)p q + M q + N$</td>
<td>$\sigma_{5,2}$</td>
</tr>
<tr>
<td>$\sigma_{5,3}$</td>
<td>$t p^2 q + (t - 5)p q + M q + N$</td>
<td>$\sigma_{5,4}$</td>
</tr>
<tr>
<td>$\sigma_{5,5}$</td>
<td>$t p^2 q + (t - 6)p q + M q + N$</td>
<td>$\sigma_{5,6}$</td>
</tr>
</tbody>
</table>

Similarly to $\sigma_2$, we can get that $\sigma_{5,i} = 0(i = 1, 2 \cdots 5)$. As for $\sigma_{5,6}$, from the Corollary 2.2, there exist two factors $h_{3,1}$, so $\sigma_{5,6} = 0$. Thus, we can get $\sigma = 0$.

**Case 2** When $t = 5$, $E_{t+10,5}^{(t+10)(t+5)p^2 q + (t-1)(p+1)q + t-3, * } = E_{1}^{(15,10)p^2 q + 5pq + 3pq + q + 2, * }$, the generator contains $(q + 2)$ factors $a_i$. Therefore, the first degree $q + 2 > 15$, it’s a contradiction. So, we get $\sigma = 0$. When $t = 4, t = 3$, the proofs are the similar to $t = 5$. Summarize the above Case 1 and Case 2, $E_{t+10,5}^{(t+10)(t+5)p^2 q + (t-1)(p+1)q + t-3, * } = 0$. That is $0 \Rightarrow h_{0,b} \in Ext_{A}^{(t+11, (t+5)p^2 q + (t-1)(p+1)q + t-3)}(Z_p, Z_p)(3 \leq t < p - 5)$.

**Proposition 3.4** Let $r \geq 2, 3 \leq t < p - 5$, then $Ext_{A}^{(t+11-r, (t+5)p^2 q + (t-1)(p+1)q + t-r-2, * } (Z_p, Z_p) = 0$.

It is sufficient if we can show that $E_{t+11-r, (t+5)p^2 q + (t-1)(p+1)q + t-r-2, * } = 0$. 


Case 1 If $r > 6$, $t + 11 - r < t + 5$, so we have

$$E_1^{t+11-r,(t+5)p^2q+(t-1)q+t-r-2,*} = 0.$$ 

Case 2 If $r = 6$, then

$$E_1^{t+11-r,(t+5)p^2q+(t-1)q+t-r-2,*} = E_1^{t+5,(t+5)p^2q+(t-1)q+t-8,*}.$$ 

Subcase 2.1 When $t \geq 8$, from the Corollary 2.2, there exist six factors $h_{1,2}$, so $\sigma = 0$.

Subcase 2.2 When $t = 7$, $E_1^{t+5,(t+5)p^2q+(t-1)q+t-8,*} = E_1^{12,12p^2q+6pq+5q+q-1,*}$. The generator contains $(q - 1)$ factors $a_i$. Therefore, the first degree $q - 1 > 12$, it's a contradiction. So, the generator is impossible to exist.

Subcase 2.3 When $3 \leq t \leq 6$, by the similar argument in Subcase 2.2, the generator is impossible to exist.

Case 3 If $r = 5$, then

$$E_1^{t+11-r,(t+5)p^2q+(t-1)q+t-r-2,*} = E_1^{t+6,(t+5)p^2q+(t-1)q+t-7,*}.$$ 

Subcase 3.1 When $t \geq 7$, from the Lemma 2.5, we know that the generator contains $t + 5$ factors, one of which must be the factor $b_{i,j}$. Thus, the possible nontrivial forms will be $\sigma = \sigma_{3,1}b_{1,1}$, $\sigma = \sigma_{3,2}b_{2,0}$, where $\sigma_{3,1} \in E_1^{t+4,(t+4)p^2q+(t-1)q+t-7,*}$, $\sigma_{3,2} \in E_1^{t+4,(t+4)p^2q+(t-1)q+t-7,*}$. By the similar argument in Subcase 2.1, we can get that $\sigma_{3,1} = 0$, $\sigma_{3,2} = 0$.

Subcase 3.2 When $3 \leq t \leq 6$, by the similar argument in Subcase 2.2, the generator is impossible to exist.

