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GLOBAL SOLUTIONS FOR THE RATIO-DEPENDENT FOOD-CHAIN MODEL WITH CROSS-DIFFUSION

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Abstract: In this paper, a food chain model with ratio-dependent functional response is investigated under homogeneous Neumann boundary conditions. Using the energy estimate and Gagliardo-Nirenberg-type inequalities, the existence and the uniform boundedness of global solutions are proved. Meanwhile, the sufficient condition for global asymptotic stability of the positive equilibrium point for the model is given by constructing the Lyapunov function.

Keywords: ratio-dependent functional response; cross-diffusion; global solutions; uniform boundedness; stability

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1 Introduction

There is growing biological and physiological evidence, see for instance [1] and the literature cited therein, that in some situations, specially when predators have to search for food and therefore have to share or compete for food, a more suitable general predator-prey theory should be based on the so-called ratio-dependent theory, which can be roughly stated as that the per capita predator growth rate should be a function of the ratio of prey to predator abundance. This is supported by numerous field and laboratory experiments and observations, see for instance [5].

The prey-dependent food-chain models were studied in [6, 7, 9, 11, 13, 20], while mathematically interesting, inherit the mechanism that generates the factitious paradox of enrichment and fail to produce the often observed extinction dynamics resulting in the collapse of the system. Consequently, a ratio-dependent food chain model, which is an ODE system with three equations whose species are hence assumed to be spatially homogeneous, was proposed by Hsu, Hwang, and Kuang in [10] to describe the growth of plant, pest, and top predator. However, it is not enough that populations of species are considered in only time and density. To make it more realistic, different spatial locations should also be taken into consideration, which have resulted in reaction-diffusion model with ratio-dependent functional response

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[12, 14]. Despite the fact that much attention has been paid to studies of weakly-coupled reaction-diffusion system, few has been found on strong-coupled reaction-diffusion system.

This paper discusses the following three questions in the one-dimensional space: the existence and the uniform boundedness of the global solution to a ratio-dependent food chain model with self and cross-diffusion, and the global asymptotic stability of the positive equilibrium point.

Concretely, consider the following problem

$$\begin{aligned}
&u_t = (d_1u + \alpha_{11}u^2 + \alpha_{12}uv + \alpha_{13}uw)_{xx} + u(1 - u - \frac{a_1v}{u+v}), \quad 0 < x < 1, t > 0,\\
&v_t = (d_2v + \alpha_{21}uv + \alpha_{22}v^2 + \alpha_{23}vw)_{xx} + v(-b_1 + \frac{m_1u}{u+v} - \frac{a_2w}{v+w}), \quad 0 < x < 1, t > 0,\\
&w_t = (d_3w + \alpha_{31}uw + \alpha_{32}vw + \alpha_{33}w^2)_{xx} + w(-b_2 + \frac{m_2v}{v+w}), \quad 0 < x < 1, t > 0,\\
&u_x(x,t) = v_x(x,t) = w_x(x,t) = 0, \quad x = 0, 1, t > 0,\\
&u(x,0) = u_0(x) \geq 0, v(x,0) = v_0(x) \geq 0, w(x,0) = w_0(x) \geq 0, \quad 0 < x < 1,
\end{aligned} \tag{1.1}$$

where u_0, v_0, w_0 are nonnegative functions which are not identically zero, u, v and w are the respective population densities of prey, predator, top predator. $d_i, \alpha_{ij}(i,j = 1,2,3), a_i, b_i, m_i$ $(i = 1,2)$ are positive constants, d_1, d_2, d_3 are the the diffusion rates of u, v, w, respectively. $\alpha_{ii}(i = 1,2,3)$ are referred as self-diffusion pressures, and $\alpha_{ij}(i \neq j,\ \ i,j = 1,2,3)$ are cross-diffusion pressures. For more details on the biological background, see references [8, 15, 16, 17, 18]. a_i, b_i, m_i $(i = 1,2)$ which can see more explanations for the biological background, refer to [10, 12, 14]. Furthermore, to avoid the case where predator and top-predator cannot survive, even when their food is infinitely abundant, we assume that $m_i > b_i, i = 1,2$.

The ODE problem associated with (1.1) was proposed and studied by Hsu, Hwang, and Kuang in [10], and from [10], system (1.1) has a unique positive equilibrium $(\bar{u}, \bar{v}, \bar{w})$ if and only if the following are satisfied:

$$m_2 > b_2, \quad A > 1, \quad 0 < a_1 < \frac{A}{A-1}, \tag{1.2}$$

where $A = m_1/(a_2(m_2 - b_2)/m_2 + b_1)$, and

$$\bar{u} = \frac{[a_1 + A(1 - a_1)]}{A}, \quad \bar{v} = (A-1)\bar{u}, \quad \bar{w} = \frac{(m_2 - b_2)\bar{v}}{b_2}.$$

We also note that $m_2 > b_2$ and $A > 1$ imply $m_1 > b_1$.

In particular, they obtained the extinction conditions of certain species and discussed the local asymptotical stability of $(\bar{u}, \bar{v}, \bar{w})$ and various scenarios where distinct solutions can be attracted to the origin, the pest-free steady state, and the positive steady state $(\bar{u}, \bar{v}, \bar{w})$. For more detail, we refer the reader to [10]. From their results, the authors pointed out that this ODE system is very rich in dynamics.

The corresponding weakly coupled reaction-diffusion system (1.1) has received a lot of attention, see [12, 14]. But up to now, the corresponding researches chiefly concentrate on the existence and nonexistence of nonconstant positive steady-state solutions of the weakly-coupled reaction-diffusion system (1.1). To the best of our knowledge, when $\alpha_{ij}(i \neq j,\ i,j = 1,2,3)$ is positive, (1.1) is a strongly-coupled reaction-diffusion system which occurs frequently is biological and it is very difficult to analyze, there are very few results for the (1.1).

For simplicity, denote $\|\cdot\|_{W_p^k(0,1)}$ by $|\cdot|_{k,p}$ and $\|\cdot\|_{L^p(0,1)}$ by $|\cdot|_p$. For the time-dependent solutions of (1.1), the local existence is an immediate consequence of a series of important papers [2–4] by Amann. Roughly speaking, if $u_0(x), v_0(x), w_0(x)$ in $W_p^1(\Omega)$ with $p > N$, then (1.1) has a unique nonnegative solution $u, v, w \in C\left([0,T), W_p^1(\Omega)\right) \bigcap C^\infty\left((0,T), C^\infty(\Omega)\right)$, where $T \in (0,\infty]$ is the maximal existence time for the local solution. If the solution (u,v,w) satisfies the estimates

$$\sup\{\|u(\cdot,t)\|_{W_p^1(\Omega)}, \|v(\cdot,t)\|_{W_p^1(\Omega)}, \|w(\cdot,t)\|_{W_p^1(\Omega)} : 0 < t < T\} < \infty,$$

then $T = +\infty$. Moreover, if $u_0(x), v_0(x), w_0(x) \in W_p^2(\Omega)$, then $u, v, w \in C\left([0,\infty), W_p^2(\Omega)\right)$.

