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VALUE DISTRIBUTION OF q-SHIFT DIFFERENCE POLYNOMIALS

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Abstract: In this paper, we investigate the value distribution of q-shift difference polynomials of meromorphic function with zero order. By using the Nevanlinna theory, we obtain the following result. Let f be a transcendental meromorphic function with zero order, m be a non-negative integer, $q, a, c \in \mathbb{C} \setminus \{0\}, b \in \mathbb{C}$, $\alpha(z)$ be a small function of $f(z)$. If $f(qz+c)-f(z) \not\equiv 0$, $n \geq 5$, then both $f(z)^n(f(z)^m - a)[f(qz+c) - f(z)] - \alpha(z)$ and $f(z)^n + a[f(qz+c) - f(z)] - b$ have infinitely many zeros, which improve the conditions $n \geq 7$ of Theorem D and $n \geq 8$ of Theorem E.

Keywords: zero order; difference polynomial; small function; Borel exceptional value

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1 Introduction and Main Results

In this paper, $\sigma(f)$ is the order of f , $\lambda(f)$ is the convergence exponent of zeros of f.

In recent years, many papers [1, 2, 5, 6, 11, 13] focus on the difference of the complex domain and get some difference analogues of the value distribution theory of meromorphic function. The difference analogues of $f'(z)$ are shift-difference $\Delta_c f(z) = f(z+c) - f(z)$ and q-difference $\Delta_q f(z) = f(qz) - f(z)$ or $\Delta_q f(z) = f(qz+c) - f(z)$, where $\Delta_c f(z) \not\equiv 0$, $\Delta_q f(z) \not\equiv 0$. The q-shift difference has been studied by some scholars. In 2006, Halburd, Korhonen [12], Barnett and Morgan [5] obtained the difference analogues of the second main theorem of Nevanlinna theory, the lemma on the logarithmic derivative, Picard's theorem and Clunie and Mokhon'ko lemmas. Later, some researchers investigate the value distribution of difference polynomials. It is important for further study of the difference equation.

In 1959, Hayman [14] proved two famous results.

Theorem A If f is a transcendental meromorphic function, $n(\geq 3)$ is a positive integer, then $f(z)^n f'(z)$ takes every finite non-zero complex value infinitely often.

Theorem B If f is a transcendental meromorphic function, $n(\geq 5)$ is a positive integer, and $a(\neq 0)$ is a constant, then $f'(z) - af(z)^n$ takes every finite complex value b infinitely often.

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Theorem A also holds when $n = 1, 2$. The case $n = 2$ was settled by Mues [18]. Bergweiler and Eremenko [19] proved the case of $n = 1$. However, only in the case of $n = 4$ and $b = 0$ can the result of Theorem B be improved (see Mues [18]).

Zhang and Korhonen [2, Theorem 4.1] proved the following result.

Theorem C Let f be a transcendental meromorphic (resp. entire) function of zero order and q be a non-zero complex constant. Then for $n \geq 6$ (resp. $n \geq 2$), $f(z)^n f(qz)$ assumes every non-zero value $a \in \mathbb{C}$ infinitely often.

Liu and Cao [1,Theorem 1.3] proved the following result.

Theorem D Let f be a transcendental meromorphic (resp. entire) function with zero order, m, n be positive integers, a, q be non-zero complex constants. If $n \geq 7$ (resp. $n \geq 3$), then $f(z)^n(f(z)^m - a)[f(qz + c) - f(z)] - \alpha(z)$ has infinitely many zeros, where $\alpha(z)$ is a nonzero small function with respect to f.

Liu and Qi [6] proved the following result.

Theorem E Let f be a zero-order transcendental meromorphic function and a, q be nonzero complex constants. Then, for $n \geq 8, f(z)^n + a[f(qz + c) - f(z)]$ assumes every nonzero value $b \in \mathbb{C}$ infinitely often.

In this paper, we first improve the conditions of Theorems D and E in the following Theorems 1.1 and 1.2.

Theorem 1.1 Let f be a transcendental meromorphic function with zero order, m be a non-negative integer, n be a positive integer, $a, q \in \mathbb{C} \setminus \{0\}$. If $n \geq 5$, then $f(z)^n(f(z)^m - a)[f(qz + c) - f(z)] - \alpha(z)$ has infinitely many zeros, where $\alpha(z)$ is a nonzero small function with respect to f.

