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A NOTE ON THE APPROXIMATED INVERSE OF A NON-NEGATIVE SYMMETRIC MATRIX

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Abstract: This paper studies the issue of the approximated inverse of nonnegative symmetric matrices. By using the matrix $S = (s_{i,j})$ to approximate its inverse, an explicit bound on the approximation error is obtained, and one conclusion that the inverse is well approximated to the order $1/(n-1)^2$ uniformly for large n is also proved.

Keywords: approximation error; inverse; symmetric; positive elements

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1 Introduction

When solving the solution for a large system of linear equations, a good approximate inverse of the coefficient matrix is crucially important in establishing fast convergence rates for iterative algorithms. See the extensive reviews [1, 5, 7, 20]. Here, we are concerned with a $n \times n$ symmetric diagonally dominant matrices $T = (t_{i,j})$ with positive elements, i.e.,

$$t_{i,j} = t_{j,i} > 0 \text{ and } t_{i,i} \geq \sum_{j=1, j\neq i}^{n} t_{i,j},\ i = 1, \cdots, n. \tag{1.1}$$

It is easy to show that T must be positive definite. This kind of diagonally dominant nonnegative matrices has received wide attention [6, 8, 10]. In [2, 7, 9], the problems on inverses of nonnegative matrices have been investigated. Markham [13] and Martínez et al. [14] studied the sufficient conditions that the inverses of nonnegative matrices are M-matrices. We propose to approximate the inverse of T, T^{-1}, by the matrix $S = (s_{i,j})$, where

$$s_{i,j} = \frac{\delta_{i,j}}{t_{i,i}} - \frac{1}{t_{\cdot\cdot}},$$

and $t_{\cdot\cdot} = \sum\limits_{i,j=1}^{n} (1-\delta_{i,j}) t_{i,j}$. In a special case that $t_{i,i} = \sum\limits_{j\neq i} t_{i,j}$ for all i, Yan and Xu [19] have obtained the upper bound of the approximation errors when using S to approximate the

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inverse of T, which is crucially used to establish the asymptotical normality of an estimated vector in the β-model for undirected random graphs with a growing number of nodes. In this paper, we derive an explicit upper bound on the approximation error for general cases (1.1).

2 An Explicit Bound on the Approximation Error

Let $m := \min_{1\le i<j\le n} t_{i,j}$, $\Delta_i := t_{i,i} - \sum_{j\ne i} t_{i,j}$, $M := \max\{\max_{1\le i<j\le n} t_{i,j}, \max_{1\le i\le n} \Delta_i\}$, and for a matrix $A = (a_{i,j})$, define $||A|| := \max_{i,j} |a_{i,j}|$. We have the following theorem.

Theorem 2.1 If

$$C(m,M) = \frac{2(n-2)m}{nM+(n-2)m} - \frac{M}{m(n-1)} - \frac{(n-2)Mm}{[(n-2)m+M][(n-2)m+2M]} > 0, \quad (2.1)$$

then

$$||T^{-1} - S|| \le \frac{1}{(n-1)^2} \times \left[\frac{1}{C(m,M)}\left(\frac{M}{m^2} + \frac{4M}{m^2 n}\right) + \frac{n-1}{mn}\right].$$

Proof Let I_n be the $n \times n$ identity matrix. Define $F = (f_{ij}) = T^{-1} - S$, $V = (v_{ij}) = I_n - TS$ and $W = (w_{ij}) = SV$. We have the recursion