Our main results as follows:

Theorem 1 Let $u_0(x), v_0(x), w_0(x) \in W_2^2(0,1), (u,v,w)$ is the unique nonnegative solution of system (1.1) in the maximal existence interval $[0,T)$. Assume that

$$\begin{aligned} &8\alpha_{11}\alpha_{21}\alpha_{31} > \alpha_{21}\alpha_{13}^2 + \alpha_{12}^2\alpha_{31},\\ &8\alpha_{12}\alpha_{22}\alpha_{32} > \alpha_{32}\alpha_{21}^2 + \alpha_{23}^2\alpha_{12},\\ &8\alpha_{13}\alpha_{23}\alpha_{33} > \alpha_{23}\alpha_{31}^2 + \alpha_{32}^2\alpha_{13}. \end{aligned} \tag{1.3}$$

Then there exist $t_0 > 0$ and positive constants M, M' which depend only on $d_i, \alpha_{ij}(i,j = 1,2,3)$, $a_i, b_i, m_i(i = 1,2)$ such that

$$\sup\{|u(\cdot,t)|_{1,2}, |v(\cdot,t)|_{1,2}, |w(\cdot,t)|_{1,2} : t \in (t_0,T)\} \le M', \tag{1.4}$$

$$\max\{u(x,t), v(x,t), w(x,t) : (x,t) \in [0,1] \times (t_0,T)\} \le M, \tag{1.5}$$

and $T = +\infty$. Moreover, in the case that $d_1, d_2, d_3 \ge 1, \frac{d_2}{d_1}, \frac{d_3}{d_1} \in [\underline{d}, \overline{d}]$, where $\underline{d}$ and $\overline{d}$ are positive constants, M', M depend on $\underline{d}, \overline{d}$ but do not on d_1, d_2, d_3.

Theorem 2 Assume that all conditions in Theorem 1 are satisfied and (1.2) holds. Assume that the following hold :

$$a_1 < 1, \quad a_2 + b_1 < m_1, \tag{1.6}$$

$$a_1(A-1)/A < m_1K/(a_2+b_1), \tag{1.7}$$

$$a_2b_2m_1(m_2-b_2)(a_2+b_1) < b_1m_2K[m_1-(a_2+b_1)][b_1m_2+a_2(m_2-b_2)], \tag{1.8}$$

$$\begin{aligned} &4\alpha\beta\bar{u}\bar{v}\bar{w}d_1d_2d_3 > M^2\bar{u}(\alpha\alpha_{23}\bar{v}+\beta\alpha_{32}\bar{w})^2(d_1+2\alpha_{11}M+\alpha_{12}M+\alpha_{13}M)\\ &+\alpha M^2\bar{v}(\alpha_{13}\bar{u}+\beta\alpha_{31}\bar{w})^2(d_2+\alpha_{21}M+2\alpha_{22}M+\alpha_{23}M)\\ &+\beta M^2\bar{w}(\alpha_{12}\bar{u}+\alpha\alpha_{21}\bar{v})^2(d_3+\alpha_{31}M+\alpha_{32}M+2\alpha_{33}M), \end{aligned} \tag{1.9}$$

where $\alpha = \frac{a_1\bar{u}}{m_1\bar{v}}, \beta = \frac{a_1a_2\bar{u}}{m_1m_2\bar{w}}$, $K = \frac{1}{2}\left\{2 - \frac{m_1}{b_1} + \sqrt{(2-\frac{m_1}{b_1})^2 + 4(1-a_1)(\frac{m_1}{b_1}-1)}\right\}$, M is the positive constant in (1.5). Then the positive equilibrium point $(\bar{u}, \bar{v}, \bar{w})$ is global asymptotic stable.

Remark 1 Problem (1.1) has a positive solution implies (1.2) holds. From [14], the positive equilibrium point $(\bar{u}, \bar{v}, \bar{w})$ of the corresponding weakly coupled reaction-diffusion system (1.1) is also global asymptotic stable under conditions (1.6)–(1.8) hold.

Remark 2 Problem (1.1) has no non-constant positive steady-state solution if all conditions of Theorem 2 hold.

2 Global Existence and Uniform Boundedness

In order to establish the uniform W_2^1-estimate of the solution to system (1.1), the following corollaries to Gagliardo-Nirenberg-type inequality (see [16]) play important roles.

Corollary 1 There exists a positive constant C such that

$$|u|_2 \le C(|u_x|_2^{\frac{1}{3}}|u|_1^{\frac{2}{3}} + |u|_1), \qquad \forall\, u \in W_2^1(0,1), \tag{2.1}$$

$$|u|_4 \le C(|u_x|_2^{\frac{1}{2}}|u|_1^{\frac{1}{2}} + |u|_1), \qquad \forall\, u \in W_2^1(0,1), \tag{2.2}$$

$$|u|_{\frac{5}{2}} \le C(|u_x|_2^{\frac{2}{5}}|u|_1^{\frac{3}{5}} + |u|_1), \qquad \forall\, u \in W_2^1(0,1), \tag{2.3}$$

$$|u_x|_2 \le C(|u_{xx}|_2^{\frac{3}{5}}|u|_1^{\frac{2}{5}} + |u|_1), \qquad \forall\, u \in W_2^2(0,1). \tag{2.4}$$

In this section we always denote that C is Sobolev embedding constant or other kind of absolute constant, A_j, B_j, C_j are the positive constants which depend only on $\alpha_{ij}(i,j = 1,2,3)$, $a_i, b_i, m_i(i = 1,2)$ and K_j are positive constants depending on d_i and $\alpha_{ij}(i,j = 1,2,3)$, $a_i, b_i, m_i(i = 1,2)$. When $d_1, d_2, d_3 \ge 1$, $\frac{d_1}{d_2}, \frac{d_3}{d_2} \in [\underline{d}, \overline{d}]$, L_j depend only on $\underline{d}, \overline{d}$ but do not on d_1, d_2, d_3.

