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THE CONSTRUCTION OF BRAIDED T-CATEGORIES

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Abstract: In this paper, we first introduce a class of new categories ${}_A\mathcal{YD}^H_G$ as a disjoint union of family of categories $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$. Then we mainly show that the category $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ forms a braided T-category if and only if there is a map $\mathscr{Q}$ such that $(A,H,\mathscr{Q})$ is a G-double structure, generalizing the main constructions by Panaite and Staic (2005). Finally, when H is finite-dimensional we construct a quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$, such that $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ is isomorphic to the representation category of the quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$.

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1 Introduction

For a group π, Turaev [12] introduced the notion of a braided π-monoidal category, here called Turaev braided π-category, and showed that such a category gives rise to a 3-dimensional homotopy quantum field theory. Kirillov [5] found that such Turaev braided π-categories also provide a suitable mathematical tool to describe the orbifold models which arise in the study of conformal field theories. Virelizier [15] used Turaev braided π-category to construct Hennings-type invariants of flat π-bundles over complements of links in the 3-sphere. We note that a Turaev braided π-category is a braided monoidal category when π is trivial.

Starting from the category of Yetter-Drinfeld modules, Panaite and Staic [6] constructed a Turaev braided category over certain group π, generalizing the work of [7]. Turaev braided π-categories were further investigated by Panaite and Staic in [6], by Zunino [17]. In the

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present paper, let G be the semi-direct product of the opposite group π^{op} of a group π by π and A an H-bicomodule algebra. We first introduce a class of new categories ${}_A\mathcal{YD}_G^H$ as a disjoint union of family of categories $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$. Then we mainly show that the category $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ forms a braided T-category, generalizing the main constructions by Panaite and Staic [6]. Finally, when H is finite-dimensional we construct a quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$, such that $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ is isomorphic to the representation category of the quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$.

The paper is organized as follows. In Section 3, let G be the semi-direct product of the opposite group π^{op} of a group π by π and A an H-bicomodule algebra, we first introduce a class of new categories ${}_A\mathcal{YD}_G^H$ as a disjoint union of family of categories $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ and give necessary and sufficient conditions making ${}_A\mathcal{YD}_G^H$ into a braided T-category.

In Section 4, when H is finite-dimensional, as an appliction, we construct a quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$, such that $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ is isomorphic to the representation category of the quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$.

2 Preliminaries

Throughout the paper, we let $\Bbbk$ be a fixed field and denote by $\otimes$ the the tensor product over $\Bbbk$. For the comultiplication Δ in a coalgebra C, we use the Sweedler-Heyneman's notation [12]:

$$\Delta(c) = c_1 \otimes c_2$$

for any $c \in C$. For a left C-comodule (M,ρ^l) and a right C-comodule (N,ρ^r), we write

$$\rho^l(m) = m_{(-1)} \otimes m_{(0)} \quad \text{and} \quad \rho^r(n) = n_{(0)} \otimes n_{(1)},$$

respectively, for all $m \in M$ and $n \in N$. For a Hopf algebra A, we always denote by $Aut(A)$ the group of Hopf automorphisms of A.

2.1 Braided T-Categories

Let π be a group with the unit e. We recall that a Turaev π-category (see [12]) is a monoidal category $\mathcal{C}$ which consists of the following data.

A family of subcategories $\{\mathcal{C}_\alpha\}_{\alpha\in\pi}$ such that $\mathcal{C}$ is a disjoint union of this family and such that $U \otimes V \in \mathcal{C}_{\alpha\beta}$, for any $\alpha,\beta \in \pi$, if the $U \in \mathcal{C}_\alpha$ and $V \in \mathcal{C}_\beta$. Here the subcategory $\mathcal{C}_\alpha$ is called the αth component of $\mathcal{C}$.

A group homomorphism $\varphi : \pi \longrightarrow \mathrm{aut}(\mathcal{C}), \beta \mapsto \varphi_\beta$, the conjugation, (where $\mathrm{aut}(\mathcal{C})$ is the group of invertible strict tensor functors from $\mathcal{C}$ to itself) such that $\varphi_\beta(\mathcal{C}_\alpha) = \mathcal{C}_{\beta\alpha\beta^{-1}}$ for any $\alpha,\beta \in \pi$. Here the functors φ_β are called conjugation isomorphisms.

We will use the left index notation in [12] or [17]: given $\beta \in G$ and an object $V \in \mathcal{C}_\beta$, the functor φ_β will be denoted by ${}^V(\cdot)$ or ${}^\beta(\cdot)$. We use the notation ${}^{\overline{V}}(\cdot)$ for ${}^{\beta^{-1}}(\cdot)$. Then we have ${}^V id_U = id_{{}^V U}$ and ${}^V(g \circ f) = {}^V g \circ {}^V f$. We remark that since the conjugation $\varphi : G \longrightarrow \mathrm{aut}(\mathcal{C})$ is a group homomorphism, for any $V, W \in \mathcal{C}$, we have ${}^{V\otimes W}(\cdot) = {}^V({}^W(\cdot))$ and ${}^1(\cdot) = {}^V({}^{\overline{V}}(\cdot)) = {}^{\overline{V}}({}^V(\cdot)) = id_{\mathcal{C}}$ and that since, for any $V \in \mathcal{C}$, the functor ${}^V(\cdot)$ is strict, we have ${}^V(f\otimes g)={}^Vf\otimes {}^Vg$, for any morphism f and g in $\mathcal{C}$, and ${}^V1=1$. And we will use $\mathcal{C}(U,V)$ for the set of morphisms (or arrows) from U to V in $\mathcal{C}$.

Recall from [12] that a braided crossed category is a crossed category $\mathcal{C}$ endowed with a braiding, i.e., with a family of isomorphisms

$$\tau=\{\tau_{U,V}\in\mathcal{C}(U\otimes V,({}^UV)\otimes U)\}_{U,V\in\mathcal{C}}$$

satisfying the following conditions:

for any arrow $f\in\mathcal{C}_\alpha(U,U')$ with $\alpha\in\pi, g\in\mathcal{C}(V,V')$, we have

$$((^\alpha g)\otimes f)\circ\tau_{U,V}=\tau_{U'V'}\circ(f\otimes g); \tag{2.1}$$

for all $U,V,W\in\mathcal{C}$, we have

$$\tau_{U\otimes V,W}=a_{{}^{U\otimes V}W,U,V}\circ(\tau_{U,{}^VW\otimes V})\circ a^{-1}_{U,{}^VW,V}\circ(\iota_U\otimes\tau_{V,W})\circ a_{U,V,W}, \tag{2.2}$$

$$\tau_{U,V\otimes W}=a^{-1}_{{}^UV,{}^UW,U}\circ(\iota_{({}^UV)}\otimes\tau_{U,W})\circ a_{{}^UV,U,W}\circ(\tau_{U,V}\otimes\iota_W)\circ a^{-1}_{U,V,W}, \tag{2.3}$$

for any $U,V\in\mathcal{C},\alpha\in\pi,\varphi_\alpha(\tau_{U,V})=\tau_{\varphi_\alpha(U),\varphi_\alpha(V)}$. $\qquad$ (2.4)

In this paper, we use terminology as in Zunino [17]; for the subject of Turaev categories, see also the original paper of Turaev [12]. If $\mathcal{C}$ is a braided crossed category then we call $\mathcal{C}$ a braided T-category.

2.2 T-Coalgebras

Let π be a group with unit e. Recall from Turaev [12] that a π-coalgebra is a family of k-spaces $C=\{C_\alpha\}_{\alpha\in\pi}$ together with a family of k-linear maps $\Delta=\{\Delta_{\alpha,\beta}:C_{\alpha\beta}\longrightarrow C_\alpha\otimes C_\beta\}_{\alpha,\beta\in\pi}$ (called the comultiplication) and a k-linear map $\varepsilon:C_e\longrightarrow\Bbbk$ (called the counit), such that Δ is coassociative in the sense that,

$$(\Delta_{\alpha,\beta}\otimes id_{C_\lambda})\Delta_{\alpha\beta,\lambda}=(id_{C_\alpha}\otimes\Delta_{\beta,\lambda})\Delta_{\alpha,\beta\lambda}\text{ for any }\alpha,\beta,\lambda\in\pi, \tag{2.5}$$

$$(id_{C_\alpha}\otimes\varepsilon)\Delta_{\alpha,e}=id_{C_\alpha}=(\varepsilon\otimes id_{C_\alpha})\Delta_{e,\alpha}\text{ for all }\alpha\in\pi. \tag{2.6}$$

We use the Sweedler-like notation (see [14]) for a comultiplication in the following way: for any $\alpha,\beta\in\pi$ and $c\in C_{\alpha\beta}$, we write $\Delta_{\alpha,\beta}(c)=c_{(1,\alpha)}\otimes c_{(2,\beta)}$.