$$F = T^{-1} - S = (T^{-1} - S)(I_n - TS) + S(I_n - TS) = FV + W. \quad (2.2)$$

Note that

$$\begin{aligned} v_{i,j} &= \delta_{i,j} - \sum_{k=1}^n t_{i,k} s_{k,j} \\ &= \delta_{i,j} - \sum_{k=1}^n t_{i,k}\left(\frac{\delta_{k,j}}{t_{k,k}} - \frac{1}{t_{..}}\right) \\ &= (\delta_{i,j} - 1)\frac{t_{i,j}}{t_{j,j}} + \frac{2t_{i,i} - \Delta_i}{t_{..}}, \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} w_{i,j} &= \sum_{k=1}^n s_{i,k} v_{k,j} = \sum_{k=1}^n \left(\frac{\delta_{i,k}}{t_{i,i}} - \frac{1}{t_{..}}\right)\left[(\delta_{k,j}-1)\frac{t_{k,j}}{t_{j,j}} + \frac{2t_{k,k} - \Delta_k}{t_{..}}\right] \\ &= \sum_{k=1}^n \frac{\delta_{i,k}}{t_{i,i}}\left[(\delta_{k,j}-1)\frac{t_{k,j}}{t_{j,j}} + \frac{2t_{k,k}-\Delta_k}{t_{..}}\right] - \frac{1}{t_{..}}\sum_{k=1}^n\left[(\delta_{k,j}-1)\frac{t_{k,j}}{t_{j,j}} + \frac{2t_{k,k}-\Delta_k}{t_{..}}\right] \\ &= \left[\frac{(\delta_{i,j}-1)}{t_{i,i}}\left(\frac{t_{i,j}}{t_{j,j}}\right) + \frac{2t_{i,i}-\Delta_i}{t_{i,i}t_{..}}\right] - \frac{1}{t_{..}}\left[\frac{-(t_{j,j}-\Delta_j)}{t_{j,j}} + 2 + \frac{\sum_k \Delta_k}{t_{..}}\right] \\ &= \frac{(\delta_{i,j}-1)t_{i,j}}{t_{i,i}t_{j,j}} + \frac{1}{t_{..}} - \frac{\Delta_i}{t_{i,i}t_{..}} - \frac{\Delta_j}{t_{j,j}t_{..}} - \frac{\sum_k \Delta_k}{t_{..}^2}. \end{aligned} \quad (2.4)$$

Furthermore, when $i \neq j$,

$$0 < \frac{1}{t_{..}} \leq \frac{1}{mn(n-1)},$$
$$0 < \frac{t_{i,j}}{t_{i,i}t_{j,j}} \leq \frac{M}{m^2(n-1)^2},$$
$$0 < \frac{\Delta_i}{t_{i,i}t_{..}} \leq \frac{M}{m^2n(n-1)^2},$$

and it is easy to show, when i, j, k are different from each other,

$$\begin{aligned}
|w_{i,i}| &\leq \max\{\frac{1}{mn(n-1)}, \frac{3M}{m^2n(n-1)^2}\}, \\
|w_{i,j}| &\leq \max\{\frac{1}{mn(n-1)}, \frac{M}{m^2(n-1)^2} + \frac{3M}{m^2n(n-1)^2}\}, \\
|w_{i,j} - w_{i,k}| &\leq \frac{M}{m^2(n-1)^2} + \frac{M}{m^2n(n-1)^2}, \\
|w_{i,i} - w_{i,k}| &\leq \frac{M}{m^2(n-1)^2} + \frac{M}{m^2n(n-1)^2}.
\end{aligned}$$

It follows that

$$\max(|w_{i,j}|, |w_{i,j} - w_{i,k}|) \leq \frac{M}{m^2(n-1)^2} + \frac{3M}{m^2n(n-1)^2} \quad \text{for all } i, j, k. \tag{2.5}$$

Next we use the recursion (2.2) to obtain a bound of the approximate error $||F||$. By (2.2) and (2.3), for any i, we have

$$f_{i,j} = \sum_{k=1}^{n} f_{i,k}[(\delta_{k,j} - 1)\frac{t_{k,j}}{t_{j,j}} + \frac{2t_{k,k} - \Delta_k}{t_{..}}] + w_{i,j}, \quad j = 1, \cdots, n. \tag{2.6}$$

Thus, to prove Theorem 2.1, it is sufficient to show that $|f_{i,j}| \leq C(M,m)/(n-1)^2$ for any i, j. Fixing any i, let $f_{i,\alpha} = \max_{1\leq k\leq n} f_{i,k}$ and $f_{i,\beta} = \min_{1\leq k\leq n} f_{i,k}$.

First, we will show that $f_{i,\beta} \leq 1/t_{..} \leq 1/(m(n-1)^2)$. A direct calculation gives that

$$\begin{aligned}
\sum_{k=1}^{n} f_{i,k}t_{k,i} &= \sum_{k=1}^{n}(T^{-1}_{i,k} - (\frac{\delta_{i,k}}{t_{i,i}} - \frac{1}{t_{..}}))t_{k,i} \\
&= 1 - (1 - \sum_{k=1}^{n}\frac{t_{k,i}}{t_{..}}) = \sum_{k=1}^{n}\frac{t_{k,i}}{t_{..}}.
\end{aligned} \tag{2.7}$$

Thus, $f_{i,\beta}\sum_{k=1}^{n} t_{k,i} \leq \sum_{k=1}^{n} f_{i,k}t_{k,i} = \sum_{k=1}^{n}\frac{t_{k,i}}{t_{..}}$. It follows that $f_{i,\beta} \leq 1/t_{..}$ and, similarly, $f_{i,\alpha} \geq 1/t_{..}$.