Theorem 1.2 Let f be a zero-order transcendental meromorphic function and a, q be nonzero complex constants. Then, for $n \geq 5, f(z)^n + a[f(qz + c) - f(z)]$ assumes every value $b \in \mathbb{C}$ infinitely often.

Then we consider the value distribution of $H_n(z) = f(z)^n[f(qz) - f(z)]$ in Theorems $1.3 - 1.6$, where f is a transcendental entire function with finite order.

Theorem 1.3 Let f be a transcendental entire function with finite order, n be a positive integer, $q \in \mathbb{C} \setminus \{0, 1\}$, $\Delta_q f(z) = f(qz) - f(z) \not\equiv 0$. If $\sigma(f)$, the order of $f(z)$, satisfies $q^{\sigma(f)} \neq 1$, then $H_n(z) = f(z)^n \Delta_q f(z)$ has infinitely many zeros.

Example 1.3.1 Let $f(z) = ze^{z^2}, q = -1$, then $H_n(z) = f(z)^n \Delta_q f(z) = -2z^{n+1}e^{(n+1)z^2}$ has only one zero, so the condition $q^{\sigma(f)} \neq 1$ in Theorem 3 is necessary.

Theorem 1.4 Let f be a transcendental entire function with finite order, $d(\neq 0)$ is a Borel exceptional value of f, $q \in \mathbb{C} \setminus \{0, 1\}$, $\Delta_q f(z) = f(qz) - f(z) \not\equiv 0$. Then $H(z) = f(z)[f(qz) - f(z)]$ assumes every value $a \in \mathbb{C}$ infinitely often, and $\lambda(H - a) = \sigma(f)$, where $\lambda(H - a)$ is the convergence exponent of zeros of $H(z) - a$.

Theorem 1.5 Let f be a transcendental entire function with finite order, $q \in \mathbb{C} \setminus \{0, 1\}$, $\Delta_q f(z) = f(qz) - f(z) \not\equiv 0$. If f has infinitely many multiple zeros, then $H(z) = f(z)[f(qz) - f(z)]$ assumes every value $a \in \mathbb{C}$ infinitely often.

Theorem 1.6 Let f be a transcendental entire function with finite order, $q \in \mathbb{C} \setminus \{0, 1\}$,

$\Delta_q f(z) = f(qz) - f(z) \not\equiv 0$. If there exists an infinite sequence $\{z_n\}$ satisfying $f(z_n) = f(qz_n) = 0$, then $H(z) = f(z)[f(qz) - f(z)]$ assumes every value $a \in \mathbb{C}$ infinitely often.

2 Some Lemmas

In the following lemmas, the logarithmic density of set E is defined by

$$\limsup_{r\to\infty} \frac{1}{\log r} \int_{[1,r]\cap E} \frac{1}{t} dt.$$

Similarly, the lower logarithmic density of set E is defined by

$$\liminf_{r\to\infty} \frac{1}{\log r} \int_{[1,r]\cap E} \frac{1}{t} dt.$$

For the proof of Theorem 1.1, we require the following Lemma 2.1 [6, Theorem 2.1] and Lemma 2.2 [10, Lemma 3.4].

Lemma 2.1 Let $f(z)$ be a meromorphic function of zero order, and let $c \in \mathbb{C}$. Then

$$m(r, \frac{f(qz+c)}{f(z)}) = o(T(r,f))$$

on a set of logarithmic density 1.

Lemma 2.2 If $f(z)$ is a non-constant zero order meromorphic function and $q \in \mathbb{C}\setminus\{0\}$, then

$$T(r, f(qz+c)) = (1+o(1))T(r, f(z)) + O(\log r)$$

on a set of lower logarithmic density 1.

For the proof of Theorem 1.2, we require the following Lemma 2.3 [10, Lemma 3.6].

Lemma 2.3 If $f(z)$ is a non-constant zero order meromorphic function and $q \in \mathbb{C}\setminus\{0\}$, then

$$N(r, f(qz+c)) = (1+o(1))N(r, f(z)) + O(\log r)$$

on a set of lower logarithmic density 1.

For the proof of Theorem 1.3 and Theorem 1.4, we require the following Lemma 2.4 [8, p.75–76] and Lemma 2.5 [8, Theorem 1.36].