A *T-coalgebra* is a π-coalgebra $H=(\{H_\alpha\},\Delta,\varepsilon)$ together with a family of k-linear maps $S=\{S_\alpha:H_\alpha\longrightarrow H_{\alpha^{-1}}\}_{\alpha\in\pi}$ (called the antipode), and a family of algebra isomorphisms $\varphi=\{\varphi_\beta:H_\alpha\longrightarrow H_{\beta\alpha\beta^{-1}}\}_{\alpha,\beta\in\pi}$ (called the crossing) such that

each H_α is an algebra with multiplication m_α and unit $1_\alpha\in H_\alpha$, $\qquad$ (2.7)

for all $\alpha,\beta\in\pi$, $\quad\Delta_{\alpha,\beta}$ and $\varepsilon:H_e\longrightarrow k$ are algebra maps, $\qquad$ (2.8)

for $\alpha\in\pi$, $m_\alpha(S_{\alpha^{-1}}\otimes id_{H_\alpha})\Delta_{\alpha^{-1},\alpha}=\varepsilon 1_\alpha=m_\alpha(id_{H_\alpha}\otimes S_{\alpha^{-1}})\Delta_{\alpha,\alpha^{-1}}$, $\qquad$ (2.9)

for all $\alpha,\beta,\gamma\in\pi$, $\quad(\varphi_\beta\otimes\varphi_\beta)\Delta_{\alpha,\gamma}=\Delta_{\beta\alpha\beta^{-1},\beta\alpha\beta^{-1}}\varphi_\beta$, $\qquad$ (2.10)

for all $\beta\in\pi$, $\quad\varepsilon\varphi_\beta=\varepsilon$, $\qquad$ (2.11)

for all $\alpha,\beta\in\pi$, $\quad\varphi_\alpha\varphi_\beta=\varphi_{\alpha\beta}$. $\qquad$ (2.12)

2.3 Twisted Semi-Direct Square of Groups

Let G, L be two groups and G act on the left the group L by automorphisms. Then $L \times G$ is a group with the multiplication

$$(l, g)(l', g') = (l(g \triangleright l'), gg'),$$

which is called a semi-direct product of L by G and denoted by $L \ltimes G$. A group π is a semi-direct product of L by G if and only if L is a normal subgroup of π, G is a subgroup of π, $L \cap G = 1$, and $\pi = LG$ (see [16]).

Let π be a group and let $L = \pi^{op}$, the opposite group of a group π. Consider the adjoint action of π on L by defining: $\gamma \triangleright \alpha = \gamma\alpha\gamma^{-1}$ for all $\alpha, \gamma \in \pi$. Then we have the semi-direct product $\pi^{op} \ltimes \pi$. The opposite group $(\pi^{op} \ltimes \pi)^{op}$ of the group $\pi^{op} \ltimes \pi$ is denoted by G with the multiplication, for all $\alpha, \beta, \lambda, \gamma \in \pi$:

$$(\alpha, \beta)\#(\lambda, \gamma) = (\gamma\alpha\gamma^{-1}\lambda, \gamma\beta), \tag{2.13}$$

which was called a twisted semi-direct square of group π (see [16]). Moreover π is a subgroup of G and $(\alpha, \beta)^{-1} = (\beta^{-1}\alpha^{-1}\beta, \beta^{-1})$.

3 A Braided T-Category ${}_A\mathcal{YD}^H_G$

Definition 3.1 Let H be a Hopf algebra with bijective antipode S and π a group with the unit e. Let A be an H-bicomodule algebra and $\zeta_\alpha, \zeta_\beta \in Aut(H)$. An (α, β)-quantum Yetter-Drinfeld module is a vector space M, such that M is a left A-module (with notation $a \otimes m \mapsto a \cdot m$) and right H-comodule (with notation $M \to M \otimes H, m \mapsto m_{(0)} \otimes m_{(1)}$) with the following compatibility condition:

$$\rho(a \cdot m) = a_{(0)} \cdot m_{(0)} \otimes \zeta_\beta(a_{(1)})m_{(1)}\zeta_\alpha(S^{-1}(a_{(-1)})) \tag{3.1}$$

for all $a \in A$ and $m \in M$. We denote by ${}_A\mathcal{YD}^H(\alpha, \beta)$ the category of (α, β)-quantum Yetter-Drinfeld module, morphisms being A-linear and H-colinear maps.

Remark 3.2 Let H be a Hopf algebra with bijective antipode S and $\zeta_\alpha, \zeta_\beta \in Aut(H)$. Let A be an H-bicomodule algebra. Then $A \otimes H$ is an object in ${}_A\mathcal{YD}^H(\alpha, \beta)$ with the following structures:

$$a \cdot (b \otimes h) = ab \otimes h, \qquad \rho(b \otimes h) = b_{(0)} \otimes h_1 \otimes \zeta_\beta(b_{(1)})h_2\zeta_\alpha(S^{-1}(b_{(-1)}))$$

for all $a, b \in A$ and $h \in H$. Furthermore, if A and H are bialgebras, then it is easy to check $\Bbbk$ is an object in ${}_A\mathcal{YD}^H(\alpha, \beta)$ with structures: $a \cdot x = \varepsilon_A(a)x$ and $\rho(x) = x \otimes 1_H$, for all $x \in \Bbbk$ if and only if the following condition holds:

$$\varepsilon_A(a)1_H = \varepsilon_A(a_{(0)})\zeta_\alpha(a_{(-1)})\zeta_\beta(S^{-1}(a_{(1)})) \tag{3.2}$$

for all $a \in A$.

Thus we have a Turaev G-category ${}_A\mathcal{YD}^H_G$ as a disjoint union of family of categories $\{{}_H\mathcal{YD}^H_G(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ over the family of the left-right (α,β)-Yetter-Drinfeld modules, with $(\alpha,\beta)\in G$.

Example 3.3 (1) Let H be a Hopf algebra with a bijective antipode and $\zeta:\pi\longrightarrow Aut(H)$ a group homomorphism. Then category ${}_H\mathcal{YD}^H(\alpha,\beta)$ is actually the category of (α,β)-Yetter-Drinfel'd modules studied in Panaite and Staic [6].

(2) For $\zeta_\alpha=\zeta_\beta=id_H$, we have ${}_H\mathcal{YD}^H(id,id)={}_H\mathcal{YD}^H$, the usual quantum Yetter-Drinfel'd module category in the sense of Caenepeel et al. [2].

(3) For $\zeta_\alpha=S^2,\zeta_\beta=id_H$, the compatibility condition (2.1) becomes

$$(a\cdot m)_{(0)}\otimes(a\cdot m)_{(1)}=a_{(0)}\cdot m_{(0)}\otimes a_{(-1)}m_{(1)}S(a_{(1)}),$$

hence ${}_H\mathcal{YD}^H(S^2,id)$ is the usual anti-quantum Yetter-Drinfeld module category in the sense of Caenepeel et al. [2].

The following notion is a generalization of one in Panaite and Staic in [6].

Proposition 3.4 For any $M\in{}_A\mathcal{YD}^H(\alpha,\beta)$ and $N\in{}_A\mathcal{YD}^H(\gamma,\delta)$, then we have $M\otimes N\in{}_A\mathcal{YD}^H(\delta\alpha\delta^{-1}\gamma,\delta\beta)$ with structures:

$$\begin{aligned}&a\triangleright(m\otimes n)=a_2\cdot m\otimes a_1\cdot n,\\&\rho_{M\otimes N}(m\otimes n):=(m\otimes n)_{\langle 0\rangle}\otimes(m\otimes n)_{\langle 1\rangle}=m_{(0)}\otimes n_{(0)}\otimes\zeta_\delta(m_{(1)})\zeta_{\delta\alpha\delta^{-1}}(n_{(1)})\end{aligned}$$

if and only if the following condition holds:

$$\begin{aligned}&a_{2(0)}\otimes a_{1(0)}\otimes\zeta_{\delta\beta}(a_{2(1)})\zeta_\delta(c)\zeta_{\delta\alpha}(S^{-1}(a_{2(-1)}))\zeta_{\delta\alpha}(a_{1(1)})\zeta_{\delta\alpha\delta^{-1}}(d)\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(a_{1(-1)})))\\=\;&a_{(0)2}\otimes a_{(0)1}\otimes\zeta_{\delta\beta}(a_{(1)})\zeta_\delta(c)\zeta_{\delta\alpha\delta^{-1}}(d)\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(a_{-1})).\end{aligned}\tag{3.3}$$