Proof of Theorem 1 First, we establish L^1-estimates of the solution (u, v, w) of (1.1). Taking integrations of the first three equations in (1.1) over the domain [0,1], respectively, and then combining the three integration equalities linearly, we have

$$\frac{d}{dt}\int_0^1 \left[m_1 u + a_1 v + \frac{a_1 a_2}{m_2} w\right] dx \le -\int_0^1 \left[\frac{b_1}{a_1}(a_1 v) + b_2\left(\frac{a_1 a_2}{m_2} w\right)\right] dx + m_1\int_0^1 (u - u^2) dx.$$

Let $m_1\int_0^1 u dx - m_1\int_0^1 u^2 dx \le C_1 - C_2\int_0^1 u dx$, where $C_2 = \min\{\frac{b_1}{a_1}, b_2\}$, by Young inequality,

$$m_1(C_2+1)\int_0^1 u dx \le \frac{1}{2\varepsilon}(m_1(C_2+1))^2 + \frac{\varepsilon}{2}\int_0^1 u^2 dx.$$

Let $\varepsilon = 2m_1$, then $C_1 = \frac{1}{4}m_1(C_2+1)^2$. Thus

$$\frac{d}{dt}\int_0^1 \left[m_1 u + a_1 v + \frac{a_1 a_2}{m_2} w\right] dx \le C_1 - C_2\int_0^1 \left[m_1 u + a_1 v + \frac{a_1 a_2}{m_2} w\right] dx. \tag{2.5}$$

Then there exists a positive constant τ_0 such that

$$\int_0^1 u dx, \int_0^1 v dx, \int_0^1 w dx \le M_0, \qquad t \ge \tau_0, \tag{2.6}$$

where $M_0 = \frac{3C_1}{2C_2}\max\{(a_1)^{-1}, (m_1)^{-1}, \frac{m_2}{a_1 a_2}\}$. Moreover, there exists a positive constant M_0' which depends on $a_i, b_i, m_i (i = 1, 2)$ and the L^1-norm of u_0, v_0 and w_0, such that

$$\int_0^1 u dx, \int_0^1 v dx, \int_0^1 w dx \le M_0', \qquad t \ge 0. \tag{2.6$'$}$$

Second, we will obtain L^2-estimates of u, v, w. We multiply the first three equations in (1.1) by u, v, w, respectively, and integrate over [0,1] to have

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\int_0^1 u^2 dx &\le -d_1 \int_0^1 u_x^2 dx - \int_0^1 [(2\alpha_{11}u + \alpha_{12}v + \alpha_{13}w)u_x^2 + \alpha_{12}uu_xv_x + \alpha_{13}uu_xw_x]dx \\
&\quad + \int_0^1 u^2 dx, \\
\frac{1}{2}\frac{d}{dt}\int_0^1 v^2 dx &\le -d_2 \int_0^1 v_x^2 dx - \int_0^1 [(\alpha_{21}u + 2\alpha_{22}v + \alpha_{23}w)v_x^2 + \alpha_{21}vu_xv_x + \alpha_{23}vv_xw_x]dx \\
&\quad + m_1 \int_0^1 v^2 dx, \\
\frac{1}{2}\frac{d}{dt}\int_0^1 w^2 dx &\le -d_3 \int_0^1 w_x^2 dx - \int_0^1 [(\alpha_{31}u + \alpha_{32}v + 2\alpha_{33}w)w_x^2 + \alpha_{31}wu_xw_x + \alpha_{32}wv_xw_x]dx \\
&\quad + m_2 \int_0^1 w^2 dx.
\end{aligned}$$

Let $d^* = \min\{d_1, d_2, d_3\}$. We proceed in the following two cases.

(1) $t \ge \tau_0$.

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\int_0^1 (u^2 + v^2 + w^2)dx &\le -d^* \int_0^1 (u_x^2 + v_x^2 + w_x^2)dx - \int_0^1 q(u_x, v_x, w_x)dx \\
&\quad + A\int_0^1 (u^2 + v^2 + w^2)dx,
\end{aligned}$$

where $A = \max\{1, m_1, m_2\}$, and

$$\begin{aligned}
q(u_x, v_x, w_x) &= (2\alpha_{11}u + \alpha_{12}v + \alpha_{13}w)u_x^2 \\
&\quad + (\alpha_{21}u + 2\alpha_{22}v + \alpha_{23}w)v_x^2 + (\alpha_{31}u + \alpha_{32}v + 2\alpha_{33}w)w_x^2 \\
&\quad + (\alpha_{12}u + \alpha_{21}v)u_xv_x + (\alpha_{13}u + \alpha_{31}w)u_xw_x + (\alpha_{23}v + \alpha_{32}w)v_xw_x
\end{aligned}$$

is positive semi-definite quadratic form of u_x, v_x, w_x if (1.3) holds. Then

$$\frac{1}{2}\frac{d}{dt}\int_0^1 (u^2 + v^2 + w^2)dx \le -d^* \int_0^1 (u_x^2 + v_x^2 + w_x^2)dx + A\int_0^1 (u^2 + v^2 + w^2)dx. \tag{2.7}$$

Notice by (2.1) and (2.6) that $|u|_2^6 \le C(|u_x|_2^2 |u|_1^4 + |u|_1^6) \le CM_0^4(|u_x|_2^2 + M_0^2)$. Therefore

$$-d^* \int_0^1 (u_x^2 + v_x^2 + w_x^2)dx \le 3d^* M_0^2 - C_3 d^* \left[\int_0^1 (u^2 + v^2 + w^2)dx\right]^3. \tag{2.8}$$

Substituting (2.8) into (2.7), we have

$$\frac{1}{2}\frac{d}{dt}\int_0^1(u^2+v^2+w^2)dx \le -C_3d^*\left[\int_0^1(u^2+v^2+w^2)dx\right]^3+A\int_0^1(u^2+v^2+w^2)dx+3d^*M_0^2. \tag{2.9}$$

This means that there exist positive constants τ_1 and M_1 depending on $d_i, a_{ij}(i,j=1,2,3)$, $a_i, b_i, m_i(i=1,2)$ such that

$$\int_0^1 u^2dx, \int_0^1 v^2dx, \int_0^1 w^2dx \le M_1, t\ge\tau_1. \tag{2.10}$$

When $d^*\ge 1$, M_1 is independent of d^* since the zero point of the right-hand side in (2.10) can be estimated by positive constants independent on d^*

(2) $t\ge 0$. Replacing M_0 with M' and repeating estimates (2.7)–(2.10), one can obtain a new inequality which is similar to (2.10). The coefficients of this new inequality depend not only on $d_i, a_{ij}(i=1,2,3)$, $a_i, b_i, m_i(i=1,2)$ but also on initial functions u_0, v_0 and w_0. Then there exists positive constant M_1' depending on $d_i, a_{ij}(i,j=1,2,3)$, $a_i, b_i, m_i(i=1,2)$ and the L^2-norm of u_0, v_0, w_0 such that

$$\int_0^1 u^2dx, \int_0^1 v^2dx, \int_0^1 w^2dx \le M_1', \qquad t\ge 0. \tag{2.10$'$}$$

For $d\ge 1$, M_1' is independent of d^*.