Lemma 2.4 Suppose that $f_1(z), f_2(z), \cdots, f_n(z) (n \geq 2)$ are meromorphic functions and $g_1(z), g_2(z), \cdots, g_n(z)$ are entire functions satisfying the following conditions

(i) $\sum\limits_{j=1}^{n} f_j(z)e^{g_j(z)} \equiv 0$;

(ii) $g_j(z) - g_k(z)$ are not constants for $1 \leq j < k \leq n$;

(iii) For $1 \leq j \leq n, 1 \leq h < k \leq n$,

$$T(r, f_j) = o\{T(r, e^{g_h - g_k})\} (r \to \infty, r \notin E),$$

then $f_j(z) \equiv 0 (j = 1, 2, \cdots, n)$.

Lemma 2.5 If $f(z)$ is a meromorphic function in the complex plane, and $a_1(z), a_2(z), a_3(z)$ are three distinct small functions of $f(z)$. Then

$$T(r,f) \leq \sum_{j=1}^{3} \bar{N}(r, \frac{1}{f-a_j(z)}) + S(r,f).$$

3 Proofs of Theorem 1.1 and Theorem 1.2

Proof of Theorem 1.1 We set

$$F(z) = f(z)^n (f(z)^m - a)[f(qz+c) - f(z)]$$

and $\Delta_q f(z) = f(qz+c) - f(z)$, then

$$\begin{aligned}
(n+m)m(r,f) &\leq m(r, f^n(f^m - a)) + S(r,f) \\
&= m(r, \frac{F}{\Delta_q f}) + S(r,f) \\
&\leq m(r,F) + m(r, \frac{1}{\Delta_q f}) + S(r,f) \\
&= m(r,F) + T(r, \Delta_q f) - N(r, \frac{1}{\Delta_q f}) + S(r,f), \qquad (3.1) \\
(n+m)N(r,f) &= N(r, f^n(f^m - a)) = N(r, \frac{F}{\Delta_q f}) \\
&\leq N(r,F) + N(r, \frac{1}{\Delta_q f}) - \bar{N}_0(r) - \bar{N}_1(r), \qquad (3.2)
\end{aligned}$$

where $\bar{N}_0(r)$ is the counting function of zeros of both $F(z)$ and $\Delta_q f(z)$,$\bar{N}_1(r)$ is the counting function of poles of both $F(z)$ and $\Delta_q f(z)$. (3.1) and (3.2) yield

$$(n+m)T(r,f) \leq T(r,F) + T(r, \Delta_q f) - \bar{N}_0(r) - \bar{N}_1(r) + S(r,f). \qquad (3.3)$$

By Lemma 2.5, we have

$$T(r,F) \leq \bar{N}(r,F) + \bar{N}(r, \frac{1}{F}) + \bar{N}(r, \frac{1}{F - \alpha(z)}) + S(r,F). \qquad (3.4)$$

From $F(z) = f(z)^n (f(z)^m - a)\Delta_q f(z)$ and $\bar{N}_0(r)$ is the counting function of zeros of both $F(z)$ and $\Delta_q f(z)$, we have

$$\begin{aligned}
\bar{N}(r, \frac{1}{F}) &\leq \bar{N}(r, \frac{1}{f}) + \bar{N}(r, \frac{1}{f^m - a}) + \bar{N}_0(r) \\
&\leq (m+1)T(r,f) + \bar{N}_0(r) + O(1).
\end{aligned} \qquad (3.5)$$

From $F(z) = f(z)^n (f(z)^m - a)\Delta_q f(z)$ and $\bar{N}_1(r)$ is the counting function of poles of both $F(z)$ and $\Delta_q f(z)$, we have

$$\bar{N}(r,F) \leq \bar{N}(r,f) + \bar{N}_1(r) \leq T(r,f) + \bar{N}_1(r). \qquad (3.6)$$

Substituting (3.5) and (3.6) into (3.4), we have

$$\begin{aligned} T(r,F) &\le \bar{N}(r,F) + \bar{N}(r,\frac{1}{F}) + \bar{N}(r,\frac{1}{F-\alpha(z)}) + S(r,F) \\ &\le (m+2)T(r,f) + \bar{N}_0(r) + \bar{N}_1(r) + \bar{N}(r,\frac{1}{F-\alpha(z)}) + S(r,f). \end{aligned} \tag{3.7}$$