Proof It is easy to see that $M\otimes N$ is a left A-module and $M\otimes N$ is a right H-comodule. We compute the compatibility condition:

$$\begin{aligned}&\rho_{M\otimes N}(a\triangleright(m\otimes n))\\=\;&(a_2\cdot m)_{(0)}\otimes(a_1\cdot n)_{(0)}\otimes\zeta_\delta((a_2\cdot m)_{(1)})\zeta_{\delta\alpha\delta^{-1}}((a_1\cdot n)_{(1)})\\=\;&a_{2(0)}\cdot m_{(0)}\otimes a_{1(0)}\cdot n_{(0)}\otimes\zeta_\delta(\zeta_\beta(a_{2(1)})m_{(1)}\zeta_\alpha(S^{-1}(a_{2(-1)})))\\&\zeta_{\delta\alpha\delta^{-1}}(\zeta_\delta(a_{1(1)})n_{(1)}\zeta_\gamma(S^{-1}(a_{1(-1)})))\\=\;&a_{2(0)}\cdot m_{(0)}\otimes a_{1(0)}\cdot n_{(0)}\otimes\zeta_{\delta\beta}(a_{2(1)})\zeta_\delta(m_{(1)})\zeta_{\delta\alpha}(S^{-1}(a_{2(-1)}))\zeta_{\delta\alpha}(a_{1(1)})\\&\zeta_{\delta\alpha\delta^{-1}}(n_{(1)})\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(a_{1(-1)})))\\\overset{(3.3)}{=}\;&a_{(0)2}\cdot m_{(0)}\otimes a_{(0)1}\cdot n_{(0)}\otimes\zeta_{\delta\beta}(a_{(1)})\zeta_\delta(m_{(1)})\zeta_{\delta\alpha\delta^{-1}}(n_{(1)})\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(a_{-1}))\\=\;&a_{(0)}\cdot(m\otimes n)_{\langle 0\rangle}\otimes\zeta_{\delta\beta}(a_{(1)})(m\otimes n)_{\langle 1\rangle}\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(a_{-1}))\end{aligned}$$

and this shows that $M\otimes N\in{}_A\mathcal{YD}^H(\delta\alpha\delta^{-1}\gamma,\delta\beta)$.

Conversely, by Remark 3.2, since $A\otimes H\in{}_A\mathcal{YD}^H(\alpha,\beta)$, we let $m=1\otimes c$ and $n=1\otimes d$ for any $c,d\in H$ and easily get eq. (3.3).

The following proposition is straightforward.

Proposition 3.5 For any $N \in {}_A\mathcal{YD}^H(\gamma,\delta)$ and $(\alpha,\beta) \in G$. Define ${}^{(\alpha,\beta)}N = N$ as vector space, with structures

$$a\blacklozenge n = \xi_{\alpha^{-1}\beta}(a)\cdot n,$$
$$\rho(n) := n_{[0]}\otimes n_{[1]} = n_{(0)}\otimes \zeta_{\beta^{-1}\delta\alpha\delta^{-1}}(n_{(1)}).$$

Then we have that ${}^{(\alpha,\beta)}N \in {}_A\mathcal{YD}^H((\alpha,\beta)\#(\gamma,\delta)\#(\alpha,\beta)^{-1})$ if and only if the following condition holds:

$$\begin{aligned} & \xi_{\alpha^{-1}\beta}(a)_{(0)}\otimes \zeta_{\beta^{-1}\delta\alpha\delta^{-1}}(\zeta_\delta(\xi_{\alpha^{-1}\beta}(a)_{(-1)})c\zeta_\gamma(S^{-1}(\xi_{\alpha^{-1}\beta}(a)_{(-1)}))) \\ = \ & \xi_{\alpha^{-1}\beta}(a_{(0)})\otimes \zeta_{\beta^{-1}\delta\beta}(a_{(-1)})\xi_{\beta^{-1}\delta\alpha\delta^{-1}}(c)\zeta_{\beta^{-1}\delta\alpha\delta^{-1}\gamma\delta^{-1}\beta}(S^{-1}(a_{(1)})). \end{aligned} \tag{3.4}$$

Furthermore, let $M \in {}_A\mathcal{YD}^H(\alpha,\beta)$ and $(\mu,\nu)\in G$. Then we have

$${}^{(\alpha,\beta)\#(\mu,\nu)}N = {}^{(\alpha,\beta)}({}^{(\mu,\nu)}N), \qquad {}^{(\mu,\nu)}(M\otimes N) = {}^{(\mu,\nu)}M\otimes {}^{(\mu,\nu)}N.$$

Proof We only show the first claim as follows.

By Remark 3.2, $A\otimes H \in {}_A\mathcal{YD}^H(\gamma,\delta)$ for any $(\gamma,\delta)\in G$. For any $d\in H$, then we have

$$(a\blacklozenge(1\otimes d))_{[0]}\otimes(a\blacklozenge(1\otimes d))_{[1]} = a_{[0]}\blacklozenge(1\otimes d)_{[0]}\otimes\zeta_{\beta^{-1}\delta\beta}(a_{(-1)})(1\otimes d)_{[1]}\zeta_{\beta^{-1}\delta\alpha\delta^{-1}\gamma\delta^{-1}\beta}(S^{-1}(a_{(1)})),$$

which implies eq. (3.4).

Conversely, one has

$$\begin{aligned} & (a\blacklozenge n)_{[0]}\otimes(a\blacklozenge n)_{[1]} \\ = \ & (\xi_{\alpha^{-1}\beta}(a)\cdot n)_{(0)}\otimes\zeta_{\beta^{-1}\delta\alpha\delta^{-1}}((\xi_{\alpha^{-1}\beta}(a)\cdot n)_{(1)}) \\ = \ & \xi_{\alpha^{-1}\beta}(a)_{(0)}\cdot n_{(0)}\otimes\zeta_{\beta^{-1}\delta\alpha\delta^{-1}}(\zeta_\delta(\xi_{\alpha^{-1}\beta}(a)_{(-1)})n_{(1)}\zeta_\gamma(S^{-1}(\xi_{\alpha^{-1}\beta}(a)_{(-1)}))) \\ \overset{(3.4)}{=} \ & \xi_{\alpha^{-1}\beta}(a_{(0)})\cdot n_{(0)}\otimes\zeta_{\beta^{-1}\delta\beta}(a_{(-1)})\xi_{\beta^{-1}\delta\alpha\delta^{-1}}(n_{(1)})\zeta_{\beta^{-1}\delta\alpha\delta^{-1}\gamma\delta^{-1}\beta}(S^{-1}(a_{(1)})) \\ = \ & a_{(0)}\blacklozenge n_{[0]}\otimes\zeta_{\beta^{-1}\delta\beta}(a_{(-1)})n_{[1]}\zeta_{\beta^{-1}\delta\alpha\delta^{-1}\gamma\delta^{-1}\beta}(S^{-1}(a_{(1)})). \end{aligned}$$

Now define a group homomorphism $\varphi: G \longrightarrow Aut({}_A\mathcal{YD}^H_G)$, $(\alpha,\beta)\mapsto\varphi_{(\alpha,\beta)}$, as

$$\varphi_{(\alpha,\beta)}: {}_A\mathcal{YD}^H(\gamma,\delta)\longrightarrow {}_A\mathcal{YD}^H((\alpha,\beta)\#(\gamma,\delta)\#(\alpha,\beta)^{-1}), \varphi_{\alpha,\beta}(N) = {}^{(\alpha,\beta)}N,$$

and the functor $\varphi_{(\alpha,\beta)}$ acts as identity on morphisms.

Consider now a map $\mathscr{Q}:\ H\otimes H\to A\otimes A$ with a twisted convolution inverse $\mathscr{R}$, that means that

$$\mathscr{Q}(h_2\otimes g_2)\mathscr{R}(h_1\otimes g_1) = \varepsilon(h)1_A\otimes\varepsilon(g)1_A$$

for all $h,g\in H$. Sometimes, we write $\mathscr{Q}(h\otimes g) := \mathscr{Q}^1(h\otimes g)\otimes\mathscr{Q}^2(h\otimes g)$ for all $h,g\in H$.

