Vol. 35 (2015)
No. 1

数 学 杂 志
J. of Math. (PRC)

POSITIVE SOLUTIONS FOR $(n-1,1)$–TYPE FRACTIONAL CONJUGATE BOUNDARY VALUE PROBLEM

WANG Yong, YANG Yang
(*School of Science, Jiangnan University, Wuxi 214122, China*)

Abstract: This paper studies the existence and multiplicity of positive solutions for $(n-1,1)$–type fractional conjugate boundary value problem. By virtue of Krasnoselskii–Zabreiko fixed point theorem, several results are formulated in terms of some inequalities associated with Green's function. The results obtained here improve some existing results in the literature.

Keywords: fractional boundary value problem; Green function; positive solution; fixed point theorem

2010 MR Subject Classification: 34B10; 34B18

Document code: A **Article ID:** 0255-7797(2015)01-0035-08

1 Introduction

In this paper, we study the existence and multiplicity of positive solutions for the fractional boundary value problem

$$\begin{cases} \mathbf{D}_{0+}^{\alpha} u(t) + f(t,u(t)) = 0,\ 0 < t < 1, \\ u^{(i)}(0) = u(1) = 0,\ 0 \le i \le n-2, \end{cases} \tag{1.1}$$

where $n \in \mathbb{N}$ and $n \ge 3$, $\alpha \in (n-1,n]$ is a real number, $\mathbf{D}_{0+}^{\alpha}$ is the standard Riemann–Liouville fractional derivative of order α and $f \in C([0,1]\times[0,+\infty),[0,+\infty))$.

In view of fractional differential equation's modeling capabilities in engineering, science, economy, and other fields, the last few decades has resulted in a rapid development of the theory of fractional differential equation, see the recent books [1–5]. This may explain the reason that the last few decades have witnessed an overgrowing interest in the research of such problems, with many papers in this direction published. Recently, there are some papers dealing with the existence of solutions (or positive solutions) of nonlinear fractional differential equation by the use of techniques of nonlinear analysis (fixed point theorems,

* **Received date:** 2013-05-12 **Accepted date:** 2013-07-30

Foundation item: Supported by the NNSF-China (11202084); NSFC-Tian Yuan Special Foundation (11226116); Natural Science Foundation of Jiangsu Province of China for Young Scholar (BK2012109); the China Scholarship Council (201208320435).

Biography: Wang Yong(1976–), male, born at Tengzhou, Shandong, lecturer, major in partial differential equations.

Leray–Schauder theory, upper and lower solution method, etc.), see for example [6–11] and the references therein.

In [6], Zhao et al. considered the existence on multiple positive solutions for the nonlinear fractional differential equation boundary value problem

$$\begin{cases} \mathbf{D}_{0+}^{\alpha}u(t)+f(t,u(t))=0,\ 0<t<1, \\ u(0)=u'(0)=u'(1)=0, \end{cases} \tag{1.2}$$

where $2<\alpha\leq 3$ is a real number, $\mathbf{D}_{0+}^{\alpha}$ is the Riemann-Liouville fractional derivative. By the properties of the Green function, the lower and upper solution method and fixed point theorem, some new existence criteria for singular and nonsingular fractional differential equation boundary value problem are established.

Meanwhile, we also note that conjugate boundary value problem of integer order differential equation has been extensively studied, see [12–15] and the references therein. However, to the best of our knowledge, there is very little known about the existence of positive solutions for fractional conjugate boundary value problem (1.1). To our knowledge, only [9–11] were devoted to this direction. In [9], Yuan considered the semipositone conjugate fractional boundary value problem (1.1) with a parameter λ and $f:[0,1]\times[0,+\infty)\to(-\infty,+\infty)$ is a sign-changing continuous function. He first given the properties of Green's function of (1.1), and then derived an interval of λ such that any λ lying in this interval, the semipositone boundary value problem (1.1) has multiple positive solutions. In [10, 11], He adopted the same method in [9] to discuss the existence of multiple positive solutions for $(n-1,1)$–type semipositone conjugate and integral boundary value problems for coupled systems of nonlinear fractional differential equations, respectively.

In this paper, we utilize Krasnoselskii–Zabreiko fixed point theorem to establish our main results based on a priori estimates achieved by developing some inequalities associated with fractional Green's function. It is well known that a cone plays a very important role involving the existence of solutions (positive solutions) for differential equations. It is difficult to structure a cone for speciality of Green's function for fractional equation. In this work, we first study the properties of the Green's function and obtain an inequality about it, and then structure a cone associated with the inequality. Based on this, we obtained some easily verifiable sufficient criteria to ensure the existence and multiplicity of positive solutions for (1.1). Thus our results improve and extend the corresponding ones in [6–11].