By Lemma 2.2, we have

$$T(r,\Delta_q f) \le T(r,f) + T(r,f(qz+c)) \le 2T(r,f) + S(r,f). \tag{3.8}$$

Substituting (3.7) and (3.8) into (3.3), we have

$$\begin{aligned} (n+m)T(r,f) &\le T(r,F) + T(r,\Delta_q f) - \bar{N}_0(r) - \bar{N}_1(r) + S(r,f) \\ &\le (m+4)T(r,f) + \bar{N}(r,\frac{1}{F-\alpha(z)}) + S(r,f) \end{aligned} \tag{3.9}$$

on a set of lower logarithmic density 1.

(3.9) implies that $F(z)-\alpha(z)$ has infinitely many zeros when $n \ge 5$. Thus the proof of Theorem 1.1 is complete.

Proof of Theorem 1.2 Denote $\Delta_q f(z) = f(qz+c) - f(z)$ and $\varphi = \frac{b-a\Delta_q f}{f^n}$.

We consider two cases

Case 1 $b - a\Delta_q f \equiv 0$. If $q \ne 1$, then $\Delta_q f = 0$ at the point $z_0 = \frac{-c}{q-1}$, contradicting $\Delta_q f \equiv \frac{b}{a}$. If $q = 1$, then any z_0 cannot be the pole of $f(z)$. Otherwise all points $z_0 + kc(k = 0, \pm1, \pm2, \cdots)$ are the poles of $f(z)$, which implies that the order of $f(z)$ is at least one, also contradicting the assumption of the Theorem 1.2. If $f(z)$ is a transcendental entire function with order zero, then $f(z)$ has infinitely many zeros. Hence $f(z)^n + a[f(qz+c) - f(z)]$ assumes value b infinitely often.

Case 2 $b - a\Delta_q f \not\equiv 0$. Then

$$nm(r,f) = m(r,f^n) = m(r,\frac{b-a\Delta_q f}{\varphi}) \le m(r,\frac{1}{\varphi}) + m(r, b - a\Delta_q f), \tag{3.10}$$

$$nN(r,f) = N(r,f^n) = N(r,\frac{b-a\Delta_q f}{\varphi}) \le N(r,\frac{1}{\varphi}) + N(r,b-a\Delta_q f) - \bar{N}_0(r) - \bar{N}_1(r), \tag{3.11}$$

where $\bar{N}_0(r)$ is the counting function of zeros of both φ and $b - a\Delta_q f$, $\bar{N}_1(r)$ is the counting function of poles of both φ and $b - a\Delta_q f$.

By (3.10) and (3.11), we have

$$\begin{aligned} nT(r,f) &\le T(r,\frac{1}{\varphi}) + T(r,b-a\Delta_q f) - \bar{N}_0(r) - \bar{N}_1(r) + O(1) \\ &\le T(r,\varphi) + T(r,b-a\Delta_q f) - \bar{N}_0(r) - \bar{N}_1(r). \end{aligned} \tag{3.12}$$

From $\varphi = \frac{b-a\Delta_q f}{f^n}$, we know that the poles of φ are generated by the zeros of f and the poles of both φ and $b - a\Delta_q f$, thus

$$\bar{N}(r,\varphi) \le \bar{N}(r,\frac{1}{f}) + \bar{N}_1(r). \tag{3.13}$$

From $\varphi = \frac{b-a\Delta_q f}{f^n}$, we know that the zeros of φ are generated by the poles of f and the zeros of both φ and $b - a\Delta_q f$, thus

$$\bar{N}(r, \frac{1}{\varphi}) \leq \bar{N}(r, f) + \bar{N}_0(r). \tag{3.14}$$

Applying the Nevanlinna second main theorem, thus from (3.13) and (3.14), we get

$$\begin{aligned} T(r,\varphi) &\leq \bar{N}(r,\varphi) + \bar{N}(r, \frac{1}{\varphi}) + \bar{N}(r, \frac{1}{\varphi - 1}) + S(r,f) \\ &\leq \bar{N}(r, \frac{1}{f}) + \bar{N}(r,f) + \bar{N}(r, \frac{1}{\varphi - 1}) + \bar{N}_0(r) + \bar{N}_1(r) + S(r,f). \end{aligned} \tag{3.15}$$