For any $M\in {}_A\mathcal{YD}^H(\alpha,\beta)$, $N\in {}_A\mathcal{YD}^H(\gamma,\delta)$ and $P\in {}_A\mathcal{YD}^H(\mu,\nu)$. Define a map as follows:

$$c_{M,N}: M\otimes N\to {}^MN\otimes M,$$
$$c_{M,N}(m\otimes n) = \mathscr{Q}(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))(n_{(0)}\otimes m_{(0)}). \tag{3.5}$$

In what follows, our main aim is to give some necessary and sufficient conditions on $\mathscr{Q}$ such that the $c_{M,N}$ defines a braiding on ${}_A\mathcal{YD}_G^H$. For this, we will find conditions under which $c_{M,N}$ is both A-linear and H-colinear, and the following conditions hold:

$$c_{M\otimes N,P} = (c_{M,\ {}^N P} \otimes id_N) \circ (id_M \otimes c_{N,P}), \tag{3.6}$$

$$c_{M,N\otimes P} = (id_{{}^M N} \otimes c_{M,P}) \circ (c_{M,N} \otimes id_P). \tag{3.7}$$

Furthermore, if $M \in {}_A\mathcal{YD}^H(\alpha,\beta)$ and $N \in {}_A\mathcal{YD}^H(\gamma,\delta)$, then we want to show the following:

$$c_{{}^{(\mu,\nu)}M,\ {}^{(\mu,\nu)}N} = c_{M,N} \tag{3.8}$$

holds, for any $(\mu,\nu) \in G$.

In order to approach to our main result we need some lemmas.

Lemma 3.6 For any $M \in {}_A\mathcal{YD}^H(\alpha,\beta)$ and $N \in {}_A\mathcal{YD}^H(\gamma,\delta)$. Then $c_{M,N}$ is A-linear if and only if the following condition is satisfied:

$$\begin{aligned} &\mathscr{Q}(\zeta_\delta(a_{1(-1)})d\zeta_\gamma(S^{-1}(a_{1(1)})) \otimes \zeta_{\alpha^{-1}}(\zeta_\beta(a_{2(-1)})c\zeta_\alpha(S^{-1}(a_{2(1)})))(a_{1(0)} \otimes a_{2(0)}) \\ = \ &[(\xi_{\alpha^{-1}\beta} \otimes 1)\Delta^{cop}(a)]\mathscr{Q}(d \otimes \zeta_{\alpha^{-1}}(c)) \end{aligned} \tag{3.9}$$

for all $a \in A$ and $c, d \in H$.

Proof If $c_{M,N}$ is A-linear then it is easy to get

$$a \cdot c_{M,N}(m \otimes n) = [(\xi_{\alpha^{-1}\beta} \otimes 1)\Delta^{cop}(a)]\mathscr{Q}(n_{(1)} \otimes \zeta_{\alpha^{-1}}(m_{(1)}))(n_{(0)} \otimes m_{(0)})$$

and

$$\begin{aligned} &c_{M,N}(a \cdot (m \otimes n)) \\ = \ &\mathscr{Q}(\zeta_\delta(a_{1(-1)})n_{(1)}\zeta_\gamma(S^{-1}(a_{1(1)})) \otimes \zeta_{\alpha^{-1}}(\zeta_\beta(a_{2(-1)})m_{(1)}\zeta_\alpha(S^{-1}(a_{2(1)}))) \\ &(a_{1(0)} \cdot n_{(0)} \otimes a_{2(0)} \cdot m_{(0)}). \end{aligned}$$

Considering these equations and taking $M = N = A \otimes C$ and $m = 1 \otimes c$ and $n = 1 \otimes d$ for all $c, d \in H$. Then we can get eq. (3.9).

Conversely, by the above formulas it is easy to see that $c_{M,N}$ is A-linear.

Lemma 3.7 For any $M \in {}_A\mathcal{YD}^H(\alpha,\beta)$ and $N \in {}_A\mathcal{YD}^H(\gamma,\delta)$. Then $c_{M,N}$ is H-colinear if and only if the following condition is satisfied:

$$\begin{aligned} &\mathscr{Q}(d_1 \otimes \zeta_{\alpha^{-1}}(c_1)) \otimes \zeta_\delta(c_2)\zeta_{\delta\alpha\delta^{-1}}(d_2) = \mathscr{Q}^1(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(0)} \otimes \mathscr{Q}^2(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(0)} \\ &\otimes \zeta_{\delta\alpha\delta^{-1}}(\zeta_\delta(\mathscr{Q}^1(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(-1)})d_1\zeta_\gamma(S^{-1}(\mathscr{Q}^1(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(1)}))) \\ &\otimes \zeta_{\delta\alpha\delta^{-1}\gamma\alpha^{-1}}(\zeta_\alpha(\mathscr{Q}^2(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(-1)})c_1\zeta_\alpha(S^{-1}(\mathscr{Q}^2(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(1)}))) \end{aligned} \tag{3.10}$$

for all $c, d \in H$.

Proof If $c_{M,N}$ is H-colinear then we do the following calculations:

$$\begin{aligned}
& \rho\circ c_{M,N}(m\otimes n)\\
= & (\mathscr{Q}^1(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))\cdot n_{(0)})_{\langle 0\rangle}\otimes(\mathscr{Q}^2(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))\cdot m_{(0)})_{(0)}\\
& \otimes(\zeta_\beta(\mathscr{Q}^1(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))\cdot n_{(0)})_{\langle 1\rangle})(\zeta_{\delta\alpha\delta^{-1}\gamma\alpha^{-1}}(\mathscr{Q}^2(n_{(1)}\otimes\alpha^{-1}\cdot m_{(1)})\cdot m_{(0)})_{(1)})\\
= & (\mathscr{Q}^1(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))\cdot n_{(0)})_{(0)}\otimes(\mathscr{Q}^2(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))\cdot m_{(0)})_{(0)}\\
& \otimes(\zeta_{\delta\alpha\delta^{-1}}(\mathscr{Q}^1(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))\cdot n_{(0)})_{(1)})(\zeta_{\delta\alpha\delta^{-1}\gamma\alpha^{-1}}(\mathscr{Q}^2(n_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))\cdot m_{(0)})_{(1)})\\
= & (\mathscr{Q}^1(n_{(1)2}\otimes\zeta_{\alpha^{-1}}(m_{(1)2}))_{(0)}\cdot n_{(0)})\otimes(\mathscr{Q}^2(n_{(1)2}\otimes\zeta_{\alpha^{-1}}(m_{(1)2}))_0\cdot m_{(0)})\\
& \otimes\zeta_{\delta\alpha\delta^{-1}}(\zeta_\delta(\mathscr{Q}^1(n_{(1)2}\otimes\zeta_{\alpha^{-1}}(m_{(1)2}))_{(-1)})n_{(1)1}\zeta_\gamma(S^{-1}(\mathscr{Q}^1(n_{(1)2}\otimes\zeta_{\alpha^{-1}}(m_{(1)2}))_{(1)})))\\
& \otimes\zeta_{\delta\alpha\delta^{-1}\gamma\alpha^{-1}}(\zeta_\alpha(\mathscr{Q}^2(n_{(1)2}\otimes\zeta_{\alpha^{-1}}(m_{(1)2}))_{(-1)})m_{(1)1}\zeta_\alpha(S^{-1}(\mathscr{Q}^2(n_{(1)2}\otimes\zeta_{\alpha^{-1}}(m_{(1)2}))_{(1)})))
\end{aligned}$$

and

$$\begin{aligned}
& (c_{M,N}\otimes id)\circ\rho(m\otimes n)=c_{M,N}(m\otimes n)_{\langle 0\rangle}\otimes(m\otimes n)_{\langle 1\rangle}\\
= & \mathscr{Q}(n_{(1)1}\otimes\zeta_{\alpha^{-1}}(m_{(1)1}))(n_{(0)}\otimes m_{(0)})\otimes\zeta_\delta(m_{(1)2})\zeta_{\delta\alpha\delta^{-1}}(n_{(1)2}).
\end{aligned}$$

Now we let $M=N=A\otimes H$ and take $m=1\otimes c$ and $n=1\otimes d$ for all $c,d\in H$. Then we can get eq. (3.10).

Conversely, by the above formulas it is easy to see that $c_{M,N}$ is H-colinear.

Lemma 3.8 For any $M\in {}_A\mathcal{YD}^H(\alpha,\beta)$, $N\in {}_A\mathcal{YD}^H(\gamma,\delta)$ and $P\in {}_A\mathcal{YD}^H(\mu,\nu)$. Then eq. (3.6) holds if and only if the following condition is satisfied, with $\mathscr{U}=\mathscr{Q}$:

$$\begin{aligned}
& (id\otimes\Delta^{cop})\mathscr{Q}(h\otimes\zeta_{\gamma^{-1}}(\zeta_{\delta\alpha^{-1}}(c)d))=[(\xi_{\gamma^{-1}\delta}\otimes 1)\mathscr{U}(\zeta_{\delta^{-1}\nu\delta}(\mathscr{Q}^1(h_2\otimes\zeta_{\gamma^{-1}}(d))_{(-1)})\\
& h_1\zeta_{\delta^{-1}\nu\gamma\nu^{-1}\mu\gamma^{-1}\delta}(S^{-1}(\mathscr{Q}^1(h_2\otimes\zeta_{\gamma^{-1}}(d))_{(1)}))\otimes\zeta_{\alpha^{-1}}(c))]\\
& (\mathscr{Q}^1(h_2\otimes\zeta_{\gamma^{-1}}(d))_{(0)}\otimes 1)\otimes\mathscr{Q}^2(h_2\otimes\zeta_{\gamma^{-1}}(d))
\end{aligned}\tag{3.11}$$

for all $c,d,h\in H$.

