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A NOTE ON QUASI-$*$-$A(K)$ OPERATORS

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Abstract: In this note, we introduce quasi-$*$-$A(k)$ operators and obtain their spectral properties as follows: (i) If T is quasi-$*$-$A(k)$ for $0 < k \leq 1$, then the spectral mapping theorem holds for the essential approximate point spectrum. (ii) If T is quasi-$*$-$A(k)$ for $0 < k \leq 1$, then $\sigma_{ja}(T)\backslash\{0\} = \sigma_a(T)\backslash\{0\}$. Besides, we consider tensor product of $*$-$A(k)$ operators.

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1 Introduction

Let H be an arbitrary complex Hilbert space and T be a bounded linear operator on H. We denote the $*$-algebra of all bounded linear operators on H by $B(H)$.

An operator T is $*$-paranormal if $||T^2x|| \geq ||T^*x||^2$ for unit vector x. $*$-paranormal operators have been studied by many researchers, see [5, 9, 11], etc. An operator T is said to be class $*$-A if $|T^2| \geq |T^*|^2$, where $|T| = (T^*T)^{\frac{1}{2}}$. As an easy extension of class $*$-A operators, an operator T is said to be quasi-$*$-A if $T^*|T^2|T \geq T^*|T^*|^2T$ in [17]. Moreover, for $k > 0$, an operator T belongs to $*$-$A(k)$ if $(T^*|T|^{2k}T)^{\frac{1}{k+1}} \geq |T^*|^2$. An operator T is absolute-$*$-k-paranormal if $||T^*x||^{k+1} \leq |||T|^kTx||||x||^k$ for every $x \in H$. Particularly an operator T is a class $*$-A(resp. $*$-paranormal) operator if and only if T is a $*$-$A(1)$(resp. absolute-$*$-1-paranormal) operator.

In this note we extend $*$-$A(k)$(resp.$*$-paranormal) operators and quasi-$*$-A operators to a new class of operators called quasi-$*$-$A(k)$(resp.quasi-absolute-$*$-k-paranormal) operators, and study their spectral properties.

Definition 1.1 Let $T \in B(H)$.

(i) For each $k > 0$, T belongs to quasi-$*$-$A(k)$ if

$$T^*(T^*|T|^{2k}T)^{\frac{1}{k+1}}T \geq |T|^4.$$

(ii) For each $k > 0$, T belongs to quasi-absolute-$*$-k-paranormal if

$$||T^*Tx||^{k+1} \leq |||T|^kT^2x||||Tx||^k \text{ for every } x \in H.$$

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It's known that quasi-$*$-$A(1)$ operator is quasi-$*$-A, and a quasi-$*$-$A(k)$ operator is quasi-absolute-$*$-k-paranormal (see Lemma 2.2), then we have the following implications:

$*$-$A(k)$ $\Rightarrow$ quasi-$*$-$A(k)$ $\Rightarrow$ quasi-absolute-$*$-k-paranormal.

By simple calculation, we have the following lemma as appeared in [22].

Lemma 1.2 Let $K = \oplus_{n=1}^{+\infty} H_n$, where $H_n \cong H$. For given positive operators A and B on H, define the operator $T_{A,B}$ on K as follows:

$$T_{A,B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ A & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & B & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & B & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & B & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & B & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Then the following assertions hold:

(i) $T_{A,B}$ belongs to $*$-$A(k)$ if and only if $B^2 \geq A^2$.

(ii) $T_{A,B}$ belongs to quasi-$*$-$A(k)$ if and only if $AB^2A \geq A^4$.

The following example provides an operator which is quasi-$*$-$A(k)$ but not $*$-$A(k)$.

Example 1.3 A non-$*$-$A(k)$ and quasi-$*$-$A(k)$ operator.

Take A and B as

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Then

$$B^2 - A^2 = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \ngeq 0.$$

Hence $T_{A,B}$ is not a $*$-$A(k)$ operator.

On the other hand,

$$A(B^2 - A^2)A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \geq 0.$$

Thus $T_{A,B}$ is a quasi-$*$-$A(k)$ operator.

Consider unilateral weighted shift operator as an infinite dimensional Hilbert space operator. Recall that given a bounded sequence of positive numbers $\alpha : \alpha_1, \alpha_2, \alpha_3, \cdots$ (called weights), the unilateral weighted shift W_α associated with α is the operator on $H = l_2$ defined by $W_\alpha e_n := \alpha_n e_{n+1}$ for all $n \geq 1$, where $\{e_n\}_{n=1}^{\infty}$ is the canonical orthogonal basis for l_2. Straightforward calculations show that W_α is quasi-$*$-$A(k)$ if and only if

$$W_\alpha = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ \alpha_1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \alpha_2 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \alpha_3 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \alpha_4 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where

$$(\alpha_{i+1}\alpha_{i+2}^k)^{\frac{1}{k+1}} \geq \alpha_i \ \ (i = 1, 2, 3, \cdots).$$

The following examples show that quasi-$*$-A operator and quasi-$*$-$A(2)$ operator are independent.