2 Preliminaries

As is known to all, the Riemann–Liouville fractional derivative $\mathbf{D}_{0+}^{\alpha}$ is defined by

$$\mathbf{D}_{0+}^{\alpha}y(t):=\frac{1}{\Gamma(n-\alpha)}\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^{n}\int_0^t\frac{y(s)\mathrm{d}s}{(t-s)^{\alpha-n+1}},$$

where Γ is the gamma function and $n=[\alpha]+1$. For more details of fractional calculus, we refer the reader to the recent books such as [1–5].

Lemma 2.1 (see [9, Lemma 3.1]) Let f be determined by (1.1). Then the problem (1.1) is equivalent to

$$u(t)=\int_0^1 G(t,s)f(s,u(s))\mathrm{d}s,$$

where

$$G(t,s):=\frac{1}{\Gamma(\alpha)}\begin{cases} t^{\alpha-1}(1-s)^{\alpha-1}-(t-s)^{\alpha-1}, & 0\le s\le t\le 1,\\ t^{\alpha-1}(1-s)^{\alpha-1}, & 0\le t\le s\le 1.\end{cases} \tag{2.1}$$

Here $G(t,s)$ is called the Green's function for the boundary value problem (1.1).

Lemma 2.2 (see [9, Lemma 3.2]) $G(t,s)\in C([0,1]\times[0,1],[0,+\infty))$ has the following properties:

(R1) $G(t,s)=G(1-s,1-t)$, for $(t,s)\in[0,1]\times[0,1]$,

(R2) $\Gamma(\alpha)k(t)q(s)\le G(t,s)\le(\alpha-1)q(s)$, for $(t,s)\in[0,1]\times[0,1]$,

(R3) $\Gamma(\alpha)k(t)q(s)\le G(t,s)\le(\alpha-1)k(t)$, for $(t,s)\in[0,1]\times[0,1]$, where

$$k(t)=\frac{t^{\alpha-1}(1-t)}{\Gamma(\alpha)},\quad q(s)=\frac{s(1-s)^{\alpha-1}}{\Gamma(\alpha)}. \tag{2.2}$$

Lemma 2.3 Let $\mathcal{K}_1:=\frac{\alpha\Gamma(\alpha+1)}{\Gamma(2\alpha+2)}$ and $\mathcal{K}_2:=\frac{\alpha-1}{\Gamma(\alpha+2)}$. Then the following inequality holds:

$$\mathcal{K}_1q(s)\le\int_0^1 G(t,s)q(t)\mathrm{d}t\le\mathcal{K}_2q(s),\forall s\in[0,1]. \tag{2.3}$$

Let

$$E:=C[0,1],\quad \|u\|:=\max_{t\in[0,1]}|u(t)|,\quad P:=\{u\in E:u(t)\ge 0,\forall t\in[0,1]\}.$$

Then $(E,\|\cdot\|)$ becomes a real Banach space and P is a cone on E. We denote $B_\rho:=\{u\in E:\|u\|<\rho\}$ for $\rho>0$ in the sequel. Now, note that u solves (1.1) if and only if u is a fixed point of the operator

$$(Au)(t):=\int_0^1 G(t,s)f(s,u(s))\mathrm{d}s.$$

Clearly, $A:P\to P$ is a completely continuous operator. Define the completely continuous linear operators $L:E\to E$ by

$$(Lu)(t):=\int_0^1 G(t,s)u(s)\mathrm{d}s.$$

Then L is also a positive operator, i.e. $L(P)\subset P$. Let $\omega=(\alpha-1)^{-1}\mathcal{K}_1$ and a cone on E as follows

$$P_0=\left\{u\in P:\int_0^1 u(t)q(t)\mathrm{d}t\ge\omega\|u\|\right\}.$$

Next, we shall prove that

Lemma 2.4 $L(P)\subset P_0$.

Proof By (R2) of Lemma 2.2, we have

$$(Lu)(t) = \int_0^1 G(t,s)u(s)\mathrm{d}s \le (\alpha-1)\int_0^1 q(s)u(s)\mathrm{d}s.$$

On the other hand, from (2.3), we find

$$\begin{aligned}\int_0^1 (Lu)(t)q(t)\mathrm{d}t =& \int_0^1 \left(\int_0^1 G(t,s)u(s)\mathrm{d}s\right) q(t)\mathrm{d}t \\ \ge& \mathcal{K}_1 \int_0^1 q(s)u(s)\mathrm{d}s \ge \frac{\mathcal{K}_1}{\alpha-1}\|Lu\|\end{aligned}$$

and thus $\int_0^1 (Lu)(t)q(t)\mathrm{d}t \ge \omega\|Lu\|$, that is, $L(P) \subset P_0$. This completes the proof.

