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NOTES ON THE CONVERGENCE OF ORLICZ CONVEX BODIES

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Abstract: In this paper, we investigate the characters of Orlicz projection body and Orlicz centroid body. By geometric analysis, we obtain the continuities of the Orlicz projection operator and Orlicz centroid operator.

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1 Introduction and Main Results

The classical Brunn-Minkowski theory emerged at the turn of the 19th into the 20th century, when Minkowski began his study of the volume of the Minkowski sum of convex bodies. In the early 1960's, Firey (see e.g. Schneider [13]) introduced an L_p-extension of Minkowski's addition (now known as Firey-Minkowski L_p-addition) of convex bodies. In the middle of 1990s, it was shown in [9,10], that a study of the volume of these Firey-Minkowski L_p-combinations leads to an embryonic L_p-Brunn-Minkowski theory. This theory was expanded rapidly (see e.g. [1–2, 4–6, 8–11, 14]).

The works of Haberl et al. [4–6] and the recent work of Ludwig and Reitzner [8], made it apparent that the time is ripe for the next step in the evolution of the Brunn-Minkowski theory towards the Orlicz-Brunn-Minkowski theory. Lutwak, Yang and Zhang recently introduced the notions of Orlicz projection bodies and Orlicz centroid bodies. It was shown in [11, 12] that a study of the Orlicz Petty projection inequality and Orlicz centroid inequality leads to the Orlicz Brunn-Minkowski theory which is a natural extension of the L_p-Brunn-Minkowski theory. Work of Haberl et al. [7] proved the even Orlicz Minkowski problem. Lutwak, Yang and Zhang (see [12]) established the Orlicz centroid inequality for

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convex bodies and conjectured that their inequality can be extended to star bodies. In [15], Zhu confirmed this conjecture. In [16], the reverse form of the Orlicz Busemann–Petty centroid inequalities was obtained in the two-dimensional case.

Let $\phi : \mathbb{R} \to [0, \infty)$ be an even strictly convex function such that $\phi(0) = 0$. The class of such a ϕ will be denoted by $\mathcal{C}$. Let K be a convex body (i.e., a compact, convex set with non-empty interior) in $\mathbb{R}^n$ that contains the origin in its interior. Denote by $\mid K \mid$ the volume of K. The Orlicz centroid body $\Gamma_\phi K$ of K, as defined in [12], is the convex body whose support function at $x \in \mathbb{R}^n$ is given by

$$h_{\Gamma_\phi K}(x) = \inf\{\lambda > 0 : \frac{1}{\mid K \mid} \int_K \phi\left(\frac{x \cdot y}{\lambda}\right) dy \leq 1\}, \tag{1.1}$$

where $x \cdot y$ denotes the standard inner product of x and y in $\mathbb{R}^n$ and the integration is with respect to Lebesgue measure in $\mathbb{R}^n$.

We say that a sequence $\{\phi_i\}$, where the $\phi_i \in \mathcal{C}$, is such that $\phi_i \to \phi_0 \in \mathcal{C}$ provided

$$\mid \phi_i - \phi_0 \mid_I := \max_{t \in I} \mid \phi_i(t) - \phi_0(t) \mid \to 0$$

for every compact interval $I \subset \mathbb{R}$.

We get the continuity of Orlicz centroid operator by the definition of the Orlicz centroid body as follows:

Theorem 1 Suppose $\phi_i \in \mathcal{C}$ and K_j is a star body (about the origin) in $\mathbb{R}^n$. If $\phi_i \to \phi \in \mathcal{C}$ and $K_j \to K$, then $\Gamma_{\phi_i} K_j \to \Gamma_\phi K$.

Lutwak, Yang and Zhang also established the definition of the Orlicz projection body $\Pi_\phi K$ of K, whose support function is given by (see [11])

$$h_{\Pi_\phi K}(x) = \inf\{\lambda > 0 : \int_{S^{n-1}} \phi\left(\frac{x \cdot u}{\lambda h_K(u)}\right) h_K(u) dS(u) \leq n \mid K \mid\}. \tag{1.2}$$

For $c > 0$, we have

$$\Pi_\phi(cK) = \frac{1}{c}\Pi_\phi K. \tag{1.3}$$

We get the continuity of Orlicz projection operator by the definition of the Orlicz projection body as follows:

Theorem 2 Suppose $\phi_i \in \mathcal{C}$ and K_j is a convex body in $\mathbb{R}^n$ that contains the origin in its interior. If $\phi_i \to \phi \in \mathcal{C}$ and $K_j \to K$, then $\Pi_{\phi_i} K_j \to \Pi_\phi K$.