Case 4 If $r = 4$, then

$$E_1^{t+11-r,(t+5)p^2q+(t-1)q+t-r-2,*} = E_1^{t+7,(t+5)p^2q+(t-1)q+t-6,*}.$$ 

Subcase 4.1 When $t \geq 6$, from Lemma 2.5, we know that the number of the factors in $\sigma$ will be $t + 5$ or $t + 6$.

Subcase 4.1.1 If $\sigma$ contains $t + 6$ factors, then there exists a factor $b_{i,j}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{4,1}b_{1,0}$, $\sigma = \sigma_{4,2}b_{1,1}$, $\sigma = \sigma_{4,3}b_{2,0}$, where

$$\sigma_{4,1} \in E_1^{t+5,(t+5)p^2q+(t-1)q+t-6,*}, \sigma_{4,2} \in E_1^{t+5,(t+4)p^2q+(t-1)q+t-6,*},$$

$$\sigma_{4,3} \in E_1^{t+5,(t+4)p^2q+(t-2)q+t-6,*}.$$ 

By the similar argument in Subcase 2.1, we know that $\sigma_{4,1} = 0$. As for $\sigma_{4,2}$, from Lemma 2.5, $\sigma_{4,2}$ must contain $t + 4$ factors, thus $\sigma = \sigma_{4,2}b_{1,1}$ contains $t + 5$ factors. It is a contradiction with that $\sigma$ contains $t + 6$ factors, then $\sigma_{4,2} = 0$. Similarly, we can get $\sigma_{4,3} = 0$.

Subcase 4.1.2 If $\sigma$ contains $t + 5$ factors, then there exist two factors $b_{i,j}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{4,4}b_{1,1}^2$, $\sigma = \sigma_{4,5}b_{2,0}^2$. 

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Due to the commutativity, the possible nontrivial forms will be

$$\sigma = \sigma_{4.6} b_{1,1} b_{2,0}, \text{ where}$$

$$\sigma_{4.4} \in E_{1}^{t+3, (t+3) p^2 q + (t-1) pq + (t-1) q + t - 6}, \sigma_{4.5} \in E_{1}^{t+3, (t+3) p^2 q + (t-3) pq + (t-1) q + t - 6},$$

$$\sigma_{4.6} \in E_{1}^{t+3, (t+3) p^2 q + (t-2) pq + (t-1) q + t - 6}.$$

By the similar argument in Subcase 2.1, we can get $\sigma_{4.4} = 0$. As for $\sigma_{4.5}$, from the Lemma 2.4 and $t - 3 < t - 1$, so $\sigma_{4.5} = 0$. Similarly, we can get $\sigma_{4.6} = 0$.

**Subcase 4.2** When $3 \leq t \leq 5$, by the similar argument in Subcase 2.2, we know that the generator is impossible to exist. Thus, we have $\sigma = 0$.

**Case 5** If $r = 3$, then

$$E_{1}^{t+11-r, (t+5)p^2 q + (t-1) pq + (t-1) q + t - r - 2} = E_{1}^{t+8, (t+5)p^2 q + (t-1) pq + (t-1) q + t - 5}.$$ 

**Subcase 5.1** When $t \geq 5$, from Lemma 2.5, the number of the factors in $\sigma$ will be $t + 5, t + 6$ or $t + 7$.

**Subcase 5.1.1** If $\sigma$ contains $t + 7$ factors, then there exists a factor $b_{i,j}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{5.1} b_{1,0}, \sigma = \sigma_{5.2} b_{1,1}, \sigma = \sigma_{5.3} b_{2,0},$ where

$$\sigma_{5.1} \in E_{1}^{t+6, (t+5) p^2 q + (t-2) pq + (t-1) q + t - 5}, \sigma_{5.2} \in E_{1}^{t+6, (t+4) p^2 q + (t-1) pq + (t-1) q + t - 5},$$

$$\sigma_{5.3} \in E_{1}^{t+6, (t+4) p^2 q + (t-2) pq + (t-1) q + t - 5}.$$

Similarly to $\sigma_{4.2}$, we can get $\sigma_{5.1} = 0 (i = 1, 2, 3)$.