From now on, let $r(L)$ denote the spectral radii of L. By Gelfand's theorem, we could easily have the following result.

Lemma 2.5 $0 < r(L) \le \mathcal{K}_2$.

Lemma 2.6 (see [16]) Let E be a real Banach space and W a cone of E. Suppose that $A : (\overline{B}_R \backslash B_r) \cap W \to W$ is a completely continuous operator with $0 < r < R$. If either

(1) $Au \nleq u$ for each $\partial B_r \cap W$ and $Au \ngeq u$ for each $\partial B_R \cap W$ or

(2) $Au \ngeq u$ for each $\partial B_r \cap W$ and $Au \nleq u$ for each $\partial B_R \cap W$.

Then A has at least one fixed point on $(\overline{B}_R \backslash B_r) \cap W$.

For convenience, we now list our hypotheses on f. Let $\lambda_1 := 1/\mathcal{K}_1 > 0$, $\lambda_2 := 1/\mathcal{K}_2 > 0$.

(H1) $\liminf\limits_{u\to\infty} \frac{f(t,u)}{u} > \lambda_1$ uniformly with respect to $t \in [0,1]$.

(H2) $\limsup\limits_{u\to 0^+} \frac{f(t,u)}{u} < \lambda_2$ uniformly with respect to $t \in [0,1]$.

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(H4) $\limsup\limits_{u\to\infty} \frac{f(t,u)}{u} < \lambda_2$ uniformly with respect to $t \in [0,1]$.

(H5) There exists a number $\rho > 0$ such that the inequality $f(t,u) \le \zeta\rho$ holds whenever $u \in [0,\rho]$, $\zeta \in (0, (\alpha-1)^{-1}\Gamma(\alpha+2))$, and $t \in [0,1]$.

3 Existence of Positive Solutions

Theorem 3.1 Suppose that (H1) and (H2) are satisfied, then (1.1) has at least one positive solution.

Proof By (H1), there exist $\varepsilon > 0$ and $b > 0$ such that $f(t,u) \ge (\lambda_1+\varepsilon)u - b$ for all $u \ge 0$ and $t \in [0,1]$. This implies

$$(Au)(t) \ge (\lambda_1+\varepsilon)\int_0^1 G(t,s)u(s)\mathrm{d}s - b\int_0^1 G(t,s)\mathrm{d}s \tag{3.1}$$

for all $u \in P$. Let

$$\mathbf{M}_1 := \{u \in P : u \ge Au\}.$$

We shall prove that $\mathbf{M}_1$ is bounded in P. Indeed, $u \in \mathbf{M}_1$, along with (3.1), leads to

$$u(t) \ge (\lambda_1+\varepsilon)\int_0^1 G(t,s)u(s)\mathrm{d}s - b\int_0^1 G(t,s)\mathrm{d}s.$$

Multiply by $q(t)$ on both sides of the above and integrate over $[0,1]$ and use (2.3) to obtain

$$\int_0^1 u(t)q(t)\mathrm{d}t \geq (\lambda_1+\varepsilon)\lambda_1^{-1}\int_0^1 u(t)q(t)\mathrm{d}t - b\lambda_2^{-1}\int_0^1 q(t)\mathrm{d}t$$

and thus $\int_0^1 u(t)q(t)\mathrm{d}t \leq \frac{\varepsilon^{-1}b(\alpha-1)\Gamma(2\alpha+2)}{\alpha\Gamma(\alpha+1)\Gamma^2(\alpha+2)}$ for all $u \in \mathbf{M}_1$. Note that we have $\mathbf{M}_1 \subset P_0$ by Lemma 2.4. This together with the preceding inequality implies

$$\|u\| \leq \frac{\varepsilon^{-1}b(\alpha-1)^2\Gamma^2(2\alpha+2)}{\alpha^2\Gamma^2(\alpha+1)\Gamma^2(\alpha+2)}$$

for all $u \in \mathbf{M}_1$, which establishes the boundedness of $\mathbf{M}_1$, as required. Taking $R > \sup \mathbf{M}_1$, we obtain

$$u \not\geq Au,\ \forall u \in \partial B_R \cap P. \tag{3.2}$$

On the other hand, by (H2), there exist $r \in (0,R)$ and $\varepsilon \in (0,\lambda_2)$ such that $f(t,u) \leq (\lambda_2-\varepsilon)u$ for all $u \in [0,r]$ and $t \in [0,1]$. This implies