Finally, L^2-estimates of u_x, v_x and w_x will be obtained. We introduce the scaling that

$$\tilde u=\frac{u}{d_2}, \tilde v=\frac{v}{d_2}, \tilde w=\frac{w}{d_2}, \tilde t=d_1t, \tag{2.11}$$

denoting $\xi=\frac{d_2}{d_1}, \eta=\frac{d_3}{d_1}$, and using u,v,w,t instead of $\tilde u,\tilde v,\tilde w,\tilde t$, respectively, then system (1.1) reduces to

$$\begin{aligned}
&u_t=P_{xx}+uf(u,v,w),\ 0<x<1, t>0,\\
&v_t=Q_{xx}+vg(u,v,w),\ 0<x<1, t>0,\\
&w_t=R_{xx}+wh(u,v,w),\ 0<x<1, t>0,\\
&u_x(x,t)=v_x(x,t)=w_x(x,t)=0,\ x=0,1,t>0,\\
&u(x,0)=\tilde u_0(x)\ge0, v(x,0)=\tilde v_0(x)\ge 0, w(x,0)=\tilde w_0(x)\ge 0, 0<x<1,
\end{aligned} \tag{2.12}$$

where

$$\begin{aligned}
&P=u+\alpha_{11}\xi u^2+\alpha_{12}\xi uv+\alpha_{13}\xi uw,\\
&Q=\xi(v+\alpha_{21}uv+\alpha_{22}v^2+\alpha_{23}vw),\\
&R=\eta w+\alpha_{31}\xi uw+\alpha_{32}\xi vw+\alpha_{33}\xi w^2,\\
&f(u,v,w)=d_1^{-1}(1-d_2u-\tfrac{a_1v}{u+v}),\\
&g(u,v,w)=d_1^{-1}(-b_1+\tfrac{m_1u}{u+v}-\tfrac{a_2w}{v+w}),\\
&h(u,v,w)=d_1^{-1}(-b_2+\tfrac{m_2v}{v+w}).
\end{aligned}$$

We still divide the subsequent discuss into two cases.

(1) $t \geq \tau_1^*(= d_2\tau_1)$ (namely, $t \geq \tau_1$ in original scale). It is clearly that

$$\begin{aligned}
&\int_0^1 u dx, \int_0^1 v dx, \int_0^1 w dx \leq M_0 d_2^{-1}, \\
&\int_0^1 u^2 dx, \int_0^1 v^2 dx, \int_0^1 w^2 dx \leq M_1 d_2^{-2}, \\
&|P|_1, |Q|_1, |R|_1 \leq A_1 K_1 d_2^{-1},
\end{aligned} \tag{2.13}$$

where $K_1 = (1+\xi+\eta)M_0 + M_1\xi d_2^{-1}$. By Young inequality, one can obtain

$$\begin{aligned}
\int_0^1 u^4 dx &\leq \left(\int_0^1 u^2 dx\right)^{\frac{1}{3}} \left(\int_0^1 u^5 dx\right)^{\frac{2}{3}} \leq M_1^{\frac{1}{3}} d_1^{-\frac{2}{3}} \left(\int_0^1 u^5 dx\right)^{\frac{2}{3}}, \\
\int_0^1 u^2 v^2 dx &\leq \left(\int_0^1 u^2 dx\right)^{\frac{1}{6}} \left(\int_0^1 v^2 dx\right)^{\frac{1}{6}} \left(\int_0^1 u^5 dx\right)^{\frac{1}{3}} \left(\int_0^1 v^5 dx\right)^{\frac{1}{3}} \\
&\leq M_1^{\frac{1}{3}} d_1^{-\frac{2}{3}} \left(\int_0^1 u^5 dx\right)^{\frac{1}{3}} \left(\int_0^1 v^5 dx\right)^{\frac{1}{3}}, \\
\int_0^1 u^3 dx &\leq \left(\int_0^1 u^2 dx\right)^{\frac{2}{3}} \left(\int_0^1 u^5 dx\right)^{\frac{1}{3}} \leq M_1^{\frac{2}{3}} d_1^{-\frac{4}{3}} \left(\int_0^1 u^5 dx\right)^{\frac{1}{3}}, \\
\int_0^1 uv^2 dx &\leq \left(\int_0^1 u^2 dx\right)^{\frac{1}{2}} \left(\int_0^1 v^2 dx\right)^{\frac{1}{6}} \left(\int_0^1 v^5 dx\right)^{\frac{1}{3}} \leq M_1^{\frac{2}{3}} d_1^{-\frac{4}{3}} \left(\int_0^1 v^5 dx\right)^{\frac{1}{3}}
\end{aligned} \tag{2.14}$$

Multiply the first three equations in (2.12) by P_t, Q_t, R_t, and integrating them over the domain [0,1], respectively, then adding up the three integration equalities, we have

$$\begin{aligned}
\frac{1}{2}\bar{y}'(t) \leq\ & -\int_0^1 u_t^2 dx - \xi\int_0^1 v_t^2 dx - \eta\int_0^1 w_t^2 dx - \xi\int_0^1 q(u_t, v_t, w_t) dx \\
& + \int_0^1 [(1 + 2\alpha_{11}\xi u + \alpha_{12}\xi v + \alpha_{13}\xi w) u u_t f + \alpha_{12}\xi u^2 v_t f + \alpha_{13}\xi u^2 w_t f] dx \\
& + \xi\int_0^1 [\alpha_{21} v^2 u_t g + (1 + \alpha_{21} u + 2\alpha_{22} v + \alpha_{23} w) v v_t g + \alpha_{23} v^2 w_t g] dx \\
& + \int_0^1 [\alpha_{31}\xi w^2 u_t h + \alpha_{32}\xi w^2 v_t h + (\eta + \alpha_{31}\xi u + \alpha_{32}\xi v + 2\alpha_{33}\xi w) w w_t h] dx,
\end{aligned}$$