Note that $(1 - \Delta_\alpha/t_{\alpha,\alpha})f_{i,\beta} = -\sum_{k=1}^{n} f_{i,\beta}(\delta_{k,\alpha} - 1)\frac{t_{k,\alpha}}{t_{\alpha,\alpha}}$. Thus,

$$f_{i,\alpha} + (1 - \frac{\Delta_\alpha}{t_{\alpha,\alpha}})f_{i,\beta} = \sum_{k=1}^{n}(f_{i,k} - f_{i,\beta})(\delta_{k,\alpha} - 1)\frac{t_{k,\alpha}}{t_{\alpha,\alpha}} + \sum_{k=1}^{n} f_{i,k}(\frac{2t_{k,k} - \Delta_k}{t_{..}}) + w_{i,\alpha}. \tag{2.8}$$

Similarly, by $(1-\Delta_\beta/t_{\beta,\beta})f_{i,\beta}=-\sum\limits_{k=1}^{n}f_{i,\beta}(\delta_{k,\beta}-1)\frac{t_{k,\beta}}{t_{\beta,\beta}}$, we have that

$$f_{i,\beta}+(1-\frac{\Delta_\beta}{t_{\beta,\beta}})f_{i,\beta}=\sum_{k=1}^{n}(f_{i,k}-f_{i,\beta})(\delta_{k,\beta}-1)\frac{t_{k,\beta}}{t_{\beta,\beta}}+\sum_{k=1}^{n}f_{i,k}(\frac{2t_{k,k}-\Delta_k}{t_{..}})+w_{i,\beta}. \quad (2.9)$$

Combining the above two equations, it yields

$$\begin{array}{rl} & f_{i,\alpha}-f_{i,\beta}+(\frac{\Delta_\beta}{t_{\beta,\beta}}-\frac{\Delta_\alpha}{t_{\alpha,\alpha}})f_{i,\beta} \\ = & \sum\limits_{k=1}^{n}(f_{i,k}-f_{i,\beta})[(\delta_{k,\alpha}-1)\frac{t_{k,\alpha}}{t_{\alpha,\alpha}}-(\delta_{k,\beta}-1)\frac{t_{k,\beta}}{t_{\beta,\beta}}]+w_{i,\alpha}-w_{i,\beta}. \end{array} \quad (2.10)$$

Let $\Omega=\{k:(1-\delta_{k,\beta})t_{k,\beta}/t_{\beta,\beta}\geq(1-\delta_{k,\alpha})t_{k,\alpha}/t_{\alpha,\alpha}\}$ and let $|\Omega|=\lambda$. Note that $1\leq\lambda\leq n-1$. Then,

$$\begin{array}{rl} & \sum\limits_{k=1}^{n}(f_{i,k}-f_{i,\beta})[(\delta_{k,\alpha}-1)\frac{t_{k,\alpha}}{t_{\alpha,\alpha}}-(\delta_{k,\beta}-1)\frac{t_{k,\beta}}{t_{\beta,\beta}}] \\ \leq & \sum\limits_{k\in\Omega}(f_{i,k}-f_{i,\beta})[(1-\delta_{k,\beta})\frac{t_{k,\beta}}{t_{\beta,\beta}}-(1-\delta_{k,\alpha})\frac{t_{k,\alpha}}{t_{\alpha,\alpha}}] \\ \leq & (f_{i,\alpha}-f_{i,\beta})[\frac{\sum_{k\in\Omega}t_{k,\beta}}{t_{\beta,\beta}}-\frac{\sum_{k\in\Omega}(1-\delta_{k,\alpha})t_{k,\alpha}}{t_{\alpha,\alpha}}] \\ \leq & (f_{i,\alpha}-f_{i,\beta})[\frac{\lambda M}{\lambda M+(n-1-\lambda)m}-\frac{(\lambda-1)m}{(\lambda-1)m+(n-\lambda)M+M}]. \end{array} \quad (2.11)$$

Let

$$f(\lambda)=\frac{\lambda M}{\lambda M+(n-1-\lambda)m}-\frac{(\lambda-1)m}{(\lambda-1)m+(n-\lambda)M}.$$

There are two cases to consider the maximum of $f(\lambda)$ in the range of $\lambda\in[1,n-1]$.