Example 1.4 A non-quasi-$*$-A and quasi-$*$-$A(2)$ operator.

Let T be a unilateral weighted shift operator with weighted sequence (α_i), given $\alpha_1 = 3, \alpha_2 = 1, \alpha_3 = 8, \alpha_3 = \alpha_4 = \alpha_5 = \cdots$. Simple calculations show that T is quasi-$*$-$A(2)$ and a non-quasi-$*$-A operator.

Example 1.5 A non-quasi-$*$-$A(2)$ and quasi-$*$-A operator.

Let T be a unilateral weighted shift operator with weighted sequence (α_i), given $\alpha_1 = 1, \alpha_2 = \frac{1}{2}, \alpha_3 = 2, \alpha_4 = \frac{1}{8}, \alpha_5 = 64, \alpha_5 = \alpha_6 = \cdots$. Simple calculations show that T is quasi-$*$-A but not a quasi-$*$-$A(2)$ operator.

2 Spectrum of Quasi-$*$-$A(k)$ Operators

In the sequel, let $\sigma(T)$, $\sigma_a(T)$, $\sigma_p(T)$, $\sigma_{ea}(T)$, $\sigma_{jp}(T)$, $\sigma_{ja}(T)$ for the spectrum of T, the approximate point spectrum of T, the point spectrum of T, the essential approximate point spectrum of T, the joint point spectrum of T, the joint approximate point spectrum of T, respectively. $\lambda \in \sigma_p(T)$ if there is a nonzero $x \in H$ such that $(T - \lambda)x = 0$. If in addition, $(T^* - \overline{\lambda})x = 0$, then $\lambda \in \sigma_{jp}(T)$. Analogously, $\lambda \in \sigma_a(T)$ if there is a sequence $\{x_n\}$ of unit vectors in H such that $(T - \lambda)x_n \to 0$. If in addition, $(T^* - \overline{\lambda})x_n \to 0$, then $\lambda \in \sigma_{ja}(T)$.

Clearly, $\sigma_{jp}(T) \subseteq \sigma_p(T)$, $\sigma_{ja}(T) \subseteq \sigma_a(T)$. In general, $\sigma_{jp}(T) \neq \sigma_p(T)$, $\sigma_{ja}(T) \neq \sigma_a(T)$.

Recently, it was shown that, for some nonnormal operator T, the nonzero points of its point spectrum and joint point spectrum are identical, the nonzero points of its approximate point spectrum and joint approximate point spectrum are identical[3, 7, 19–21]. In this section, we will extend that result to quasi-$*$-$A(k)$ for $0 < k \leq 1$.

To prove the inclusion relation between quasi-$*$-$A(k)$ operator and quasi-absolute-$*$-k-paranormal operator, we need the following lemma.

Lemma 2.1 [10] Let A be a positive linear operator on a Hilbert space H. Then

(i) $(A^\lambda x, x) \geq (Ax, x)^\lambda$ for any $\lambda > 1$ and $||x|| = 1$.

(ii) $(A^\lambda x, x) \leq (Ax, x)^\lambda$ for any $0 \leq \lambda \leq 1$ and $||x|| = 1$.

Lemma 2.2 For each $k > 0$, every quasi-$*$-$A(k)$ operator is a quasi-absolute-$*$-k-paranormal operator.

Proof Suppose that T belongs to quasi-$*$-$A(k)$ for $k>0$, i.e.,

$$T^*|T^*|^2T \le T^*(T^*|T|^{2k}T)^{\frac{1}{k+1}}T.$$

Then, for every $x \in H$,

$$\begin{aligned}||T^*Tx||^{2(k+1)} &= (T^*|T^*|^2Tx, x)^{(k+1)}\\ &\le (T^*(T^*|T|^{2k}T)^{\frac{1}{k+1}}Tx, x)^{(k+1)}\\ &= ((T^*|T|^{2k}T)^{\frac{1}{k+1}}Tx, Tx)^{(k+1)}\\ &\le (T^*|T|^{2k}T^2x, Tx)||Tx||^{2k}\\ &= |||T|^kT^2x||^2||Tx||^{2k}.\end{aligned}$$

Therefore,

$$||T^*Tx||^{k+1} \le |||T|^kT^2x||||Tx||^k \text{ for every } x \in H,$$

that is, T is quasi-absolute-$*$-k-paranormal for $k>0$.