By Lemma 2.2, we have

$$T(r, b - a\Delta_q f) \leq 2T(r,f) + S(r,f). \tag{3.16}$$

Substituting (3.15) and (3.16) into (3.12), we have

$$\begin{aligned} nT(r,f) &\leq 2T(r,f) + \bar{N}(r,f) + \bar{N}(r, \frac{1}{f}) + \bar{N}(r, \frac{1}{\varphi - 1}) + S(r,f) \\ &\leq 4T(r,f) + \bar{N}(r, \frac{1}{\varphi - 1}) + S(r,f). \end{aligned} \tag{3.17}$$

That is

$$(n-4)T(r,f) \leq \bar{N}(r, \frac{1}{\varphi - 1}) + S(r,f) = \bar{N}(r, \frac{1}{f(z)^n + a[f(qz+c) - f(z)] - b}) + S(r,f)$$

on a set of lower logarithmic density 1.

The above inequality shows that $f(z)^n + a[f(qz+c) - f(z)] - b$ has infinitely many zeros when $n \geq 5$. Thus the proof of Theorem 1.2 is completed.

4 Proofs of Theorem 1.3–Theorem 1.6

Proof of Theorem 1.3 Suppose that $f(z)$ has infinitely many zeros, then $H(z) = f(z)\Delta_q f(z)$ has infinitely many zeros. If $f(z)$ has only finitely many zeros, then we can set

$$f(z) = P(z)e^{h(z)},$$

where $P(z)(\not\equiv 0), h(z)$ are polynomials and $\sigma(f) = \deg h(z)(\geq 1)$, thus

$$\Delta_q f(z) = P(qz)e^{h(qz)} - P(z)e^{h(z)} = e^{h(z)}[P(qz)e^{h(qz)-h(z)} - P(z)].$$

By the condition $q^{\sigma(f)} \neq 1$, we obtain that $h(qz) - h(z) \not\equiv$ constant. Thus using Lemma 2.5, we get that $P(qz)e^{h(qz)-h(z)} - P(z)$ has infinitely many zeros, that is, $H(z) = f(z)\Delta_q f(z)$ has infinitely many zeros.

Thus Theorem 1.3 is proved.

Proof of Theorem 1.4 Suppose $d(\neq 0)$ is a Borel exceptional value of $f(z)$, we can set

$$f(z) = d + p(z)e^{\alpha z^k},$$

where k is a positive integer, $\alpha(\neq 0)$ is a complex constant and $p(z)(\not\equiv 0)$ is an entire function satisfying

$$\sigma(p) < \sigma(f) = k,$$

thus

$$f(qz) = d + p(qz)e^{\alpha q^k z^k} = d + p(qz)e^{\alpha(q^k-1)z^k}e^{\alpha z^k} = d + p(qz)p_1(z)e^{\alpha z^k},$$

where $p_1(z) = e^{\alpha(q^k-1)z^k}, p_1(z)(\not\equiv 0)$ is an entire function satisfying $\sigma(p_1(z)) = k$ or 0.

So

$$H(z) = p(z)[p(qz)p_1(z) - p(z)]e^{2\alpha z^k} + d[p(qz)p_1(z) - p(z)]e^{\alpha z^k}. \tag{4.1}$$

Since $f(qz) - f(z) \not\equiv 0$, this gives

$$p(qz)p_1(z) - p(z) \not\equiv 0. \tag{4.2}$$

On combining (4.1) with (4.2), we deduce

$$\sigma(H) = \sigma(f) = k. \tag{4.3}$$

If $d^*(\in \mathbb{C})$ is a Borel exceptional value of $H(z)$, then we can set

$$H(z) = d^* + p^*(z)e^{\beta z^k}, \tag{4.4}$$

where $\beta(\neq 0)$ is a complex constant, $p^*(z)(\not\equiv 0)$ is an entire function satisfying

$$\sigma(p^*(z)) < \sigma(H(z)) = k,$$

(4.1) and (4.4) give that

$$p(z)[p(qz)p_1(z) - p(z)]e^{2\alpha z^k} + d[p(qz)p_1(z) - p(z)]e^{\alpha z^k} - p^*(z)e^{\beta z^k} - d^* = 0. \tag{4.5}$$

If $\beta \neq 2\alpha$ or $\beta \neq \alpha$, combining Lemma 2.4 with (4.5), we get

$$p(qz)p_1(z) - p(z) \equiv 0,$$

this contradicts with (4.2).