where $\bar{y} = \int_0^1 (P_x^2 + Q_x^2 + R_x^2) dx$. It is not hard to verify by (1.3) that there exists a positive constant C_4 depending only on α_{ij} $(i, j = 1, 2, 3)$, such that

$$q(u_t, v_t, w_t) \geq C_4(u + v + w)(u_t^2 + v_t^2 + w_t^2).$$

Thus

$$\begin{aligned}
\frac{1}{2}\bar{y}'(t) \le & -\int_0^1 u_t^2dx - \xi\int_0^1 v_t^2dx - \eta\int_0^1 w_t^2dx - C_4\xi\int_0^1 (u+v+w)(u_t^2+v_t^2+w_t^2)dx \\
& + \int_0^1 (1+2\alpha_{11}\xi u + \alpha_{12}\xi v + \alpha_{13}\xi w)uu_tfdx + \int_0^1 \xi(1+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)v_tgvdx \\
& + \int_0^1 (\eta+\alpha_{31}\xi u+\alpha_{32}\xi v+2\alpha_{33}\xi w)w_thwdx + \int_0^1 \alpha_{12}\xi u^2v_tfdx + \int_0^1 \alpha_{13}\xi u^2w_tfdx \\
& + \int_0^1 \alpha_{21}\xi v^2u_tgdx + \int_0^1 \alpha_{23}\xi v^2w_tgdx + \int_0^1 \alpha_{31}\xi w^2u_thdx + \int_0^1 \alpha_{32}\xi w^2v_thdx. \qquad (2.15)
\end{aligned}$$

By the estimates (2.13), (2.14), one can obtain the following estimates for the terms on the right-hand side of (2.15)

$$\begin{aligned}
& -\int_0^1 u_t^2dx \le -\frac{1}{2}\int_0^1 P_{xx}^2dx + \int_0^1 u^2f^2dx, \\
& -\xi\int_0^1 v_t^2dx \le -\frac{\xi}{2}\int_0^1 Q_{xx}^2dx + \xi\int_0^1 v^2g^2dx, \\
& -\eta\int_0^1 w_t^2dx \le -\frac{\eta}{2}\int_0^1 R_{xx}^2dx + \eta\int_0^1 w^2h^2dx, \\
& \int_0^1 u^2f^2dx \le d_1^{-2}(1+a_1^2)\int_0^1 u^2dx + d_1^{-2}d_2^2\int_0^1 u^4dx + a_1d_1^{-2}d_2\int_0^1 u^3dx \\
\le\ & (1+a_1^2)d_1^{-2}d_2^{-2}M_1 + d_1^{-\frac{8}{3}}d_2^2M_1^{\frac{1}{3}}\left(\int_0^1 u^5dx\right)^{\frac{2}{3}} + a_1d_1^{-\frac{10}{3}}d_2M_1^{\frac{2}{3}}\left(\int_0^1 u^5dx\right)^{\frac{1}{3}}, \\
& \xi\int_0^1 v^2g^2dx \le \int_0^1 \xi d_1^{-2}(b_1^2+a_2^2+m_1^2+a_2b_1)v^2dx \le \xi M_1d_1^{-2}d_2^{-2}(b_1^2+a_2^2+m_1^2+a_2b_1), \\
& \eta\int_0^1 w^2h^2dx \le d_1^{-2}(b_2^2+m_2^2)\eta\int_0^1 w^2dx \le \eta d_1^{-2}(b_2^2+m_2^2)M_1d_2^{-2}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
& -\int_0^1 u_t^2dx - \xi\int_0^1 v_t^2dx - \eta\int_0^1 w_t^2dx \\
\le\ & -\frac{1}{2}\int_0^1 P_{xx}^2dx - \frac{\xi}{2}\int_0^1 Q_{xx}^2dx - \frac{\eta}{2}\int_0^1 R_{xx}^2dx \\
& + C_5(1+\xi+\eta)M_1d_1^{-2}d_2^{-2} + C_6\xi^2(1+\eta)M_1^{\frac{1}{3}}d_2^{-\frac{2}{3}}\left[\int_0^1 u^5dx\right]^{\frac{2}{3}} \\
& + C_7\xi d_1^{-1}M_1^{\frac{2}{3}}d_2^{-\frac{4}{3}}\left(\int_0^1 u^5dx\right)^{\frac{1}{3}}. \qquad (2.16)
\end{aligned}$$

For $\int_0^1 uu_t f dx$, one can obtain

$$
\begin{aligned}
&\int_0^1 uu_t f dx \\
\leq\ & d_1^{-1}(1+a_1)\left|\int_0^1 u_t u dx\right| + \xi\left|\int_0^1 u^2 u_t dx\right| \\
\leq\ & d_1^{-1}(1+a_1)\left(\frac{1}{2\epsilon}\int_0^1 u dx + \frac{\epsilon}{2}\int_0^1 uu_t^2 dx\right) + \xi\left(\frac{1}{2\epsilon}\int_0^1 u^3 dx + \frac{\epsilon}{2}\int_0^1 uu_t^2 dx\right) \\
\leq\ & \frac{1+a_1}{2\epsilon} M_0 d_1^{-1} d_2^{-1} + \frac{1}{2\epsilon}\xi M_1^{\frac{2}{3}} d_1^{-\frac{4}{3}}\left(\int_0^1 u^5 dx\right)^{\frac{1}{3}} + \frac{(d_1^{-1}+\xi)}{2}\epsilon\int_0^1 uu_t^2 dx.
\end{aligned}
$$