Proof If eq. (3.6) holds. Then we compute as follows:

$$\begin{aligned}
& (c_{M,\ ^NP}\otimes id_N)\circ(id_M\otimes c_{N,P})(m\otimes n\otimes p)\\
= & \mathscr{U}((\mathscr{Q}^1(p_{(1)}\otimes\zeta_{\gamma^{-1}}(n_{(1)}))\cdot p_{(0)})_{(1)}\otimes\zeta_{\alpha^{-1}}(m_{(1)}))((\mathscr{Q}^1(p_{(1)}\otimes\zeta_{\gamma^{-1}}(n_{(1)}))\cdot p_{(0)})_{(0)}\\
& \otimes m_{(0)}\otimes\mathscr{Q}^2(p_{(1)}\otimes\zeta_{\gamma^{-1}}(n_{(1)}))\cdot n_{(0)})\\
= & [(\xi_{\gamma^{-1}\delta}\otimes 1)\mathscr{U}(\zeta_{\delta^{-1}\nu\delta}(\mathscr{Q}^1(p_{(1)2}\otimes\zeta_{\gamma^{-1}}(n_{(1)}))_{(-1)})p_{(1)1}\zeta_{\delta^{-1}\nu\gamma\nu^{-1}\mu\gamma^{-1}\delta}\\
& (S^{-1}(\mathscr{Q}^1(p_{(1)2}\otimes\zeta_{\gamma^{-1}}(n_{(1)}))_{(1)}))\otimes\zeta_{\alpha^{-1}}(m_{(1)}))]\\
& (\mathscr{Q}^1(p_{(1)2}\otimes\zeta_{\gamma^{-1}}(n_{(1)}))_{(0)}\cdot p_{(0)}\otimes m_{(0)})\otimes\mathscr{Q}^2(p_{(1)2}\otimes\zeta_{\gamma^{-1}}(n_{(1)}))
\end{aligned}$$

and

$$\begin{aligned}
& c_{M\otimes N,P}(m\otimes n\otimes p)\\
= & \mathscr{Q}(p_{(1)}\otimes\zeta_{\gamma^{-1}\delta\alpha^{-1}\delta^{-1}}((m\otimes n)_{\langle 1\rangle}))(p_{(0)}\otimes(m\otimes n)_{\langle 0\rangle})\\
= & \mathscr{Q}(p_{(1)}\otimes\zeta_{\gamma^{-1}}(\zeta_{\delta\alpha^{-1}}(m_{(1)})n_{(1)}))(p_{(0)}\otimes(m_{(0)}\otimes n_{(0)})).
\end{aligned}$$

Take $M = N = P = A \otimes H$ and $m = 1 \otimes c$, and $n = 1 \otimes d$, and $p = 1 \otimes h$ for all $c, d, h \in H$. Then we obtain eq. (3.11).

Conversely, the proof is straightforward. We omit the details.

Lemma 3.9 For any $M \in {}_A\mathcal{YD}^H(\alpha, \beta)$, $N \in {}_A\mathcal{YD}^H(\gamma, \delta)$ and $P \in {}_A\mathcal{YD}^H(\mu, \nu)$. Then eq. (3.7) holds if and only if the following condition is satisfied, with $\mathscr{U} = \mathscr{Q}$:

$$\begin{aligned}&(\Delta^{cop} \otimes id)\mathscr{Q}(\zeta_\mu(d\zeta_{\gamma\mu^{-1}}(h))) \otimes \zeta_{\alpha^{-1}}(c) = \mathscr{Q}^1(d \otimes \zeta_{\alpha^{-1}}(c_2)) \otimes \mathscr{U}(h \otimes \zeta_{\alpha^{-1}}\\&(\mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(\zeta_\beta(\mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(c))_{(-1)})m_{(1)1}\zeta_\alpha(S^{-1}\\&(\mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(c))_{(1)})))))(1 \otimes \mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(c))_{(0)})\end{aligned} \tag{3.12}$$

for all $c, d, h \in H$.

Proof If eq.(3.7) holds, then we have

$$\begin{aligned}&(id_{{}^M N} \otimes c_{M,P}) \circ (c_{M,N} \otimes id_P)(m \otimes n \otimes p)\\=\;& \mathscr{Q}^1(n_{(1)} \otimes \zeta_{\alpha^{-1}}(m_{(1)})) \cdot n_{(0)} \otimes \mathscr{U}(p_{(1)} \otimes \zeta_{\alpha^{-1}}[(\mathscr{Q}^2(n_{(1)} \otimes \zeta_{\alpha^{-1}}(m_{(1)})) \cdot m_{(0)})_{(1)}])\\&(p_{(0)} \otimes (\mathscr{Q}^2(n_{(1)} \otimes \zeta_{\alpha^{-1}}(m_{(1)})) \cdot m_{(0)})_{(0)})\\=\;& \mathscr{Q}^1(n_{(1)} \otimes \zeta_{\alpha^{-1}}(m_{(1)2})) \cdot n_{(0)} \otimes \mathscr{U}(p_{(1)} \otimes \zeta_{\alpha^{-1}}(\mathscr{Q}^2(n_{(1)} \otimes \zeta_{\alpha^{-1}}(\zeta_\beta(\mathscr{Q}^2(n_{(1)}\\&\otimes \zeta_{\alpha^{-1}}(m_{(1)}))_{(-1)})m_{(1)1}\zeta_\alpha(S^{-1})(\mathscr{Q}^2(n_{(1)} \otimes \zeta_{\alpha^{-1}}(m_{(1)}))_{(1)})))))(p_{(0)} \otimes \mathscr{Q}^2(n_{(1)}\\&\otimes \zeta_{\alpha^{-1}}(m_{(1)}))_{(0)} \cdot m_{(0)})\end{aligned}$$

and

$$c_{M,N\otimes P}(m \otimes n \otimes p) = \mathscr{Q}(\zeta_\mu(n_{(1)})\zeta_{\mu\gamma\mu^{-1}}(p_{(1)}) \otimes \zeta_{\alpha^{-1}}(m_{(1)}))(n_{(0)} \otimes p_{(0)} \otimes m_{(0)}).$$

Take $M = N = P = A \otimes H$ and $m = 1 \otimes c$, and $n = 1 \otimes d$, and $p = 1 \otimes h$ for all $c, d, h \in H$. Then we obtain eq. (3.12).

Conversely, it is straightforward.

Lemma 3.10 For any $M \in {}_A\mathcal{YD}^H(\alpha, \beta)$, $N \in {}_A\mathcal{YD}^H(\gamma, \delta)$ and $P \in {}_A\mathcal{YD}^H(\mu, \nu)$. Then eq. (3.8) holds if and only if the following condition holds:

$$(\xi_{\mu^{-1}\nu} \otimes \xi_{\mu^{-1}\nu})\mathscr{Q}(\zeta_{\nu^{-1}\delta\mu\delta^{-1}}(d) \otimes \zeta_{\nu^{-1}\mu\alpha^{-1}}(c)) = \mathscr{Q}(d \otimes \zeta_{\alpha^{-1}}(c)) \tag{3.13}$$

for all $c, d \in H$.

Proof Straightforward.

Therefore, we can summarize our results as follows.

Theorem 3.11 Let A and H be bialgebras and π a group with the unit e. Let ξ : $\pi \longrightarrow \mathrm{Aut}(A)$ and $\zeta : \pi \longrightarrow \mathrm{Aut}(H)$ be group homomorphisms. Let A be an H-bicomodule algebra and $\mathscr{Q} : H \otimes H \longrightarrow A \otimes A$ a twisted convolution invertible map. Then the family of maps given by eq. (3.5) defines a braiding on the category ${}_A\mathcal{YD}_G^H$ if and only if equations (3.2)–(3.4) and (3.9)–(3.13) are satisfied.