**Subcase 5.1.2** If $\sigma$ contains $t + 6$ factors, then there exist two factors $b_{i,j}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{5.4} b_{1,1}^2, \sigma = \sigma_{5.5} b_{2,0}^2, \sigma = \sigma_{5.6} b_{1,0} b_{1,1}, \sigma = \sigma_{5.7} b_{1,0} b_{2,0}, \sigma = \sigma_{5.8} b_{1,1} b_{2,0},$ where

$$\sigma_{5.4} \in E_{1}^{t+4, (t+3) p^2 q + (t-1) pq + (t-1) q + t - 5}, \sigma_{5.5} \in E_{1}^{t+4, (t+3) p^2 q + (t-3) pq + (t-1) q + t - 5},$$

$$\sigma_{5.6} \in E_{1}^{t+4, (t+4) p^2 q + (t-2) pq + (t-1) q + t - 5}, \sigma_{5.7} \in E_{1}^{t+4, (t+4) p^2 q + (t-3) pq + (t-1) q + t - 5},$$

$$\sigma_{5.8} \in E_{1}^{t+4, (t+3) p^2 q + (t-2) pq + (t-1) q + t - 5}.$$

Similarly to $\sigma_{4.2}$, we can get that $\sigma_{5.4} = 0, \sigma_{5.5} = 0, \sigma_{5.8} = 0$. Similarly to $\sigma_{4.5}$, we can get that $\sigma_{5.6} = 0, \sigma_{5.7} = 0$.

**Subcase 5.1.3** If $\sigma$ contains $t + 5$ factors, then there exist three factors $b_{i,j}(b_{1,0}, b_{1,1}, b_{2,0})$. Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{5.9} b_{1,1}^3, \sigma = \sigma_{5.10} b_{2,0}^2, \sigma = \sigma_{5.11} b_{1,0} b_{1,1} b_{2,0},$ where

$$\sigma_{5.9} \in E_{1}^{t+2, (t+2) p^2 q + (t-1) pq + (t-1) q + t - 5}, \sigma_{5.10} \in E_{1}^{t+2, (t+2) p^2 q + (t-4) pq + (t-1) q + t - 5},$$

$$\sigma_{5.11} \in E_{1}^{t+2, (t+2) p^2 q + (t-2) pq + (t-1) q + t - 5}, \sigma_{5.12} \in E_{1}^{t+2, (t+2) p^2 q + (t-3) pq + (t-1) q + t - 5},$$

By the similar argument in Subcase 2.1, we can know that $\sigma_{5.9} = 0$. Similarly to $\sigma_{4.5}$, we can get that $\sigma_{5.10} = 0, \sigma_{5.11} = 0, \sigma_{5.12} = 0$. 

Subcase 5.2 When $t = 4$ or $t = 3$, by the similar argument in Subcase 2.2, we know that the generator is impossible to exist. Thus, in this case, we can get $\sigma = 0$.

Case 6 If $r = 2$, then
$$E_{t+11-r,(t+5)p^2q+(t-1)pq+(t-1)q+t-r-2, \ast}^t = E_{t+9,(t+5)p^2q+(t-1)pq+(t-1)q+t-4, \ast}^t.$$ 

Subcase 6.1 When $t \geq 5$, from Lemma 2.5, the number of the factors in $\sigma$ will be $t + 5, t + 6, t + 7$ or $t + 8$.

Subcase 6.1.1 If $\sigma$ contains $t + 8$ factors, then there exists a factor $b_{i,j}(b_{1.0}, b_{1.1}, b_{2.0})$.

Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{6.1}b_{1.0}, \sigma = \sigma_{6.2}b_{1.1}, \sigma = \sigma_{6.3}b_{2.0}$, where
$$\sigma_{6.1} \in E_{t+7,(t+5)p^2q+(t-2)pq+(t-1)q+t-4, \ast}^t, \sigma_{6.2} \in E_{t+7,(t+4)p^2q+(t-1)pq+(t-1)q+t-4, \ast}^t, \sigma_{6.3} \in E_{t+7,(t+4)p^2q+(t-2)pq+(t-1)q+t-4, \ast}^t.$$

Similarly to $\sigma_{4.2}$, we can get $\sigma_{6.1} = 0, \sigma_{6.2} = 0, \sigma_{6.3} = 0$.

Subcase 6.1.2 If $\sigma$ contains $t + 7$ factors, then there exist two factors $b_{i,j}(b_{1.0}, b_{1.1}, b_{2.0})$.

Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{6.4}b_{1.0}^2, \sigma = \sigma_{6.5}b_{1.1}^2, \sigma = \sigma_{6.6}b_{2.0}^2, \sigma = \sigma_{6.7}b_{1.0}b_{1.1}, \sigma = \sigma_{6.8}b_{1.0}b_{2.0}, \sigma = \sigma_{6.9}b_{1.1}b_{2.0}$, where
$$\sigma_{6.4} \in E_{t+5,(t+5)p^2q+(t-3)pq+(t-1)q+t-4, \ast}^t, \sigma_{6.5} \in E_{t+5,(t+3)p^2q+(t-1)pq+(t-1)q+t-4, \ast}^t, \sigma_{6.6} \in E_{t+5,(t+3)p^2q+(t-3)pq+(t-1)q+t-4, \ast}^t, \sigma_{6.7} \in E_{t+5,(t+4)p^2q+(t-2)pq+(t-1)q+t-4, \ast}^t, \sigma_{6.8} \in E_{t+5,(t+4)p^2q+(t-3)pq+(t-1)q+t-4, \ast}^t, \sigma_{6.9} \in E_{t+5,(t+3)p^2q+(t-2)pq+(t-1)q+t-4, \ast}^t.$$

Similarly to $\sigma_{4.5}$, we can get $\sigma_{6.4} = 0$. Similarly to $\sigma_{4.2}$, we can get that $\sigma_{6,i} = 0$ $(i = 5, 6 \cdots 9)$.

Subcase 6.1.3 If $\sigma$ contains $t + 6$ factors, then there exist three factors $b_{i,j}(b_{1.0}, b_{1.1}, b_{2.0})$.

Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{6.10}b_{1.0}^2, \sigma = \sigma_{6.11}b_{1.1}^2, \sigma = \sigma_{6.12}b_{2.0}^2, \sigma = \sigma_{6.13}b_{1.0}b_{1.1}, \sigma = \sigma_{6.14}b_{1.0}b_{2.0}, \sigma = \sigma_{6.15}b_{1.1}b_{2.0}$, where the first degrees of $\sigma_{6,i}(i = 10, 11, \cdots, 16)$ are all $t + 3$ and the second degrees of them are listed in Table 4 ($M = t - 1$, and $N = t - 4$).

<table>
<thead>
<tr>
<th>$\sigma_{6,i}$</th>
<th>the second degree</th>
<th>$\sigma_{6,i}$</th>
<th>the second degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{6.10}$</td>
<td>$(t + 2)p^2q + (t - 1)pq + Mq + N$</td>
<td>$\sigma_{6.11}$</td>
<td>$(t + 2)p^2q + (t - 4)pq + Mq + N$</td>
</tr>
<tr>
<td>$\sigma_{6.12}$</td>
<td>$(t + 3)p^2q + (t - 2)pq + Mq + N$</td>
<td>$\sigma_{6.13}$</td>
<td>$(t + 2)p^2q + (t - 2)pq + Mq + N$</td>
</tr>
<tr>
<td>$\sigma_{6.14}$</td>
<td>$(t + 3)p^2q + (t - 4)pq + Mq + N$</td>
<td>$\sigma_{6.15}$</td>
<td>$(t + 2)p^2q + (t - 3)pq + Mq + N$</td>
</tr>
<tr>
<td>$\sigma_{6.16}$</td>
<td>$(t + 3)p^2q + (t - 3)pq + Mq + N$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Similarly to $\sigma_{4.2}$, we can get that $\sigma_{6.10} = 0, \sigma_{6.11} = 0, \sigma_{6.13} = 0$. Similarly to $\sigma_{4.5}$, we can get that $\sigma_{6.12} = 0, \sigma_{6.14} = 0, \sigma_{6.15} = 0, \sigma_{6.16} = 0$.

Subcase 6.1.4 If $\sigma$ contains $t + 5$ factors, then there exist four factors $b_{i,j}(b_{1.0}, b_{1.1}, b_{2.0})$.

Due to the commutativity, the possible nontrivial forms will be $\sigma = \sigma_{6.17}b_{1.0}^2, \sigma = \sigma_{6.18}b_{1.1}^2$. 