Similarly, we estimates the rest term on the right-hand side of (2.15), we have

$$
\begin{aligned}
&\int_0^1 (1+2\alpha_{11}\xi+\alpha_{12}\xi v+\alpha_{13}\xi)uu_t f dx + \int_0^1 \xi(1+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)vv_t g dx \\
&+\int_0^1 (\eta+\alpha_{31}\xi u+\alpha_{32}\xi v+2\alpha_{33}\xi w)ww_t h dx + \int_0^1 \alpha_{12}\xi uv_t f dx + \int_0^1 \alpha_{13}\xi uw_t f dx \\
&+\int_0^1 \alpha_{21}\xi v^2 u_t g dx + \int_0^1 \alpha_{23}\xi v^2 w_t g dx + \int_0^1 \alpha_{31}\xi w^2 u_t h dx + \int_0^1 \alpha_{32}\xi w^2 v_t h dx \\
\leq\ & \lambda\epsilon\xi\int_0^1 (u+v+w)(u_t^2+v_t^2+w_t^2)dx + \frac{C_8}{\epsilon}M_0 d_1^{-1} d_2^{-2}(1+\xi+\eta) \\
&+\frac{C_9}{\epsilon}M_1^{\frac{2}{3}} d_2^{-\frac{4}{3}}\xi(1+d_1^{-1}+\eta)\left[\int_0^1 (u^5+v^5+w^5)dx\right]^{\frac{1}{3}} + \frac{C_{10}}{\epsilon}\xi^2\int_0^1 (u^5+v^5+w^5)dx,
\end{aligned}
\tag{2.17}
$$

where λ is a positive integer. Choose a small enough positive number $\epsilon = \epsilon(\alpha_{ij}(i = 1,2,3)$, $a_i, b_i, m_i, (i = 1,2)$ such that $\lambda\epsilon < C_4$. Substituting inequalities (2.16) and (2.17) into (2.15), one can obtain

$$
\begin{aligned}
\frac{1}{2}\bar{y}'(t) \quad \leq\ & -\frac{1}{2}\int_0^1 P_{xx}^2 dx - \frac{\xi}{2}\int_0^1 Q_{xx}^2 dx - \frac{\eta}{2}\int_0^1 R_{xx}^2 dx \\
&+B_1 K_2 d_1^{-1} d_2^{-1} + B_2 K_3 d_1^{-\frac{4}{3}} z^{\frac{1}{3}} + B_3 K_4 d_1^{-\frac{2}{3}} z^{\frac{2}{3}} + B_4 K_5 z,
\end{aligned}
\tag{2.18}
$$

where $z = \int_0^1 (u^5+v^5+w^5)dx$, $K_2 = (1+\xi+\eta)(M_0+M_1)$, $K_3 = M_1^{\frac{2}{3}}\xi(1+d_1^{-1}+\eta)$, $K_4 = M_1^{\frac{1}{3}}\xi^2(1+\eta)$, $K_5 = \xi^2$. Clearly,

$$P \geq \alpha_{11}\xi u^2,\ Q \geq \alpha_{22}\xi v^2,\ R \geq \alpha_{33}\xi w^2.$$

It follows from inequality (2.3) to functions P, Q, R that

$$
\begin{aligned}
&z \leq B_5\xi^{-\frac{5}{2}}\int_0^1 (P^{\frac{5}{2}}+Q^{\frac{5}{2}}+R^{\frac{5}{2}})dx \leq B_6\xi^{-\frac{5}{2}}K_1^{\frac{3}{2}} d_1^{-\frac{3}{2}}\bar{y}^{\frac{1}{2}} + B_6\xi^{-\frac{5}{2}}K_1^{\frac{5}{2}} d_1^{-\frac{5}{2}}, \\
&z^{\frac{1}{3}} \leq B_7\xi^{-\frac{5}{6}}K_1^{\frac{1}{2}} d_1^{-\frac{1}{2}}\bar{y}^{\frac{1}{6}} + B_7\xi^{-\frac{5}{6}}K_1^{\frac{5}{6}} d_1^{-\frac{5}{6}}, \\
&z^{\frac{2}{3}} \leq B_8\xi^{-\frac{5}{3}}K_1 d_1^{-1}\bar{y}^{\frac{1}{3}} + B_8\xi^{-\frac{5}{3}}K_1^{\frac{5}{3}} d_1^{-\frac{5}{3}}.
\end{aligned}
\tag{2.19}
$$

Moreover, one can obtain by (2.4) and (2.15)

$$\begin{aligned} & -\frac{\xi}{2}\int_0^1 P_{xx}^2 dx - \frac{1}{2}\int_0^1 Q_{xx}^2 dx - \frac{\eta}{2}\int_0^1 R_{xx}^2 dx \\ \leq\ & -B_9 \min\{1,\xi,\eta\} L_1^{-\frac{4}{3}} d_1^{\frac{4}{3}} \bar{J}^{\frac{5}{3}} + (1+\xi+\eta) L_1^2 d_1^{-2}. \end{aligned} \tag{2.20}$$

Combining (2.18), (2.19) and (2.20) we have

$$\begin{aligned} & \frac{1}{2}\bar{y}'(t) \\ \leq\ & -A_1 \min\{1,\xi,\eta\} K_1^{-\frac{4}{3}} d_2^{\frac{4}{3}} \bar{y}^{\frac{5}{3}} + A_2 \xi^{-\frac{5}{6}} K_1^{\frac{1}{2}} K_3 d_2^{-\frac{11}{6}} \bar{y}^{\frac{1}{6}} + A_3 \xi^{-\frac{5}{3}} K_1 K_4 d_2^{-\frac{5}{3}} \bar{y}^{\frac{1}{3}} \\ & + A_4 \xi^{-\frac{5}{2}} K_1^{\frac{3}{2}} K_5 d_2^{-\frac{3}{2}} \bar{y}^{\frac{1}{2}} \\ & + A_5 [K_1^2 d_2^{-2}(1+\xi+\eta) + K_2 d_1^{-1} d_2^{-1} + K_1^{\frac{5}{6}} K_3 \xi^{-\frac{5}{6}} d_2^{-\frac{13}{6}} + K_1^{\frac{5}{3}} K_4 \xi^{-\frac{5}{3}} d_2^{-\frac{7}{3}} + K_1^{\frac{5}{2}} K_5 \xi^{-\frac{5}{2}} d_2^{-\frac{5}{2}}]. \end{aligned} \tag{2.21}$$