$$(Au)(t) \leq (\lambda_2-\varepsilon)\int_0^1 G(t,s)u(s)\mathrm{d}s \tag{3.3}$$

for all $u \in \overline{B}_r \cap P$. Let

$$\mathbf{M}_2 := \{u \in \overline{B}_r \cap P : u \leq Au\}.$$

Now, we claim $\mathbf{M}_2 = \{0\}$. Indeed, if there exist $u_0 \in \partial B_r \cap P$, then this together with (3.3) leads to

$$u_0(t) \leq (\lambda_2-\varepsilon)\int_0^1 G(t,s)u_0(s)\mathrm{d}s.$$

Multiply by $q(t)$ on both sides of the preceding inequality and integrate over $[0,1]$ and use (2.3) to obtain

$$\int_0^1 u_0(t)q(t)\mathrm{d}t \leq (\lambda_2-\varepsilon)\lambda_2^{-1}\int_0^1 u_0(t)q(t)\mathrm{d}t$$

and thus $\int_0^1 u_0(t)q(t)\mathrm{d}t = 0$, whence $u_0(t) \equiv 0$, contradicting $u_0 \in \partial B_r \cap P$. Therefore,

$$u \not\leq Au, \forall u \in \partial B_r \cap P. \tag{3.4}$$

Now Lemma 2.6 indicates that the operator A has at least one fixed point on $(B_R \setminus \overline{B}_r) \cap P$. Therefore (1.1) has at least one positive solution, which completes the proof.

Theorem 3.2 If (H3) and (H4) are satisfied, then (1.1) has at least one positive solution.

Proof By (H3), there exist $r > 0$ and $\varepsilon > 0$ such that

$$f(t,u) \geq (\lambda_1+\varepsilon)u, \text{ for all } u \in [0,r] \text{ and } t \in [0,1]. \tag{3.5}$$

This implies

$$(Au)(t) \geq (\lambda_1+\varepsilon)\int_0^1 G(t,s)u(s)\mathrm{d}s \tag{3.6}$$

for all $u \in \overline{B}_r \cap P$. Let $\mathbf{M}_3 := \{u \in \overline{B}_r \cap P : u \geq Au\}$. We claim that $\mathbf{M}_3 = \{0\}$. Indeed, if the claim is false, then there exists $u_1 \in \partial B_r \cap P$ such that $u_1 \geq Au_1$. Combining with (3.6), we obtain

$$u_1(t) \geq (\lambda_1 + \varepsilon) \int_0^1 G(t,s)u_1(s)\mathrm{d}s.$$

Multiply by $q(t)$ on both sides of the above and integrate over [0,1] and use (2.3) to obtain

$$\int_0^1 u_1(t)q(t)\mathrm{d}t \geq (\lambda_1 + \varepsilon)\lambda_1^{-1} \int_0^1 u_1(t)q(t)\mathrm{d}t$$

and thus $\int_0^1 u_1(t)q(t)\mathrm{d}t = 0$, whence $u_1(t) \equiv 0$, contradicting $u_1 \in \partial B_r \cap P$. Consequently,

$$u \ngeq Au, \forall u \in \partial B_r \cap P. \tag{3.7}$$

In addition, by (H4), there exist $\varepsilon \in (0, \lambda_2)$ and $m > 0$ such that

$$f(t,u) \leq (\lambda_2 - \varepsilon)u + m, \text{ for all } u \geq 0 \text{ and } t \in [0,1]. \tag{3.8}$$

Let $\mathbf{M}_4 := \{u \in P : \ u \leq Au\}$. We shall prove that $\mathbf{M}_4$ is bounded in P. Indeed, if $u \in \mathbf{M}_4$, then we have

$$\begin{aligned} u(t) \leq (Au)(t) &= \int_0^1 G(t,s)f(s,u(s))\mathrm{d}s \\ &\leq \int_0^1 G(t,s)((\lambda_2 - \varepsilon)u(s) + m)\mathrm{d}s = (\lambda_2 - \varepsilon)(Lu)(t) + u_0(t), \end{aligned} \tag{3.9}$$

where $u_0 \in P \setminus \{0\}$ being defined by $u_0(t) = m\int_0^1 G(t,s)\mathrm{d}s$. Notice $r((\lambda_2 - \varepsilon)L) < 1$ by Lemma 2.5. This implies the inverse operator of $I - (\lambda_2 - \varepsilon)L$ exists and equals