σ = σ_{6,19}b_{1,1}b_{2,0}, \sigma = σ_{6,20}b_{1,1}b_{2,0}, \sigma = σ_{6,21}b_{1,1}b_{2,0}, \sigma = σ_{6,21}b_{1,1}b_{2,0}, \sigma = σ_{6,21}b_{1,1}b_{2,0}, \quad \text{where the first degrees of } σ_{6,i}(i = 17, 18, \cdots, 21) \text{ are all } t + 1 \text{ and the second degrees of them are listed in Table 5 (}\ M = t - 1, \text{ and } N = t - 4). \text{Table 5: The factors and second degrees.}

<table>
<thead>
<tr>
<th>σ_{6,1}</th>
<th>the second degree</th>
<th>σ_{6,1}</th>
<th>the second degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ_{6,17}</td>
<td>(t + 1)p^2q + (t - 1)pq + Mq + N</td>
<td>σ_{6,18}</td>
<td>(t + 1)p^2q + (t - 5)pq + Mq + N</td>
</tr>
<tr>
<td>σ_{6,19}</td>
<td>(t + 1)p^2q + (t - 2)pq + Mq + N</td>
<td>σ_{6,20}</td>
<td>(t + 1)p^2q + (t - 4)pq + Mq + N</td>
</tr>
<tr>
<td>σ_{6,21}</td>
<td>(t + 1)p^2q + (t - 3)pq + Mq + N</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Similarly to Subcase 2.1, we can get that σ_{6,17} = 0. Similarly to σ_{3,2}, we can get that σ_{6,i} = 0(i = 18, 19, 20, 21).

**Subcase 6.2** When t = 3, t = 4, by the similar argument in Subcase 2.2, we know that the total degree is impossible to exist. Thus, we have σ = 0.

Therefore, we can get that E_{t+1}^{11-r-3} is equal to that of \( \sigma = \sigma_{6,1} \). Moreover, we have additively

From \[4, Lemma 2.2\], we know that the rank of Ext_{s}^{11,5p^2q + q + s - 1}(H^*V(2), Z_p) = 0.

**Proposition 3.5** Let s ≥ 2, p ≥ 11, then Ext_{s}^{11,5p^2q + q + s - 1}(H^*V(2), Z_p) = 0.

From Corollary 3.2, we have

\[ \text{Ext}_{s}^{11,5p^2q + q + s - 1}(H^*V(3), Z_p) \cong \text{Ext}_{s}^{11,5p^2q + q + s - 1}(Z_p, Z_p). \]

From [4, Lemma 2.2], we know that the rank of Ext_{s}^{11,5p^2q + q + s - 1}(Z_p, Z_p) is less than or equal to that of \( |P(b'_j) \otimes H^*(U(L))|^{11,5p^2q + q + s - 1} \), and \( |P(b'_j) \otimes H^*(U(L))|^{s,t} \) is the E_2-term of the MSS, where \( P(\cdot) \) is the polynomial algebra. Up to the total degree \( t - s < (p^3 + 3p^2 + 2p + 1)q - 4 \), \( H^*(U(L)) \) is multiplicative by the following cohomology classes

\[ h_i = \{ R_i \}, \quad g_i = \{ R_i^0 R_i^1 \}, \quad k_i = \{ R_i^0 R_i^1 \} (i ≥ 0), \]
\[ l_1 = \{ R_1^0 R_1^2 R_1^3 \}, \quad l_2 = \{ R_2^0 R_2^1 R_1^1 \}, \quad l_3 = \{ R_3^0 R_2^1 R_1^1 \}, \]
\[ l_4 = \{ R_4^0 R_2^1 R_1^1 \}, \quad l_5 = \{ R_5^0 R_2^1 R_1^1 \}, \quad l_6 = \{ R_6^0 R_2^1 R_1^1 \}, \]
\[ m_1 = \{ R_1^0 R_2^0 R_1^1 R_1^1 \}, \quad m_2 = \{ R_2^0 R_2^0 R_1^0 R_1^1 \}, \]
\[ m_3 = \{ R_3^0 R_2^0 R_1^1 R_1^1 \}, \quad m_4 = \{ R_4^0 R_2^0 R_1^1 R_1^1 \}. \]