Multiplying inequality (2.21) by d_2^2, we have

$$\begin{aligned} \frac{1}{2}y'(t) \ \leq\ & -A_1 \min\{1,\xi,\eta\} K_1^{-\frac{4}{3}} y^{\frac{5}{3}} + A_2 \xi^{-\frac{5}{6}} K_1^{\frac{1}{2}} K_3 d_2^{-\frac{1}{6}} y^{\frac{1}{6}} \\ & + A_3 \xi^{-\frac{5}{3}} K_1 K_4 d_2^{-\frac{1}{3}} y^{\frac{1}{3}} + A_4 \xi^{-\frac{5}{2}} K_1^{\frac{3}{2}} K_5 d_2^{-\frac{1}{2}} y^{\frac{1}{2}} \\ & + A_5 [K_1^2 (1+\xi+\eta) + K_2 \xi + K_1^{\frac{5}{6}} K_3 \xi^{-\frac{5}{6}} d_2^{-\frac{1}{6}} + K_1^{\frac{5}{3}} K_4 \xi^{-\frac{5}{3}} d_2^{-\frac{1}{3}} + K_1^{\frac{5}{2}} K_5 \xi^{-\frac{5}{2}} d_2^{-\frac{1}{2}}], \end{aligned} \tag{2.22}$$

where $y = \int_0^1 [(d_2 P_x)^2 + (d_2 Q_x)^2 + (d_2 R_x)^2] dx$. Inequality (2.22) implies that there exist $\tilde{\tau}_2 > 0$ and positive constant $\tilde{M}_2$ depending only on $d_i, \alpha_{ij}(i,j=1,2,3)$, $a_i, b_i, m_i (i=1,2)$ such that

$$\int_0^1 (d_2 P_x)^2 dx, \int_0^1 (d_2 Q_x)^2 dx, \int_0^1 (d_2 R_x)^2 dx \leq \tilde{M}_2, \qquad t \geq \tilde{\tau}_2. \tag{2.23}$$

In the case that $d_1, d_2, d_3 \geq 1, \frac{d_2}{d_1}, \frac{d_3}{d_1} \in [\underline{d}, \overline{d}]$, the coefficients of inequality (2.22) can be estimated by some constants depending on $\underline{d}, \overline{d}$ but not on d_1, d_2, d_3. So $\tilde{M}_2$ depends on $\alpha_{ij}(i,j=1,2,3), a_i, b_i, m_i, (i=1,2), \underline{d}, \overline{d}$ and is irrelevant to d_1, d_2, d_3 when $d_1, d_2, d_3 \geq 1$ and $\frac{d_2}{d_1}, \frac{d_3}{d_1} \in [\underline{d}, \overline{d}]$. Since

$$\begin{pmatrix} u_x \\ v_x \\ w_x \end{pmatrix} = \begin{pmatrix} P_u & P_v & P_w \\ Q_u & Q_v & Q_w \\ R_u & R_v & R_w \end{pmatrix}^{-1} \begin{pmatrix} P_x \\ Q_x \\ R_x \end{pmatrix},$$

we can transform the formulations of u_x, v_x, w_x into fraction representations, then distribute the denominators of the absolute value of the fractions to the numerators term by term and enlarge the term concerning with u_x, v_x or w_x to obtain

$$|d_2 u_x| + |d_2 v_x| + |d_2 w_x| \leq K(|d_2 P_x| + |d_2 Q_x| + |d_2 R_x|), 0 < x < 1, t > 0, \tag{2.24}$$

where K is a constant depending only on $\xi, \eta, \alpha_{ij}(i,j=1,2,3)$, After scaling back and contacting estimates (2.23) and (2.24), there exist positive constant M_2 depending on $d_i,\ \alpha_{ii},\ (i=1,2,3), \alpha_{12},\ \alpha_{21},\ \alpha_{23},\ \alpha_{32}, a_i,\ b_i, m_i(i=1,2)$ and $\tau_2>0$, such that

$$\int_0^1 u_x^2 dx, \int_0^1 v_x^2 dx, \int_0^1 w_x^2 dx \le M_2, \qquad t \ge \tau_2. \tag{2.25}$$

When $d_1, d_2, d_3 \ge 1$ and $\frac{d_2}{d_1}, \frac{d_3}{d_1} \in [\underline{d}, \overline{d}]$, M_2 independent on d_1, d_2, d_3.

(2) $t \ge 0$. Modifying the dependency of the coefficients in inequalities (2.15)–(2.17), namely replacing M_0, M_1 with M_0', M_1', there exists positive constant M_2' depending on $d_i, \alpha_{ij}(i,j=1,2,3), a_i, b_i, m_i, (i=1,2)$ and the W_2^1 -norm of u_0, v_0, w_0 such that

$$\int_0^1 u_x^2 dx, \int_0^1 v_x^2 dx, \int_0^1 w_x^2 dx \le M_2', \qquad t \ge 0. \tag{2.25$'$}$$

Furthermore, in the case that $d_1, d_2, d_3 \ge 1, \frac{d_2}{d_1}, \frac{d_3}{d_1} \in [\underline{d}, \overline{d}]$, M_2' depends on $\underline{d}, \overline{d}$ but not on d_1, d_2, d_3.

Summarizing estimates (2.6), (2.10), (2.25) and using Sobolev embedding theorem, there exist positive constants M, M' depending only on $d_i,\ \alpha_{ij}(i,j=1,2,3),\ a_i, b_i, m_i(i=1,2)$ such that (1.4) and (1.5) hold. In particular, M, M' depend only on $\alpha_{ij}(i,j=1,2,3),\ a_i, b_i, m_i(i=1,2)$ $\underline{d}, \overline{d}$ but do not depend on d_1, d_2, d_3 when $d_1, d_2, d_3 \ge 1, \frac{d_2}{d_1}, \frac{d_3}{d_1} \in [\underline{d}, \overline{d}]$.

Similarly, there exist positive constant M'' depending on $d_i,\ \alpha_{ij}(i,j=1,2,3),\ a_i, b_i, m_i(i=1,2)$ and the initial functions u_0, v_0, w_0 such that

$$|u(\cdot,t)|_{1,2}, |v(\cdot,t)|_{1,2}, |w(\cdot,t)|_{1,2} \le M'', t \ge 0.$$

Further, in the case that $d_1, d_2, d_3 \ge 1, \frac{d_2}{d_1}, \frac{d_3}{d_1} \in [\underline{d}, \overline{d}]$, M'' depends only on $\underline{d}, \overline{d}$ but not on d_1, d_2, d_3. Thus $T=+\infty$. This is complete proof of Theorem 1.