$$(I - (\lambda_2 - \varepsilon)L)^{-1} = I + (\lambda_2 - \varepsilon)L + (\lambda_2 - \varepsilon)^2L^2 + \cdots + (\lambda_2 - \varepsilon)^nL^n + \cdots,$$

from which we obtain $(I - (\lambda_2 - \varepsilon)L)^{-1}(P) \subset P$. Applying this to (3.9) gives $u \leq (I - (\lambda_2 - \varepsilon)L)^{-1}u_0$ for all $u \in \mathbf{M}_4$. This proves the boundedness of $\mathbf{M}_4$, as required. Choosing $R > \sup\{\|u\| : u \in \mathbf{M}_4\}$ and $R > \rho$, we have

$$u \nleq Au, \ \forall u \in \partial B_R \cap P. \tag{3.10}$$

Now Lemma 2.6 implies that A has at least one fixed point on $(B_R \setminus \overline{B}_r) \cap P$. Therefore (1.1) has at least one positive solution, which completes the proof.

Theorem 3.3 Suppose that (H1), (H3) and (H5) hold, then (1.1) has at least two positive solutions.

Proof By (H5), we have

$$\begin{aligned} \|Au\| &= \max_{t\in[0,1]} \int_0^1 G(t,s)f(s,u(s))\mathrm{d}s \\ &\leq (\alpha - 1)\zeta\rho \int_0^1 q(s)\mathrm{d}s = \frac{(\alpha-1)\zeta\rho}{\Gamma(\alpha+2)} < \rho = \|u\| \end{aligned}$$

for all $u \in \partial B_\rho \cap P$, from which we obtain

$$u \nleq Au, \forall u \in \partial B_\rho \cap P. \tag{3.11}$$

On the other hand, by (H1) and (H3) , we may take $R > \rho$ and $r \in (0, \rho)$ so that (3.2) and (3.7) hold (see the proofs of Theorems 3.1 and 3.2). Combining (3.2), (3.7) and (3.11), we conclude, together with Lemma 2.6, A has at least two fixed points, one on $(B_R \backslash \overline{B}_\rho) \cap P$ and the other on $(B_\rho \backslash \overline{B}_r) \cap P$. Hence (1.1) has at least two positive solutions in $P \backslash \{0\}$. This completes the proof.

4 Examples

In this section, we offer some interesting examples to illustrate our main results.

Example 4.1 Let $f(t, u) = u^\alpha, t \in [0, 1], u \in \mathbb{R}^+$, where $\alpha \in (0, 1) \cup (1, \infty)$. If $\alpha \in (1, \infty)$, then (H1) and (H2) are satisfied. If $\alpha \in (0, 1)$, then (H3) and (H4) are satisfied. By Theorems 3.1 or 3.2, (1.1) has at least one positive solution.

Example 4.2 Let

$$f(t, u) = \begin{cases} \dfrac{\lambda_2}{2} u^\alpha, & 0 \leq u \leq 1, \\ 2\lambda_1 u^\alpha - 2\lambda_1 + \dfrac{\lambda_2}{2}, & u \geq 1, \end{cases}$$

where $\alpha \geq 1$. Now (H1) and (H2) are satisfied. By Theorem 3.1, (1.1) has at least one positive solution.

Example 4.3 Let

$$f(t, u) = \begin{cases} 2\lambda_1 u^\beta, & 0 \leq u \leq 1, \\ \dfrac{\lambda_2}{2} u^\beta + 2\lambda_1 - \dfrac{\lambda_2}{2}, & u \geq 1, \end{cases}$$

where $0 < \beta \leq 1$. Now (H3) and (H4) are satisfied. By Theorem 3.2, (1.1) has at least one positive solution.

Example 4.4 Let $f(t, u) = \lambda(u^a + u^b)$, where $0 < a < 1 < b$, $0 < \lambda < (\alpha - 1)^{-1}\Gamma(\alpha + 2)$. (H1), (H3) and (H5) are satisfied with $\rho = 1$. By Theorem 3.3, (1.1) has at least two positive solutions.

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$(n-1,1)$–型分数阶共轭边值问题的正解

王 勇，杨 阳

(江南大学理学院, 江苏 无锡 214122)

摘要: 本文研究了$(n-1,1)$–型分数阶共轭边值问题正解的存在性与多解性问题. 利用Krasnoselskii–Zabreiko不动点定理, 结合与Green函数相关的不等式, 获得了几个存在性结果, 推广了一些现有的结果.

关键词: 分数阶边值问题; Green函数; 正解; 不动点定理

MR(2010)**主题分类号**: 34B10; 34B18 **中图分类号**: O175.8