Lemma 2.3 [6] Let H be a complex Hilbert space. Then there exists a Hilbert space K such that $H \subset K$ and a map $\varphi : B(H) \to B(K)$ such that

(i) φ is a faithful $*$-representation of the algebra $B(H)$ on K;

(ii) $\varphi(A) \ge 0$ for any $A \ge 0$ in $B(H)$;

(iii) $\sigma_a(T) = \sigma_a(\varphi(T)) = \sigma_p(\varphi(T))$ for any $T \in B(H)$.

Lemma 2.4 [21] Let $\varphi : B(H) \to B(K)$ be Berberian's faithful $*$-representation. Then $\sigma_{ja}(T) = \sigma_{jp}(\varphi(T))$.

Lemma 2.5 Let T be a quasi-$*$-$A(k)$ operator for $0<k\le 1$ and $\lambda \ne 0$. Then $Tx = \lambda x$ implies $T^*x = \bar{\lambda}x$.

Proof Let $\lambda \ne 0$ and suppose $x \in N(T-\lambda)$, we get $Tx = \lambda x$. Since T is quasi-$*$-$A(k)$, for every unit vector $x \in H$, $||T^*Tx||^{k+1} \le |||T|^kT^2x||||Tx||^k$, then

$$\begin{aligned}||T^*x||^{k+1}|\lambda|^{k+1} &\le |\lambda|^{k+2}|||T|^kx|| = |\lambda|^{k+2}(|T|^{2k}x, x)^{\frac{1}{2}}\\ &\le |\lambda|^{k+2}(|T|^2x, x)^{\frac{k}{2}} = |\lambda|^{k+2}||Tx||^k\\ &= |\lambda|^{2k+2}.\end{aligned}$$

Hence, for all $||x|| = 1$, $||T^*x||^2 \le |\lambda|^2$, and then

$$\begin{aligned}||T^*x - \overline{\lambda}x|| &= ((T-\lambda)^*x, (T-\lambda)^*x)\\ &= ||T^*x||^2 - (x, \overline{\lambda}Tx) - (\overline{\lambda}Tx, x) + |\lambda|^2||x||^2\\ &\le |\lambda|^2 - |\lambda|^2 - |\lambda|^2 + |\lambda|^2\\ &= 0.\end{aligned}$$

Therefore, we have $||T^*x - \overline{\lambda}x|| = 0$, that is, $T^*x = \bar{\lambda}x$.

Remark 2.6 The condition "$\lambda \ne 0$" cannot be omitted in Lemma 2.5. In fact, Example 1.3 shows that $T_{A,B}$ is a quasi-$*$-$A(k)$ operator, however for the vector $x =$

$(0, 0, 1, -1, 0, 0, \cdots)$, $T_{A,B}(x) = 0$, but $T^*_{A,B}(x) \neq 0$. Therefore, the relation $N(T_{A,B}) \subseteq N(T^*_{A,B})$ does not always hold.

Theorem 2.7 Let $T \in B(H)$ be quasi-$*$-$A(k)$ for $0 < k \leq 1$. Then

(i) $\sigma_{jp}(T)\backslash\{0\} = \sigma_p(T)\backslash\{0\}$;

(ii) If $(T - \lambda)x = 0, (T - \mu)y = 0$, and $\lambda \neq \mu$, then we have $< x, y >= 0$;

(iii) $\sigma_{ja}(T)\backslash\{0\} = \sigma_a(T)\backslash\{0\}$.

Proof (i) Clearly by Lemma 2.5.

(ii) Without loss of generality, we assume $\mu \neq 0$. Then $(T - \mu)^*y = 0$ by Lemma 2.5. Thus we have $\mu < x, y >=< x, T^*y >=< Tx, y >= \lambda < x, y >$. Since $\lambda \neq \mu$, $< x, y >= 0$.

(iii) Let φ: $B(H) \to B(K)$ be Berberian's faithful $*$-representation of Lemma 2.3. In the following, we shall show that $\varphi(T)$ is also a quasi-$*$-$A(k)$ operator.