Moreover, we have additively

\[ H^*(U(L)) \cong \{ 1, l_4, h_3 \} \otimes \{ 1, h_0, h_1, g_0, k_0, k_0 h_0 \} \]
\[ + \{ h_2, h_2 h_0, g_1, l_1, l_2, l_1 h_1, k_1, k_1 l_3, k_1 h_1, l_1 h_2, m_1, m_1 h_0, g_2, g_2 h_0, l_5, m_2, m_3, l_6, m_4 \}, \]

and the bidegrees of \( R_i^0 \), \( b_j^1 \) are (1, 2(p^{i+j} - p^i)), (2, 2(p^{i+j-1} - p^{i+1})), respectively. In the MSS, \( b_1^0 \) converges to \( b_0 \in \text{Ext}_{t}^{2, pq}(Z_p, Z_p) \), and the total degree of \( b_1^0 \) is \( |b_1^0| = pq - 2 \). The generators whose total degrees are less than or equal to \( 5p^2q + q - 12 \) in \( |P(b'_j) \otimes H^*(U(L))|^{s,t} \) and the total degrees \(|\lambda| \mod pq - 2 \) are listed in Table 6 (\( t = 1, 2, 3, 4, 5, t' = 1, 2, 3, 4 \)).
Table 6: The generators $\lambda$ and total degrees $|\lambda|$ mod $pq - 2$

| $|\lambda|$ | $tq, t'(q + 2)$ | $q - 1$, 1 | 2q, $q + 2$, 2q + 1 | $q + 1$, 2q, $q + 4$, 4q + 3, 2q + 5, 4q + 4, 2q + 4, 4q + 3, 2q + 5, 5q + 4, 4q + 8, 5q + 7 |
|------------|-----------------|-------------|----------------|----------------|----------------|
| $l_1$, $l_2$, $l_1 h_1$, $k_1$, $l_3$, $k_1 h_1$, $l_1 h_2$, $m_1$, $m_1 h_0$ |

Let $x$ be a generator of $\text{Ext}^{s+11,5p^2q+q+s-1}_A(H^*(V(2),\mathbb{Z}_p))$, then we have

$$\left(i_3\right)_*(x) \in \text{Ext}^{s+11,5p^2q+q+s-1}_A(H^*(V(3),\mathbb{Z}_p)).$$

The total degree of $(i_3)_*(x)$ is $5p^2q + q + s - 1 - (s + 11) = 5p^2q + q - 12 \equiv 6q - 2 \pmod{pq - 2}$. From the above Table, we know that the generator $\lambda$ with total degree mod $pq - 2$ being equal to $6q - 2$ in $[P(b_j^i) \otimes H^{*,*}(U(L))]^{s,t}$ doesn’t exist. So, we can get that $(i_3)_*(x) = 0$. Consider the following exact sequence:

$$\cdots \xrightarrow{(j_3)_*} \text{Ext}^{s+10,5p^2q+q+s-1-(2p^3-1)}_A(H^*(V(2),\mathbb{Z}_p)) \xrightarrow{(\alpha_3)_*} \text{Ext}^{s+11,5p^2q+q+s-1}_A(H^*(V(2),\mathbb{Z}_p)) \xrightarrow{(j_3)_*} \cdots,$$

there exists an element $x_1 \in \text{Ext}^{s+10,5p^2q+q+s-1-(2p^3-1)}_A(H^*(V(2),\mathbb{Z}_p))$ satisfying $(\alpha_3)_*(x_1) = x$. The total degree of $(i_3)_*(x_1)$ is $5p^2q + q + s - 1 - (2p^3 - 1) - (s + 10) \equiv 4q - 6 \pmod{pq - 2}$. From the above Table, we know that

$$0 = (i_3)_*(x_1) \in \text{Ext}^{s+10,5p^2q+q+s-1-(2p^3-1)}_A(H^*(V(3),\mathbb{Z}_p)).$$

Using the exactness repeatedly, there exists an element $x_k \in \text{Ext}^{s+11-k,5p^2q+q+s-1-k(2p^3-1)}_A(H^*(V(2),\mathbb{Z}_p))$ satisfying $(\alpha_3)_*(x_k) = x_{k-1}$. But the total degree of $(i_3)_*(x_k) \pmod{pq - 2}$ is different from that in the above table, so we know that

$$0 = (i_3)_*(x_k) \in \text{Ext}^{s+11-k,5p^2q+q+s-1-k(2p^3-1)}_A(H^*(V(3),\mathbb{Z}_p)).$$

Let $k = 5$, then

$$x_5 \in \text{Ext}^{s+6,5p^2q+q+s-1-5(2p^3-1)}_A(H^*(V(2),\mathbb{Z}_p)) = \text{Ext}^{s+6,10p^2q+q+s+4}_A(H^*(V(2),\mathbb{Z}_p)) = 0.$$

Therefore, we have $x = (\alpha_3)_* \cdots (\alpha_3)_*(x_5) = 0$, that is

$$\text{Ext}^{s+11,5p^2q+q+s-1}_A(H^*(V(2),\mathbb{Z}_p)) = 0(s \geq 2, p \geq 11).$$

**Proposition 3.6** Let $r \geq 2$, $p \geq 11$, then

$$\text{Ext}^{11-r,5p^2q+q-r+1}_A(H^*(V(2),\mathbb{Z}_p)) = 0.$$
The proposition is evident for \( r \geq 11 \). Thus, we need only to consider the case of \( 2 \leq r < 11 \).