Definition 3.12 Let A and H be bialgebras and π a group with the unit e. Let $\xi : \pi \longrightarrow \mathrm{Aut}(A)$ and $\zeta : \pi \longrightarrow \mathrm{Aut}(H)$ be group homomorphisms. Let A be an H-bicomodule algebra. We say that a G-double structure is (A, H) together with a linear map $\mathscr{Q} : H \otimes H \longrightarrow A \otimes A$ such that the following conditions hold:

(1) $\varepsilon_A(a)1_H = \varepsilon_A(a_{(0)})\zeta_\alpha(a_{(-1)})\zeta_\beta(S^{-1}(a_{(1)}))$;

(2) $a_{2(0)} \otimes a_{1(0)} \otimes \zeta_{\delta\beta}(a_{2(1)})\zeta_\delta(c)\zeta_{\delta\alpha}(S^{-1}(a_{2(-1)}))\zeta_{\delta\alpha}(a_{1(1)})\zeta_{\delta\alpha\delta^{-1}}(d)\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(a_{1(-1)})))$
$= a_{(0)2} \otimes a_{(0)1} \otimes \zeta_{\delta\beta}(a_{(1)})\zeta_\delta(c)\zeta_{\delta\alpha\delta^{-1}}(d)\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(a_{-1}))$;

(3) $\xi_{\alpha^{-1}\beta}(a)_{(0)} \otimes \zeta_{\beta^{-1}\delta\alpha\delta^{-1}}(\zeta_\delta(\xi_{\alpha^{-1}\beta}(a)_{(-1)})c\zeta_\gamma(S^{-1}(\xi_{\alpha^{-1}\beta}(a)_{(-1)})))$
$= \xi_{\alpha^{-1}\beta}(a_{(0)}) \otimes \zeta_{\beta^{-1}\delta\beta}(a_{(-1)})\xi_{\beta^{-1}\delta\alpha\delta^{-1}}(c)\zeta_{\beta^{-1}\delta\alpha\delta^{-1}\gamma\delta^{-1}\beta}(S^{-1}(a_{(1)}))$;

(4) $\mathscr{Q}(\zeta_\delta(a_{1(-1)})d\zeta_\gamma(S^{-1}(a_{1(1)})) \otimes \zeta_{\alpha^{-1}}(\zeta_\beta(a_{2(-1)})c\zeta_\alpha(S^{-1}(a_{2(1)}))))(a_{1(0)} \otimes a_{2(0)})$
$= [(\xi_{\alpha^{-1}\beta} \otimes 1)\Delta^{cop}(a)]\mathscr{Q}(d \otimes \zeta_{\alpha^{-1}}(c))$;

(5) $\mathscr{Q}(d_1 \otimes \zeta_{\alpha^{-1}}(c_1)) \otimes \zeta_\delta(c_2)\zeta_{\delta\alpha\delta^{-1}}(d_2) = \mathscr{Q}^1(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(0)} \otimes \mathscr{Q}^2(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(0)}$
$\otimes\zeta_{\delta\alpha\delta^{-1}}(\zeta_\delta(\mathscr{Q}^1(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(-1)})d_1\zeta_\gamma(S^{-1}(\mathscr{Q}^1(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(1)})))$
$\otimes\zeta_{\delta\alpha\delta^{-1}\gamma\alpha^{-1}}(\zeta_\alpha(\mathscr{Q}^2(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(-1)})c_1\zeta_\alpha(S^{-1}(\mathscr{Q}^2(d_2 \otimes \zeta_{\alpha^{-1}}(c_2))_{(1)})))$;

(6) $(id \otimes \Delta^{cop})\mathscr{Q}(h \otimes \zeta_{\gamma^{-1}}(\zeta_{\delta\alpha^{-1}}(c)d)) = [(\xi_{\gamma^{-1}\delta} \otimes 1)\mathscr{U}(\zeta_{\delta^{-1}\nu\delta}(\mathscr{Q}^1(h_2 \otimes \zeta_{\gamma^{-1}}(d))_{(-1)})$
$h_1\zeta_{\delta^{-1}\nu\gamma\nu^{-1}\mu\gamma^{-1}\delta}(S^{-1}(\mathscr{Q}^1(h_2 \otimes \zeta_{\gamma^{-1}}(d))_{(1)})) \otimes \zeta_{\alpha^{-1}}(c))]$
$(\mathscr{Q}^1(p_{(1)2} \otimes \zeta_{\gamma^{-1}}(d))_{(0)} \otimes 1) \otimes \mathscr{Q}^2(p_{(1)2} \otimes \zeta_{\gamma^{-1}}(d))$;

(7) $(\Delta^{cop} \otimes id)\mathscr{Q}(\zeta_\mu(d\zeta_{\gamma\mu^{-1}}(e))) \otimes \zeta_{\alpha^{-1}}(c) = \mathscr{Q}^1(d \otimes \zeta_{\alpha^{-1}}(c_2)) \otimes \mathscr{U}(h \otimes \zeta_{\alpha^{-1}}$
$(\mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(\zeta_\beta(\mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(c))_{(-1)})m_{(1)1}\zeta_\alpha(S^{-1}$
$(\mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(c))_{(1)})))))(1 \otimes \mathscr{Q}^2(d \otimes \zeta_{\alpha^{-1}}(m_{(1)}))_{(0)} \cdot m_{(0)})$;

(8) $(\xi_{\mu^{-1}\nu} \otimes \xi_{\mu^{-1}\nu})\mathscr{Q}(\zeta_{\nu^{-1}\delta\mu\delta^{-1}}(d) \otimes \zeta_{\nu^{-1}\mu\alpha^{-1}}(c)) = \mathscr{Q}(d \otimes \zeta_{\alpha^{-1}}(c))$;

(9) There exists a map: $\mathscr{R} : H \otimes H \longrightarrow A \otimes A$ such that
$\mathscr{Q} * \mathscr{R}(c \otimes d) = \mathscr{R} * \mathscr{Q}(c \otimes d) = \varepsilon(c)\varepsilon(d)1 \otimes 1$.

Proposition 3.13 Let $M \in {}_A\mathcal{YD}^H(\alpha, \beta)$ and assume that M is finite-dimensional. Then

(1) If the following condition holds:

$$S^{-1}(a_{(0)})_{(0)} \otimes \zeta_{\beta^{-1}}(a_{(-1)})\zeta_{\beta^{-1}\alpha^{-1}}S(\zeta_\beta(S^{-1}(a_{(0)})_{(-1)})h\zeta_\alpha S^{-1}$$
$$((S^{-1}(a_{(0)})_{(1)})))\zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(a_{(-1)})) = S^{-1}(a) \otimes \zeta_{\beta^{-1}\alpha^{-1}}S(h) \qquad (3.14)$$

for all $a \in A$ and $h \in H$, then M^* is an object in $M \in {}_A\mathcal{YD}^H(\beta^{-1}\alpha^{-1}\beta, \beta^{-1})$, with the module action and comodule coaction as follows:

$$(a \bullet f)(m) = f(S^{-1}(h) \cdot m),$$
$$\rho(f)(m) = f_{\langle 0\rangle}(m) \otimes f_{\langle 1\rangle} = f(m_{(0)}) \otimes \zeta_{\beta^{-1}\alpha^{-1}}S(m_{(1)})$$

for $a \in A, f \in M^*$ and $m \in M$.

(2) The maps $b_M : k \to M \otimes M^*,\ b_M(1) = \sum_i e_i \otimes e^i$ (where e_i and e^i are dual bases in M and M^*) and $d_M : M^* \otimes M \to k,\ d_M(f \otimes m) = f(m)$ are morphisms in ${}_A\mathcal{YD}^H_G$ and we have

$$(id_M \otimes d_M)(b_M \otimes id_M) = id_M; \quad (d_M \otimes id_{M^*})(id_{M^*} \otimes b_M) = id_{M^*}.$$

Proof (1) For all $a \in A$ and $f \in M^*$, we compute

$$\begin{aligned}
& (a_{(0)} \bullet f_{\langle 0\rangle})(m) \otimes \zeta_{\beta^{-1}}(a_{(-1)}) f_{\langle 1\rangle} \zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(a_{(-1)})) \\
= \ & f_{\langle 0\rangle}(S^{-1}(a_{(0)}) \cdot m) \otimes \zeta_{\beta^{-1}}(a_{(-1)}) f_{\langle 1\rangle} \zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(a_{(-1)})) \\
= \ & f((S^{-1}(a_{(0)}) \cdot m)_{(0)}) \otimes \zeta_{\beta^{-1}}(a_{(-1)}) \zeta_{\beta^{-1}\alpha^{-1}} S((S^{-1}(a_{(0)}) \cdot m)_{(1)}) \zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(a_{(-1)})) \\
= \ & f(S^{-1}(a_{(0)})_{(0)} \cdot m_{(0)})) \otimes \zeta_{\beta^{-1}}(a_{(-1)}) \zeta_{\beta^{-1}\alpha^{-1}} S(\zeta_\beta(S^{-1}(a_{(0)})_{(-1)}) m_{(1)} \\
& \zeta_\alpha S^{-1}((S^{-1}(a_{(0)})_{(1)}))) \zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(a_{(-1)})) \\
\overset{(3.14)}{=} \ & f(S^{-1}(a) \cdot m_{(0)}) \otimes \zeta_{\beta^{-1}\alpha^{-1}} S(m_{(1)}) \\
= \ & (a \bullet f)_{\langle 0\rangle}(m) \otimes (a \bullet f)_{\langle 1\rangle}
\end{aligned}$$

and as required.