In fact, since T is a quasi-$*$-$A(k)$ operator, by Lemma 2.3, we have

$$\begin{aligned}&(\varphi(T))^*[((\varphi(T))^*|\varphi(T)|^{2k}\varphi(T))^{\frac{1}{k+1}} - |(\varphi(T))^*|^2]\varphi(T)\\&= \varphi(T^*[(T^*|T|^{2k}T)^{\frac{1}{k+1}} - |T^*|^2]T) \geq 0.\end{aligned}$$

Then

$$\begin{aligned}\sigma_a(T)\backslash\{0\} &= \sigma_a(\varphi(T))\backslash\{0\} \quad \text{by Lemma 2.3}\\&= \sigma_p(\varphi(T))\backslash\{0\} \quad \text{by Lemma 2.3}\\&= \sigma_{jp}(\varphi(T))\backslash\{0\} \quad \text{by (i)}\\&= \sigma_{ja}(T)\backslash\{0\} \quad \text{by Lemma 2.4.}\end{aligned}$$

Recall that $T \in B(H)$ is said to have finite ascent if $N(T^n) = N(T^{n+1})$ for some positive integer n, where $N(T)$ for the null space of T.

Theorem 2.8 If T is quasi-$*$-$A(k)$ for $0 < k \leq 1$, then $T - \lambda$ has finite ascent for each $\lambda \in \mathbb{C}$.

Proof If $\lambda \neq 0$, then $N(T-\lambda) \subseteq N(T^* - \overline{\lambda})$ by Lemma 2.5, thus $N(T-\lambda) = N(T-\lambda)^2$. If $\lambda = 0$, let $x \in N(T^2)$, since T is quasi-$*$-$A(k)$, then

$$||T^*Tx||^{k+1} \leq |||T|^kT^2x||||Tx||^k \text{ for every } x \in H.$$

We have

$$\begin{aligned}|||T|^2x||^{k+1} &= ||T^*Tx||^{k+1} \leq |||T|^kT^2x||||Tx||^k\\&= (|T|^{2k}T^2x, T^2x)^{\frac{1}{2}}||Tx||^k\\&\leq (|T|^2T^2x, T^2x)^{\frac{k}{2}}||Tx||^k||T^2x||^{(1-k)}\\&= ||T^3x||^k||Tx||^k||T^2x||^{(1-k)} \text{ for every } x \in H.\end{aligned}$$

Hence $|T|^2x = 0$ implies $Tx = 0$. This shows that $T - \lambda$ has finite ascent for each $\lambda \in \mathbb{C}$.

Recall that $T \in B(H)$ has the single valued extension property (abbrev. SVEP), if for every open set U of $\mathbb{C}$, the only analytic solution $f\colon U \to H$ of the equation

$$(T-\lambda)f(\lambda)=0$$

for all $\lambda \in U$ is the zero function on U.

Theorem 2.9 If T is quasi-$*$-$A(k)$ for $0<k\leq 1$, then T has SVEP.

Proof Clearly by Theorem 2.8 and [14, Proposition 1.8].

As a simple consequence of the preceding result, we obtain

Corollary 2.10 If T is quasi-$*$-$A(k)$ for $0<k\leq 1$, then

(i) $\sigma_{ea}(f(T)) = f(\sigma_{ea}(T))$ for every $f \in H(\sigma(T))$, where $H(\sigma(T))$ is the space of functions analytic on an open neighborhood of $\sigma(T)$;

(ii) T obeys a-Browder's theorem, that is $\sigma_{ea}(T)=\sigma_{ab}(T)$, where $\sigma_{ab}(T) := \cap\{\sigma_a(T+K) : TK=KT \text{ and } K \text{ is a compact operator}\}$;

(iii) a-Browder's theorem holds for $f(T)$ for every $f \in H(\sigma(T))$.

Proof Note that T has SVEP, Corollary 2.10 follows by [1].

3 Tensor Products of $*$-$A(k)$ Operators

Given non-zero T, $S \in B(H)$, let $T\otimes S$ denote the tensor product on the product space $H\otimes H$. The operation of taking tensor products $T\otimes S$ preserves many properties of T, $S \in B(H)$, but by no means all of them. The normaloid property is invariant under tensor products [16, p.623], $T\otimes S$ is normal if and only if T and S are normal [12, 18], however, there exist paranormal operators T and S such that $T\otimes S$ is not paranormal [4]. Duggal [8] showed that for non-zero T, $S\in B(H)$, $T\otimes S \in H(p)$ if and only if T, $S\in H(p)$. Recently, this result was extended to class $*$-A operators and class A operators in [9, 13], respectively.

In this section we consider the tensor products of $*$-$A(k)$ operators. The following key lemma is due to J. Stochel.

Lemma 3.1 [18, Proposition 2.2] Let $A_1, A_2 \in B(H)$, $B_1, B_2 \in B(K)$ be non-negative operators. If A_1 and B_1 are non-zero, then the following assertions are equivalent:

(i) $A_1\otimes B_1 \leq A_2 \otimes B_2$.