For any \( y \in \text{Ext}_A^{11-r,5p^2q+q-r+1}(H^*V(2),\mathbb{Z}_p) \), we can know \( 0 < q - r + 1 < q \) from \( 2 \leq r < 11 \). Since \( Sdim((i_3)_*(y)) = 5p^2q + q - r + 1 = q - r + 1 \neq 0 \) (mod \( q \)) and \( \text{Ext}_p^*(\mathbb{Z}_p,\mathbb{Z}_p) = 0 \) (\( t \neq 0 \) mod \( q \)), from Proposition 3.1, we can get that

\[
0 = (i_3)_*(y) \in \text{Ext}_A^{11-r,5p^2q+q-r+1}(H^*V(3),\mathbb{Z}_p).
\]

According to the exactness, there exists an element

\[
y_1 \in \text{Ext}_A^{10-r,5p^2q+q-r+1-(2p^3-1)}(H^*V(2),\mathbb{Z}_p)
\]

such that \( (\alpha_3)_*(y_1) = y \), and

\[
Sdim((i_3)_*(y_1)) = 5p^2q + q - r + 1 - (2p^3 - 1) = 4p^2q - pq - r = q - r \neq 0 \) (mod \( q \)),
\]

so \( 0 = (i_3)_*(y_1) \in \text{Ext}_A^{10-r,5p^2q+q-r+1-(2p^3-1)}(H^*V(3),\mathbb{Z}_p) \).

Similarly, there exists an element \( y_k \in \text{Ext}_A^{11-k-r,5p^2q+q-r+1-k(2p^3-1)}(H^*V(2),\mathbb{Z}_p) \) satisfying \( (\alpha_3)_*(y_k) = y_{k-1} \), and \( Sdim((i_3)_*(y_k)) \neq 0 \) (mod \( q \)). Let \( k = 5 \), then

\[
y_5 \in \text{Ext}_A^{6-r,5p^2q+q-r+1-5(2p^3-1)}(H^*V(2),\mathbb{Z}_p) = \text{Ext}_A^{6-r,-10p^2q+q-r+6}(H^*V(2),\mathbb{Z}_p) = 0.
\]

Thus, we get that \( y = (\alpha_3)_* \cdots (\alpha_3)_*(y_5) = 0 \), that is

\[
\text{Ext}_A^{11-r,5p^2q+q-r+1}(H^*V(2),\mathbb{Z}_p) = 0 \quad (r \geq 2, p \geq 11).
\]

**The Proof of Theorem 1.2** First, we consider the ASS with \( E_2 \)-term:

\[
\text{Ext}_A^{*,*}(H^*V(2),\mathbb{Z}_p) \Rightarrow \pi_{*-s}V(2),
\]

and its differential is \( d_r : E_r^{s,t} \to E_r^{s+r,t+r-1} \).

From Proposition 3.5,

\[
E_2^{r+11,5p^2q+q+r-1} = \text{Ext}_A^{r+11,5p^2q+q+r-1}(H^*V(2),\mathbb{Z}_p),
\]

we can get that \( E_2^{r+11,5p^2q+q+r-1} = 0 \) (\( r \geq 2 \)). Let \( h_0b_5^1 \) be the image of \( h_0b_5^1 \in \text{Ext}_A^{11,5p^2q+q}(\mathbb{Z}_p,\mathbb{Z}_p) \) under the map

\[
(i_2)_*(i_1)_*(i_0)_* : \text{Ext}_A^{11,5p^2q+q}(\mathbb{Z}_p,\mathbb{Z}_p) \to \text{Ext}_A^{11,5p^2q+q}(H^*V(2),\mathbb{Z}_p),
\]

then \( d_r(h_0b_5^1) \in E_r^{r+11,5p^2q+q+r-1} = 0 \) (\( r \geq 2 \)). Furthermore, we can get that