(2) Straightforward.

Similarly, one has the following result.

Proposition 3.14 Let $M \in {}_A\mathcal{YD}^H(\alpha, \beta)$ and assume that M is finite dimensional. Then

(1) If the following condition holds:

$$\begin{aligned}
& S(a_{(0)})_{(0)} \otimes \zeta_{\beta^{-1}}(a_{(-1)}) \zeta_{\beta^{-1}\alpha^{-1}} S^{-1}(\zeta_\beta(S(a_{(0)})_{(-1)}) h \zeta_\alpha S^{-1} \\
& ((S(a_{(0)})_{(1)}))) \zeta_{\beta^{-1}\alpha^{-1}\beta}(S(a_{(-1)})) = S(a) \otimes \zeta_{\beta^{-1}\alpha^{-1}} S^{-1}(h)
\end{aligned} \tag{3.15}$$

for all $a \in A$ and $h \in H$, then ${}^*M$ is an object in $M \in {}_A\mathcal{YD}^H(\beta^{-1}\alpha^{-1}\beta, \beta^{-1})$, with the module action and comodule coaction as follows:

$$\begin{aligned}
& (a \bullet f)(m) = f(S(h) \cdot m), \\
& \rho(f)(m) = f_{\langle 0\rangle}(m) \otimes f_{\langle 1\rangle} = f(m_{(0)}) \otimes \zeta_{\beta^{-1}\alpha^{-1}} S^{-1}(m_{(1)})
\end{aligned}$$

for $a \in A, f \in M^*$ and $m \in M$.

(2) The maps $b_M : k \to {}^*M \otimes M,\ b_M(1) = \sum\limits_i e^i \otimes e_i$ (where e_i and e^i are dual bases in M and ${}^*M$) and $d_M : M \otimes {}^*M \to k,\ d_M(m \otimes f) = f(m)$ are morphisms in ${}_A\mathcal{YD}^H_G$ and we have

$$(d_M \otimes id_M)(id_M \otimes b_M) = id_M; \quad (id_{\,{}^*M} \otimes d_M)(b_M \otimes id_{\,{}^*M}) = id_{\,{}^*M}.$$

Now, we consider ${}_A\mathcal{YD}^H_{G;fd}$, the subcategory of ${}_A\mathcal{YD}^H_G$ consisting of finite dimensional objects, then by Proposition 3.13 and Proposition 3.14, we get

Theorem 3.15 If equations (3.14) and (3.15) hold then ${}_A\mathcal{YD}^H_{G;fd}$ is a braided T-category with left and right dualities being given as in Proposition 3.13 and Proposition 3.14, respectively.

4 Application

In this section we construct a quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$, such that $\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ is isomorphic to the representation category of the quasitriangular T-coalgebra $\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$.

Theorem 4.1 Let G be a twisted semi-direct square group and $\mathcal{Q}: H\otimes H \longrightarrow A\otimes A$ a linear map. Let $\xi:\pi\longrightarrow Aut(A)$ and $\zeta:\pi\longrightarrow Aut(H)$ be group homomorphisms. Let $(A,H,\mathcal{Q})$ be a G-double structure and assume H is finite-dimensional with a dual basis $(e_i)_i\in H$ and $(e^i)_i\in H^*$. Then $A\#H^*=\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ is a T-coalgebra with the following structures:

The multiplication $m_{(\alpha,\beta)}$ and the unit of $A\#H^*(\alpha,\beta)$ are given, for any $a,b\in A$ and $h^*,g^*\in H^*$, by

$$(a\#h^*)(b\#g^*)=\sum_i\langle h^*,\zeta_\beta(b_{(-1)})e_i\zeta_\alpha(S^{-1}(b_{(1)}))\rangle ab_{(0)}\#e^ig^*, \tag{4.1}$$

$$1_{A\#H^*(\alpha,\beta)}=1_A\otimes\varepsilon_H. \tag{4.2}$$

The comultiplication and the counit of $A\#H^*$ are given by

$$\Delta_{(\alpha,\beta),(\gamma,\delta)}:A\#H^*((\alpha,\beta)\#(\gamma,\delta))\longrightarrow A\#H^*(\alpha,\beta)\otimes A\#H^*(\gamma,\delta),$$

$$\Delta_{(\alpha,\beta),(\gamma,\delta)}(a\#h^*)=(a_2\#\zeta^*_{\delta^{-1}}(h_1^*)\otimes(a_1\#\zeta^*_{\delta\alpha^{-1}\delta^{-1}}(h_2^*)), \tag{4.3}$$

$$\varepsilon_{A\#H^*}:A\#H^*\longrightarrow k,\quad \varepsilon_{A\#H^*}(a\#h^*)=(\varepsilon_A\otimes 1_H)(a\#h^*) \tag{4.4}$$

for all $a\in A$ and $h^*\in H^*$.

The antipode $S^{A\#H^*}=\{S^{A\#H^*}_{(\alpha,\beta)}:A\#H^*(\alpha,\beta)\longrightarrow A\#H^*((\alpha,\beta)^{-1})\}_{(\alpha,\beta)\in G}$ is given by

$$S^{A\#H^*}_{(\alpha,\beta)}(a\#h^*)=\sum_i\langle\zeta^*_{\beta^{-1}\alpha^{-1}}(S^*(h^*)),\zeta_{\beta^{-1}}(S^{-1}(a)_{(-1)})e_i,$$
$$\zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(S^{-1}(a)_{(1)}))>S^{-1}(a)_{(0)}\#e^i. \tag{4.5}$$

The crossing $\varphi=\{\varphi^{(\gamma,\delta)}_{(\alpha,\beta)}:A\#H^*(\gamma,\delta)\longrightarrow A\#H^*((\alpha,\beta)\#(\gamma,\delta)\#(\alpha,\beta)^{-1})\}$ is defined by

$$\varphi^{(\gamma,\delta)}_{(\alpha\beta)}(a\#c^*)=\xi_{\beta^{-1}\alpha}(a)\#\zeta^*_{\beta^{-1}\delta\alpha\delta^{-1}}(h^*). \tag{4.6}$$

Proof First, the multiplication is associative and the unit is $1_A\otimes\varepsilon_H$.

Second, it is straightforward to check that φ satisfies equation (2.10), (2.11) and (2.12), i.e., the following conditions hold:

φ is multiplicative, i.e., $\varphi_{(\alpha,\beta)}\circ\varphi_{(\gamma,\delta)}=\varphi_{(\alpha,\beta)\#(\gamma,\delta)}$, in particular $\varphi^{(\gamma,\delta)}_{(e,e)}=id$.

φ is compatible with Δ, i.e.,

$$\Delta_{(\mu,\nu)\#(\alpha,\beta)\#(\mu,\nu)^{-1},\,(\mu,\nu)\#(\gamma,\delta)\#(\mu,\nu)^{-1}}\circ\varphi^{(\alpha,\beta)\#(\gamma,\delta)}_{(\mu,\nu)}=(\varphi^{(\alpha,\beta)}_{(\mu,\nu)}\otimes\varphi^{(\alpha,\beta)}_{(\mu,\nu)})\circ\Delta_{(\alpha,\beta),\,(\gamma,\delta)}.$$

φ is compatible with ε, i.e., $\varepsilon\circ\varphi^{(e,e)}_{(\alpha,\beta)}=\varepsilon$ for any $(\alpha,\beta)\in G$.

Third, the coassociativity follows directly from the coassociativity of the comultiplication of A and H^* and the fact $\varphi_{(\alpha,\beta)}\circ\varphi_{(\gamma,\delta)}=\varphi_{(\alpha,\beta)\#(\gamma,\delta)}$. It is easy to check that $\varepsilon_{A\#H^*}$ is multiplicative.