(ii) There exists $c>0$ for which $A_1 \leq cA_2$ and $B_1 \leq c^{-1}B_2$.

Theorem 3.2 Let T, $S\in B(H)$ be non-zero operators. Then $T\otimes S$ is a $*$-$A(k)$ operator if and only if T and S are $*$-$A(k)$ operators.

Proof By simple calculation we have

$$\begin{aligned}
&T\otimes S \text{ is a } *\text{-}A(k) \text{ operator}\\
&\Leftrightarrow [(T\otimes S)^*|T\otimes S|^{2k}(T\otimes S)]^{\frac{1}{k+1}} \geq |(T\otimes S)^*|^2\\
&\Leftrightarrow [(T\otimes S)^*|T\otimes S|^{2k}(T\otimes S)]^{\frac{1}{k+1}} - |T^*|^2\otimes|S^*|^2 \geq 0\\
&\Leftrightarrow [(T^*\otimes S^*)(|T|^{2k}\otimes|S|^{2k})(T\otimes S)]^{\frac{1}{k+1}} - |T^*|^2\otimes|S^*|^2 \geq 0\\
&\Leftrightarrow (T^*|T|^{2k}T)^{\frac{1}{k+1}}\otimes(S^*|S|^{2k}S)^{\frac{1}{k+1}} - |T^*|^2\otimes|S^*|^2 \geq 0\\
&\Leftrightarrow |T^*|^2\otimes[(S^*|S|^{2k}S)^{\frac{1}{k+1}} - |S^*|^2] + [(T^*|T|^{2k}T)^{\frac{1}{k+1}} - |T^*|^2]\otimes(S^*|S|^{2k}S)^{\frac{1}{k+1}} \geq 0.
\end{aligned}$$

Thus the sufficiency is easily proved. Conversely, suppose that $T \otimes S$ belongs to $*$-$A(k)$. Without loss of generality, it is enough to show that T belongs to $*$-$A(k)$. Since $T \otimes S$ is $*$-$A(k)$, we obtain

$$(T^*|T|^{2k}T)^{\frac{1}{k+1}} \otimes (S^*|S|^{2k}S)^{\frac{1}{k+1}} \geq |(T \otimes S)^*|^2.$$

Therefore, by Lemma 3.1, there exists a positive real number l for which

$$l(T^*|T|^{2k}T)^{\frac{1}{k+1}} \geq |T^*|^2$$

and

$$l^{-1}(S^*|S|^{2k}S)^{\frac{1}{k+1}} \geq |S^*|^2.$$

Consequently, for arbitrary $x, y \in H$, using Lemma 2.1 we have

$$\begin{aligned}
||T||^2 = ||T^*||^2 &= \sup\{(|T^*|^2 x,\ x) : ||x|| = 1\} \\
&\leq \sup\{(l(T^*|T|^{2k}T)^{\frac{1}{k+1}} x,\ x) : ||x|| = 1\} \\
&\leq l \sup\{((T^*|T|^{2k}T)x,\ x)^{\frac{1}{1+k}} : ||x|| = 1\} \\
&\leq l||T^*|T|^{2k}T||^{\frac{1}{1+k}} \\
&\leq l||T||^2
\end{aligned}$$

and

$$\begin{aligned}
||S||^2 = ||S^*||^2 &= \sup\{(|S^*|^2 y,\ y) : ||y|| = 1\} \\
&\leq \sup\{[l^{-1}(S^*|S|^{2k}S)^{\frac{1}{k+1}} y,\ y] : ||y|| = 1\} \\
&\leq l^{-1} \sup\{[(S^*|S|^{2k}S)y,\ y]^{\frac{1}{1+k}} : ||y|| = 1\} \\
&\leq l^{-1}||S^*|S|^{2k}S||^{\frac{1}{1+k}} \\
&\leq l^{-1}||S||^2.
\end{aligned}$$

Clearly, we must have $l = 1$, and hence T is a $*$-$A(k)$ operator.

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关于拟-$*$-$A(k)$算子的注记

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摘要: 本文引入了拟-$*$-$A(k)$算子并研究其谱性质如下: (i) 如果T 是拟$*$-$A(k)$ 算子, 其中$0 < k \leq 1$, 则谱映射定理对T 的本质近似点谱成立. (ii) 如果T 是拟$*$-$A(k)$ 算子, 其中$0 < k \leq 1$, 则$\sigma_{ja}(T)\backslash\{0\} = \sigma_a(T)\backslash\{0\}$. 最后对$*$-$A(k)$ 算子的张量积性质也进行了讨论.

关键词: 拟-$*$-$A(k)$ 算子; 单值扩展性质; 联合近似点谱; 张量积

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