Thus, any finite value a is not the Borel exceptional value of $H(z)$. Hence $H(z)$ assumes every value $a \in \mathbb{C}$ infinitely often.

It follows from (4.3) that $\lambda(H - a) = \sigma(H) = \sigma(f)$. Thus Theorem 1.3 is proved.

Proofs of Theorem 1.5 and Theorem 1.6 In the following, we only give the proof of Theorem 1.5. Theorem 1.6 can be proved similarly, we omit its proof.

Obviously, if $a = 0$, noting that $\Delta_q f(z)$ is an entire function and $f(z)$ has infinitely many zeros, we get that $H(z)$ has infinitely many zeros.

Now assume that $a \neq 0$ and $H(z) - a$ has only finitely many zeros, then we can set

$$H(z) - a = f(z)f(qz) - f(z)^2 - a = p(z)e^{h(z)}, \tag{4.6}$$

where $p(z), h(z)$ are polynomials. It follows from the condition of Theorem 1.5 that $H(z)$ is a transcendental entire function, hence $p(z) \not\equiv 0$ and $\deg h(z) \geq 1$. Taking derivatives in both sides of (4.6) and eliminating $e^{h(z)}$, we get

$$\frac{[f(z)f(qz)]'}{f(z)f(qz)} - \frac{[2f(z)]'}{f(qz)} = \frac{p'(z) + p(z)h'(z)}{p(z)}\{1 - \frac{f(z)}{f(qz)} - \frac{a}{f(z)f(qz)}\}. \tag{4.7}$$

From $p(z), h(z)$ are polynomials satisfying $p(z) \not\equiv 0$ and $\deg h(z) \geq 1$, we get $p'(z) + p(z)h'(z) \not\equiv 0$. Since $f(z)$ has infinitely many multiple zeros, so there exists a sufficiently large point z_0, where z_0 is the zero of $f(z)$ with multiplicity $k \geq 2$, $p'(z_0) + p(z_0)h'(z_0) \neq 0$ and $p(z_0) \neq 0$. Next we discuss the following two cases.

Case 1 If z_0 is the zero of $f(qz)$ with multiplicity $k_q \geq 1$, then z_0 is the simple pole of $\frac{[f(z)f(qz)]'}{f(z)f(qz)}$ and the pole of $-\frac{[2f(z)]'}{f(qz)}$ with multiplicity $k_q - k + 1$. However, z_0 is the pole of $\frac{f(z)}{f(qz)}$ with multiplicity $k_q - k$ and the pole of $\frac{a}{f(z)f(qz)}$ with multiplicity $k_q + k$. This shows that (4.7) is a contradiction.

Case 2 If $f(qz_0) \neq 0$, then z_0 is the simple pole of $\frac{[f(z)f(qz)]'}{f(z)f(qz)}$ and the zero of $-\frac{[2f(z)]'}{f(qz)}$. However, z_0 is the zero of $\frac{f(z)}{f(qz)}$ and the pole of $\frac{a}{f(z)f(qz)}$ with multiplicity $k \geq 2$. This shows that (4.7) is also a contradiction.

Then $H(z)$ assumes every value $a \in \mathbb{C}$ infinitely often.

Thus the proof of Theorem 1.5 is completed.

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q -差分多项式的值分布

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摘要: 本文研究了零级的亚纯函数的q -差分多项式的值分布. 利用Nevanlinna理论, 得到了以下结果. 设f是零级的超越亚纯函数, m是非负整数, $q,a,c\in\mathbb{C}\setminus\{0\},b\in\mathbb{C}$, $\alpha(z)$是$f(z)$的小函数.如果$f(qz+c)-f(z)\not\equiv 0$, $n\geq 5$, 则$f(z)^n(f(z)^m-a)[f(qz+c)-f(z)]-\alpha(z)$和$f(z)^n+a[f(qz+c)-f(z)]-b$有无穷多个零点. 该结果改进了定理D中的$n\geq 7$和定理E中的$n\geq 8$.

关键词: 零级; 差分多项式; 小函数; Borel例外值

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