3 Global Stability

In this section we discuss global asymptotic stability of positive equilibrium point $(\bar{u}, \bar{v}, \bar{w})$ for (1.1), namely to prove Theorem 2. Define

$$H(u,v,w) = \int_0^1 \left(u - \bar{u} - \bar{u}\ln\frac{u}{\bar{u}}\right) dx + \alpha \int_0^1 \left(v - \bar{v} - \bar{v}\ln\frac{v}{\bar{v}}\right) dx + \beta \int_0^1 \left(w - \bar{w} - \bar{w}\ln\frac{w}{\bar{w}}\right) dx,$$

where $\alpha = \frac{a_1\bar{u}}{m_1\bar{v}}, \beta = \frac{a_1 a_2 \bar{u}}{m_1 m_2 \bar{w}}$. Obviously, $H(u,v,w)$ is nonnegative and $H(u,v,w)=0$ if and only if $(u,v,w)=(\bar{u},\bar{v},\bar{w})$. By Theorem 1, $H(u,v,w)$ is well-posed for $t \ge 0$ if (u,v,w) is a non-zero solution to system (1.1). The time derivative of $H(u,v,w)$ for system (1.1) satisfies

$$\begin{aligned}
&\frac{dH(u,v,w)}{dt}\\
= & -\int_0^1 \Bigg[\left(d_1+2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w\right)\frac{\bar u}{u^2}u_x^2+\alpha\left(d_2+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w\right)\frac{\bar v}{v^2}v_x^2\\
&+\beta\left(d_3+\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w\right)\frac{\bar w}{w^2}w_x^2+\left(\alpha_{12}\frac{\bar u}{u}+\alpha\alpha_{21}\frac{\bar v}{v}\right)u_xv_x\\
&+\left(\alpha_{13}\frac{\bar u}{u}+\beta\alpha_{31}\frac{\bar w}{w}\right)u_xw_x+\left(\alpha\alpha_{23}\frac{\bar v}{v}+\beta\alpha_{32}\frac{\bar w}{w}\right)v_xw_x\Bigg]dx\\
&-\int_0^1\Bigg[\left(1-\frac{a_1\bar v}{(\bar u+\bar v)(u+v)}\right)(u-\bar u)^2+\beta\frac{m_2\bar v}{(\bar v+\bar w)(v+w)}(w-\bar w)^2\\
&+\alpha\left(\frac{m_1\bar u}{(\bar u+\bar v)(u+v)}-\frac{a_2\bar w}{(\bar v+\bar w)(v+w)}\right)(v-\bar v)^2\Bigg]dx. \qquad (3.1)
\end{aligned}$$

The first integrand in above equality is positive semi-definite if

$$\begin{aligned}
&4\alpha\beta\frac{\bar u}{u^2}\frac{\bar v}{v^2}\frac{\bar w}{w^2}(d_1+2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w)(d_2+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)\\
&\cdot(d_3+\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w)\\
&+\left(\alpha_{12}\frac{\bar u}{u}+\alpha\alpha_{21}\frac{\bar v}{v}\right)\left(\alpha_{13}\frac{\bar u}{u}+\beta\alpha_{31}\frac{\bar w}{w}\right)\left(\alpha\alpha_{23}\frac{\bar v}{v}+\beta\alpha_{32}\frac{\bar w}{w}\right)\\
> &\ \frac{\bar u}{u^2}\left(\alpha\alpha_{23}\frac{\bar v}{v}+\beta\alpha_{32}\frac{\bar w}{w}\right)^2(d_1+2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w)\\
&+\alpha\frac{\bar v}{v^2}\left(\alpha_{13}\frac{\bar u}{u}\beta\alpha_{31}\frac{\bar w}{w}\right)^2(d_2+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)\\
&+\beta\frac{\bar w}{w^2}\left(\alpha_{12}\frac{\bar u}{u}+\alpha\alpha_{21}\frac{\bar v}{v}\right)^2(d_3+\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w). \qquad (3.2)
\end{aligned}$$

By the maximum-norm estimate in Theorem 1, condition (1.9) implies (3.2). Under our assumptions (1.6)–(1.8), we can claim that for $t \gg 1$ the following hold:

$$\frac{a_1\bar v}{(\bar u+\bar v)(u+v)}\le 1,\quad \frac{a_2\bar w}{(\bar v+\bar w)(v+w)}\le\frac{m_1\bar u}{(\bar u+\bar v)(u+v)}.$$

So, the second integrand in above equality is positive semi-definite if conditions (1.6)–(1.8) hold. Therefore, when the all conditions in Theorem 2 hold, there exists a positive constant δ such that

$$\begin{aligned}
&\frac{dH(u,v,w)}{dt}\le-\delta\int_0^1[(u-\bar u)^2+(v-\bar v)^2+(w-\bar w)^2]dx,\\
&\frac{dH(u,v,w)}{dt}<0,(u,v,w)\ne(\bar u,\bar v,\bar w). \qquad (3.3)
\end{aligned}$$

Now, we recall the following lemma which can be find in [19].

Lemma 1 Let a and b be positive constants. Assume that $\varphi,\ \psi \in C^1[a,+\infty)$, $\psi(t) \geq 0$, and φ is bounded from below. If $\varphi'(t) \leq -b\psi(t)$ and $\psi'(t)$ is bounded from above in $[a,+\infty)$, then $\lim\limits_{t\to\infty} \psi(t) = 0$.

Using partial integration, Hölder inequality and (1.5), one can easily verify that

$$\frac{d}{dt}\int_0^1 [(u-\bar{u})^2 + (v-\bar{v})^2 + (w-\bar{w})^2]dx$$

is bounded from above. Then from Lemma 1 and (3.3) we have

$$|u(\cdot,t)-\overline{u}|_2 \to 0,\ |v(\cdot,t)-\overline{v}|_2 \to 0,\ |w(\cdot,t)-\overline{w}|_2 \to 0 \quad (t\to\infty).$$

Clearly, $|u(\cdot,t)|_\infty \leq C|u|_{1,2}^{\frac{1}{2}}|u|_2^{\frac{1}{2}}$. By (1.4), we have

$$|u(\cdot,t)-\overline{u}|_\infty \to 0,\ |v(\cdot,t)-\overline{v}|_\infty \to 0,\ |w(\cdot,t)-\overline{w}|_\infty \to 0 \quad (t\to\infty).$$

Namely, (u,v,w) converges uniformly to $(\bar{u},\bar{v},\bar{w})$. By the fact that $H(u,v,w)$ is decreasing for $t \geq 0$, it is obvious that $(\bar{u},\bar{v},\bar{w})$ is global asymptotic stable. The proof of Theorem 2 is completed.

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带比例功能反应函数食物链交错扩散模型的整体解

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摘要: 本文研究了带有比例功能反应函数食物链交错扩散模型整体解的存在性和正平衡点的稳定性. 利用能量方法和Gagliardo-Nirenberg型不等式, 获得了该模型整体解的存在性和一致有界性, 同时通过构造Lyapunov 函数给出了该模型正平衡点全局渐近稳定的充分条件.

关键词: 比例依赖功能反应函数; 交错扩散; 整体解; 一致有界性; 稳定性

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