Fourth, we show that $\Delta_{(\alpha,\beta),(\gamma,\delta)}$ is an algebra morphism, i.e., axiom (2.8) is satisfied. For any $a,b\in A$ and $h^*,g^*\in H^*$, we do calculations as follows:

$$\begin{aligned}
&\Delta_{(\alpha,\beta),(\gamma,\delta)}[(a\#h^*)(b\#g^*)]\\
=\;&\langle h^*,e_i^{\psi((\alpha,\beta)\#(\gamma,\delta)}\rangle(a_\psi b)_2\#\zeta^*_{\delta^{-1}}((e^id^*)_1)\otimes(a_\psi b)_1\#\zeta^*_{\delta\alpha^{-1}\delta^{-1}}((e^id^*)_2)\\
=\;&\langle h^*,\zeta_{\delta\beta}(b_{(-1)})e_je_i\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(b_{(1)}))\rangle a_2(b_{(0)})_2\#\zeta^*_{\delta^{-1}}(e^jg_1^*)\otimes a_1(b_{(0)})_1\#\zeta^*_{\delta\alpha^{-1}\delta^{-1}}(e^ig_2^*)\\
=\;&\langle h^*,\zeta_{\delta\beta}(b_{(-1)})\zeta_\delta(e_j)\zeta_{\delta\alpha\delta^{-1}}(e_i)\zeta_{\delta\alpha\delta^{-1}\gamma}(S^{-1}(b_{(1)}))\rangle a_2(b_{(0)})_2\\
&\#e^j\zeta^*_{\delta^{-1}}(g_1^*)\otimes a_1(b_{(0)})_1\#e^i\zeta^*_{\delta\alpha^{-1}\delta^{-1}}(g_2^*)\\
=\;&\langle\zeta^*_{\delta^{-1}}(h^*),\ \zeta_\beta((b_2)_{(-1)})e_j\zeta_\alpha(S^{-1}((b_2)_{(1)}))\zeta_{\alpha\delta^{-1}}(\zeta_\delta((b_1)_{(-1)})e_i\zeta_\gamma(S^{-1}((b_1)_{(1)})))\rangle\\
&a_2\,(b_2)_{(0)}\#e^j\zeta^*_{\delta^{-1}}(d_1^*)\otimes a_1\,(b_1)_{(0)}\#e^i\zeta_{\delta\alpha^{-1}\delta^{-1}}(g_2^*)\\
=\;&(a_2\#\zeta^*_{\delta^{-1}}(h_1^*)(b_2\#\zeta^*_{\delta^{-1}}(g_1^*)\otimes(a_1\#\zeta^*_{\delta\alpha^{-1}\delta^{-1}}(h_2^*)(b_1\#\zeta^*_{\delta\alpha^{-1}\delta^{-1}}(d_2^*)\\
=\;&\Delta_{(\alpha,\beta)}(a\#h^*)\Delta_{(\gamma,\delta)}(b\#g^*).
\end{aligned}$$

Finally, for all $(\alpha,\beta)\in G$, we have to check axiom (2.9). We now prove one of them as follows:

$$\begin{aligned}
&m_{(\alpha,\beta)^{-1}}\circ(S^{A\#H^*}_{(\alpha,\beta)}\otimes id_{(\alpha,\beta)^{-1}})\circ\Delta_{(\alpha,\beta),(\alpha,\beta)^{-1}}(a\#h^*)\\
=\;&S^{A\#H^*}_{(\alpha,\beta)}(a_2\#\zeta^*_\beta(h_1^*)(a_1\#\zeta^*_{\beta^{-1}\alpha^{-1}\beta}(h_2^*)\\
=\;&\sum_i\langle\zeta^*_{\beta^{-1}\alpha^{-1}}\zeta^*_\beta S^*(h_1^*),\zeta_{\beta^{-1}}(S^{-1}(a_2)_{(-1)})e_i\zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(S^{-1}(a_2)_{(1)}))\rangle\\
&(S^{-1}(a_2)_{(0)}\#e^i)(a_1\#\zeta^*_{\beta^{-1}\alpha^{-1}\beta}(h_2^*)\\
\overset{(4.1)}{=}\;&\sum_{i,j}\langle\zeta^*_{\beta^{-1}\alpha^{-1}\beta}S^*(h_1^*),\zeta_{\beta^{-1}}(S^{-1}(a_2)_{(-1)})e_i\zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(S^{-1}(a_2)_{(1)}))\rangle\\
&\langle e^i,\zeta_{\beta^{-1}}(a_{1(-1)})e_j\zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}(a_{1(1)}))\rangle(S^{-1}(a_2)_{(0)}a_{1(0)}\#e^j\zeta^*_{\beta^{-1}\alpha^{-1}\beta}(h_2^*))\\
=\;&\sum_{i,j}\langle\zeta^*_{\beta^{-1}\alpha^{-1}\beta}S^*(h_1^*),\zeta_{\beta^{-1}}((S^{-1}(a_2)a_1)_{(-1)})e_i\zeta_{\beta^{-1}\alpha^{-1}\beta}(S^{-1}((S^{-1}(a_2)a_1)_{(1)}))\rangle\\
&((S^{-1}(a_2)a_1)_{(0)}\#e^j\zeta^*_{\beta^{-1}\alpha^{-1}\beta}(h_2^*))\\
=\;&\varepsilon_A(a)\zeta^*_{\beta^{-1}\alpha^{-1}\beta}S^*(h_1^*)\zeta^*_{\beta^{-1}\alpha^{-1}\beta}(h_2^*))\\
=\;&1_{(\alpha,\beta)}\varepsilon_{A\otimes H^*}(a\#h^*),
\end{aligned}$$

and the other one can be verified in the similar way.

Theorem 4.2 Let G be a twisted semi-direct square group and and $\mathscr{Q}:H\otimes H\longrightarrow A\otimes A$ a linear map. Let $\xi:\pi\longrightarrow\mathrm{Aut}(A)$ and $\zeta:\pi\longrightarrow\mathrm{Aut}(H)$ be group homomorphisms. Let $(A,H,\mathscr{Q})$ be a G-double structure and H a finite-dimensional with a dual basis $(e_i)_i\in H$ and $(e^i)_i\in H^*$. Then the category ${}_A\mathcal{YD}^H$ is isomorphic to the category $\mathrm{Rep}(A\#H^*)$ of representations of $A\#H^*$ as braided T-categories. Moreover, $A\#H^* = \{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ is a quasitriangular T-coalgebra with the quasitriangular structure given by

$$\begin{aligned} & R = \{R_{(\alpha,\beta),(\gamma,\delta)} \\ = \; & \sum_{i,j} e^i\#\mathscr{Q}^1(e_i\otimes\zeta_\alpha(e_j))\otimes e^j\#\mathscr{Q}^2(e_i\otimes\zeta_\alpha(e_j))\in A\#H^*(\alpha,\beta)\otimes A\#H^*(\gamma,\delta)\} \end{aligned}$$

for all $\alpha,\beta,\gamma,\delta\in\pi$.

Proof Since $(A,H,\mathscr{Q})$ is a G-double structure we have the braided T-category ${}_A\mathcal{YD}^H_G$. The braiding on ${}_A\mathcal{YD}^H_G$ translates into a braiding on the category $Rep(A\#H^*)$ of representations of $A\#H^*$. But this means that $A\#H^* = \{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$ is a quasitriangular T-coalgebra . The invertible map $\mathscr{Q}: H\otimes H \longrightarrow A\otimes A$ satisfying the conditions (4), (5), (6) and (7) in Definition 3.12 induces a map

$$\widetilde{\mathscr{Q}}: k \longrightarrow A\#H^*(\alpha,\beta)\otimes A\#H^*(\gamma,\delta).$$

Then $\widetilde{\mathscr{Q}}(1)$ is just the corresponding $R_{(\alpha,\beta),(\gamma,\delta)}\in A\#H^*(\alpha,\beta)\otimes A\#H^*(\gamma,\delta)$.

In this case, we have the braiding on the category $Rep(A\#H^*)$:

$$c_{M,N}: M\otimes N\to {}^MN\otimes M,\ m\otimes n\mapsto [\tau_{(\gamma,\delta),(\alpha,\beta)}R_{(\alpha,\beta),(\gamma,\delta)}](n\otimes m)$$

for any $M\in {}_{A\#H^*(\alpha,\beta)}\mathscr{M}$ and $N\in {}_{A\#H^*(\gamma,\delta)}\mathscr{M}$.

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辫子T -范畴的构造

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摘要: 本文首先引入了一类新的范畴${}_A\mathcal{YD}^H_G$, 这个范畴是一簇范畴$\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$的非交并, 获得了范畴$\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$是一个辫子$T$ -范畴当且仅当$(A,H,\mathscr{Q})$是一个G -偶结构, 推广了2005年Panaite和Staic的主要结论. 最后, 当H是有限维时, 构造了一个拟三角T -余代数$\{A\#H^*(\alpha,\beta)\}_{(\alpha,\beta)\in G}$, 它的表示范畴与$\{{}_A\mathcal{YD}^H(\alpha,\beta)\}_{(\alpha,\beta)\in G}$是同构的.

关键词: 量子Yetter-Drinfeld模; 辫子T -范畴; 拟三角结构

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