Case I When $M=m$, it is easy to show $f(\lambda)=1/(n-1)$.

Case II If $M\neq m$, since

$$\begin{array}{rcl} f'(\lambda) & = & \frac{(n-1)Mm}{[\lambda M+(n-1-\lambda)m]^2}-\frac{(n-1)Mm}{[(\lambda-1)m+(n-\lambda)M]^2} \\ & = & \frac{(n-1)Mm[(n-2\lambda)(M-m)][\lambda M+(n-1-\lambda)m+(\lambda-1)m+(n-\lambda)M]}{[\lambda M+(n-1-\lambda)m]^2[(\lambda-1)m+(n-\lambda)M]^2} \end{array}$$

and

$$f''(\lambda)=-2(M-m)Mm(n-1)\left(\frac{1}{[\lambda M+(n-1-\lambda)m]^3}+\frac{1}{[(\lambda-1)m+(n-\lambda)M]^3}\right),$$

$f(\lambda)$ takes its maximum at $\lambda=n/2$ when $1\leq\lambda\leq n-1$. A direct calculation gives that

$$f(\frac{n}{2}) = \frac{nM-(n-2)m}{nM+(n-2)m}. \quad (2.12)$$

Moreover, denote

$$g(\lambda)=\frac{(\lambda-1)m}{(\lambda-1)m+(n-\lambda)M}-\frac{(\lambda-1)m}{(\lambda-1)m+(n-\lambda)M+M},$$

therefore

$$g'(\lambda) = \frac{Mm[M^2((n-\lambda)^2+2(n-\lambda)(\lambda-1)+n-1)+(2Mm-m^2)(\lambda-1)^2]}{[(\lambda-1)m+(n-\lambda)M]^2[(\lambda-1)m+(n-\lambda)M+M]^2},$$

$g'(\lambda) > 0$ when $1 \leq \lambda \leq n-1$ such that for $1 \leq \lambda \leq n-1$,

$$0 \leq g(\lambda) \leq g(n-1) = \frac{(n-2)Mm}{[(n-2)m+M][(n-2)m+2M]}. \tag{2.13}$$

By (2.12) and (2.13), we have

$$\begin{aligned} & \max_{1\leq\lambda\leq n-1}\left[\frac{\lambda M}{\lambda M+(n-1-\lambda)m} - \frac{(\lambda-1)m}{(\lambda-1)m+(n-\lambda)M+M}\right] \\ \leq & \max_{1\leq\lambda\leq n-1} f(\lambda) + \max_{1\leq\lambda\leq n-1} g(\lambda) \\ \leq & \frac{1}{n-1}I(M=m) + \frac{nM-(n-2)m}{nM+(n-2)m}I(M\neq m) + \frac{(n-2)Mm}{[(n-2)m+M][(n-2)m+2M]} \\ = & \frac{nM-(n-2)m}{nM+(n-2)m} + \frac{(n-2)Mm}{[(n-2)m+M][(n-2)m+2M]}, \end{aligned} \tag{2.14}$$

where $I(\cdot)$ is an indictor function. Since $f_{i,\alpha} - f_{i,\beta} + \frac{1}{t_{\cdot\cdot}} \geq |f_{i,\beta}|$, we have

$$\begin{aligned} & f_{i,\alpha} - f_{i,\beta} + (\tfrac{\Delta_\beta}{t_{\beta,\beta}} - \tfrac{\Delta_\alpha}{t_{\alpha,\alpha}})f_{i,\beta} \\ \geq & f_{i,\alpha} - f_{i,\beta} - (f_{i,\alpha} - f_{i,\beta} + \tfrac{1}{t_{\cdot\cdot}})|\tfrac{\Delta_\beta}{t_{\beta,\beta}} - \tfrac{\Delta_\alpha}{t_{\alpha,\alpha}}| \\ \geq & (1 - \tfrac{M}{m(n-1)})(f_{i,\alpha} - f_{i,\beta}) - \tfrac{M}{m^2n(n-1)^2}. \end{aligned} \tag{2.15}$$