\[
h_0b_5^1 \in E_2^{11,5p^2q+q} = \text{Ext}_A^{11,5p^2q+q}(H^*V(2),\mathbb{Z}_p)
\]

is a permanent cycle in the ASS. Moreover, from Proposition 3.6,

\[
E_2^{11-r,5p^2q+q-r+1} = \text{Ext}_A^{11-r,5p^2q+q-r+1}(H^*V(2),\mathbb{Z}_p) = 0 \quad (r \geq 2),
\]
we have that $E^{11−r,5p^2q+r+1}_r = 0 (r > 2)$. So, $h_0b^5_1$ is impossible to be the $d_r$-boundary in the ASS, and $h_0b^5_1 \in \text{Ext}^{11,5p^2q+q}(H^*V(2), \mathbb{Z}_p)$ converges to a nontrivial element in $\pi_*(V(2))$.

**The Proof of Theorem 1.1** From Theorem 1.2, we know that there exists a nontrivial element $f$ in $\pi_*(V(2))$, which is represented by $h_0b^5_1 \in \text{Ext}^{11,5p^2q+q}(H^*V(2), \mathbb{Z}_p)$, where $h_0b^5_1$ denotes the image of $h_0b^5_1 \in \text{Ext}^{11,5p^2q+q}(\mathbb{Z}_p, \mathbb{Z}_p)$ under the homomorphism

$$(i_2)_*(i_1)_*(i_0)_*: \text{Ext}^{11,5p^2q+q}(\mathbb{Z}_p, \mathbb{Z}_p) \to \text{Ext}^{11,5p^2q+q}(H^*V(2), \mathbb{Z}_p).$$

Consider the following composition of maps

$$\tilde{f}: \Sigma^{5p^2q+q−11}S \xrightarrow{f} V(2) \xrightarrow{\gamma^t} \Sigma^{(p^2+p+1)q}V(2) \xrightarrow{j_0j_1j_2} \Sigma^{−t(p^2+p+1)q+(p+1)q+q+3}S,$$

the composed map $\tilde{f} = j_0j_1j_2\gamma^t f$ is represented by

$$(j_0j_1j_2)_*(\gamma^t)_*(i_2i_1i_0)_*(h_0b^5_1) \in \text{Ext}^{11+t, (5+t)p^2q+(t−1)(p+1)q+t−3}(\mathbb{Z}_p, \mathbb{Z}_p).$$

From [3], we have known that $\gamma_t = j_0j_1j_2 \in \pi_*(S)$ is represented by $\tilde{\gamma}_t$ in the ASS. By the knowledge of Yoneda products, we know that the following composition:

$$\text{Ext}^0_A(\mathbb{Z}_p, \mathbb{Z}_p) \xrightarrow{(i_2i_1i_0)_*} \text{Ext}^0_A(H^*V(2), \mathbb{Z}_p) \xrightarrow{(\gamma^t)_*} \text{Ext}^{t,(p^2+p+1)q+t}(H^*V(2), \mathbb{Z}_p)$$

$$\xrightarrow{(j_0j_1j_2)_*} \text{Ext}^{t,(pq+(t−1)pq+(t−2)q+t−3)}(\mathbb{Z}_p, \mathbb{Z}_p)$$

is a homomorphism which is multiplied by $\tilde{\gamma}_t$. Hence, $\tilde{f} \in \pi_*(S)$ is represented by

$$\tilde{\gamma}_t h_0b^5_1 \in \text{Ext}^{11+t, (5+t)p^2q+(t−1)(p+1)q+t−3}(\mathbb{Z}_p, \mathbb{Z}_p)$$

in the ASS.

Moreover, from the Proposition 3.4 we know that $\tilde{\gamma}_t h_0b^5_1$ can’t be hit by the differentials in the ASS, then we get that $\tilde{\gamma}_t h_0b^5_1$ converges to a nontrivial element $\tilde{f}$ in $\pi_*(S)$.

**References**


球面稳定的同伦群中的一族新元素

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摘要：本文研究了球面稳定的同伦群中元素的非平凡性。利用May谱序列，证明了在Adams谱序列$E_2$项中存在乘积元素收敛到球面稳定的同伦群中的一族阶为$p$的非零元。此非零元具有更高维数的滤子。

关键词：稳定同伦群，Toda-Smith谱，Adams谱序列，May谱序列

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