Combining (2.10), (2.11), (2.14) and (2.15), it yields

$$\begin{aligned} & (1 - \tfrac{M}{m(n-1)})(f_{i,\alpha} - f_{i,\beta}) \\ \leq & (\tfrac{nM-(n-2)m}{nM+(n-2)m} + \tfrac{(n-2)Mm}{[(n-2)m+M][(n-2)m+2M]}) \times (f_{i,\alpha} - f_{i,\beta}) + \tfrac{M}{m^2(n-1)^2} + \tfrac{4M}{m^2n(n-1)^2}, \end{aligned}$$

so that

$$C(m,M)(f_{i,\alpha} - f_{i,\beta}) \leq \frac{M}{m^2(n-1)^2} + \frac{4M}{m^2n(n-1)^2},$$

where $C(m,M)$ is defined in (2.1). Consequently, if the condition (2.1) holds, then

$$\begin{aligned} \max_{j=1,\cdots,n} |f_{i,j}| & \leq f_{i,\alpha} - f_{i,\beta} + \tfrac{1}{t_{\cdot\cdot}} \\ & \leq \tfrac{1}{C(m,M)} \times \left[\tfrac{M}{m^2(n-1)^2} + \tfrac{4M}{m^2n(n-1)^2}\right] + \tfrac{1}{mn(n-1)} \\ & = \tfrac{1}{(n-1)^2} \times \left[\tfrac{1}{C(m,M)}(\tfrac{M}{m^2} + \tfrac{4M}{m^2n}) + \tfrac{n-1}{mn}\right], \end{aligned}$$

where the first inequality holds by $\max_j |f_{i,j}| \leq f_{i,\alpha} - f_{i,\beta} + f_{i,\beta}I(f_{i,\beta} > 0)$ and $0 \leq f_{i,\beta}I(f_{i,\beta} > 0) \leq 1/t_{\cdot\cdot}$. This completes the proof.

Remark If M and m are constants, $C(m,M) \approx 2m/(M+m)$ when n is large enough. Therefore the condition (2.1) is very week.

3 Discussion

Our proposed matrices S could be used as preconditioners for solving linear systems with diagonally dominant and non-negative matrices just as [18, 20] concerned with M-matrices. The bound on the approximation error in Theorem 2.1 depends on m, M and n. When m and M are bounded by a constant, all the elements of $T^{-1}-S$ are of order $O(1/(n-1)^2)$ as $n\to\infty$, uniformly.

Finally, we illustrate by an example that the bound on the approximation error in Theorem 2.1 is optimal in the sense that any bound in the form of $C(m,M)/f(n)$ requires $f(n)=O((n-1)^2)$ as $n\to\infty$. Assume that the matrix T consists of the elements: $t_{i,i}=(n-1)M, i=1,\cdots,n-1; t_{n,n}=(n-1)m$ and $t_{i,j}=m, i,j=1,\cdots,n; i\neq j$, which satisfies (1.1). By the Sherman-Morrison formula, we have

$$
\begin{aligned}
(T^{-1})_{i,j} &= \frac{\delta_{i,j}}{(n-1)M-m}-\frac{m}{[(n-1)M-m]^2}, i,j=1,\cdots,n-1,\\
(T^{-1})_{n,j} &= \frac{\delta_{n,j}}{(n-2)m}-\frac{1}{(n-2)[(n-1)M-m]},\ \ j=1,\cdots,n.
\end{aligned}
$$

In this case, the elements of S are

$$
\begin{aligned}
S_{i,j} &= \frac{\delta_{i,j}}{(n-1)M}-\frac{1}{n(n-1)m},\ i,j=1,\cdots,n-1; i\neq j,\\
S_{n,j} &= \frac{\delta_{n,j}}{(n-1)m}-\frac{1}{n(n-1)m},\ j=1,\cdots,n.
\end{aligned}
$$

It is easy to show that the bound of $||T^{-1}-S||$ is $O(\frac{1}{(n-1)^2m})$. This suggests that the rate $1/(n-1)^2$ is optimal. On the other hand, if M and m are constants, the upper bound of $||T^{-1}-S||$ approximately equal to $(1+\frac{M}{m})\frac{M}{2(n-1)^2m^2}$. Therefore there is a gap between these two bounds that implies there might be space for improvement. It is interesting to see if the bounds in Theorem 2.1 can be further relaxed.

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关于非负对称矩阵的近似逆矩阵

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摘要: 本文研究了非负对称矩阵的近似逆矩阵问. 利用矩阵$S=(s_{i,j})$ 去近似它的逆矩阵方法, 获得了近似误差的一个显式上界, 并且证明了近似逆的误差对于很大的n一致地具有阶$1/(n-1)^2$.

关键词: 近似误差; 逆; 对称; 非负元

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