2 Preliminaries

In this section we collect some basic well-known facts that we will use in the proofs of our main results. For references about the Brunn-Minkowski theory, see [3, 13].

Let $\rho(K, \cdot) = \rho_K : \mathbb{R}^n \backslash \{0\} \to [0, \infty)$ denote the radial function of the set $K \subset \mathbb{R}^n$, star-shaped about the origin; i.e. $\rho_K(x) = \max\{\lambda > 0 : \lambda x \in K\}$. If ρ_K is strictly positive and continuous, then we call K a star body and we denote the class of star bodies (about the origin) in $\mathbb{R}^n$ by $\mathcal{S}_0^n$. If $c > 0$, then obviously for the dilate $cK = \{cx : x \in K\}$ we have

$$\rho_{cK} = c\rho_K. \tag{2.1}$$

Let $h(K, \cdot) = h_K : \mathbb{R}^n \to \mathbb{R}$ denote the support function of the convex body K in $\mathbb{R}^n$, i.e., $h_K(x) = \max\{x \cdot y : y \in K\}$, we have

$$h_{cK}(x) = ch_K(x) \quad \text{and} \quad h_K(cx) = ch_K(x). \tag{2.2}$$

For $\phi \in \mathcal{C}$ define $\phi^\star \in \mathcal{C}$ by

$$\phi^\star(t) = \int_0^1 \phi(ts)ds^n, \tag{2.3}$$

where $ds^n = ns^{n-1}ds$. Obviously, $\phi_i \to \phi_0 \in \mathcal{C}$ implies $\phi_i^\star \to \phi_0^\star$.

It will be helpful to also use the alternate definition of Orlicz centroid body (see [12]):

$$h_{\Gamma_\phi K}(x) = \inf\{\lambda > 0 : \int_{\mathcal{S}^{n-1}} \phi^\star\left(\frac{1}{\lambda}(x \cdot u)\rho_K(u)\right) dV_K^*(u) \leq 1\}, \tag{2.4}$$

where $\phi^\star$ is defined by (2.3) and dV_K^* is the volume-normalized dual conical measure of K, defined by $\mid K \mid dV_K^* = \frac{1}{n}\rho_K^n dS$, where dS is Lebesgue measure on S^{n-1} (i.e., $(n-1)$-dimensional Hausdorff measure). For $c > 0$, an immediate consequence of definitions (2.4) and (2.1) is the fact that

$$\Gamma_\phi cK = c\Gamma_\phi K. \tag{2.5}$$

Lemma 2.1 (see [12]) Suppose $K \in S_0^n$ and $u_0 \in S^{n-1}$. Then

$$\int_{S^{n-1}} \phi^\star\left(\frac{1}{\lambda_0}(u_0 \cdot v)\rho_K(v)\right) dV_K^*(v) = 1$$

if and only if $h_{\Gamma_\phi K}(u_0) = \lambda_0$.

Associated with each $\phi \in \mathcal{C}$ is $c_\phi \in (0, \infty)$ defined by

$$c_\phi = \min\{c > 0 : \max\{\phi(c), \phi(-c)\} \leq 1\}.$$

Throughout $B = \{x \in \mathbb{R}^n : \mid x \mid \leq 1\}$ will denote the unit ball centered at the origin, and $\omega_n = \mid B \mid$ will denote its n-dimensional volume. We shall make use of the trivial fact that for $u_0 \in S^{n-1}$,

$$\omega_{n-1} = \int_{\mathcal{S}^{n-1}} (u_0 \cdot u)_+ dS(u) = \frac{1}{2}\int_{\mathcal{S}^{n-1}} \mid u_0 \cdot u \mid dS(u),$$

where $(t)_+ = \max\{t, 0\}$ for $t \in \mathbb{R}$, and where S denotes Lebesgue measure on S^{n-1}, i.e., S is $(n-1)$-dimensional Hausdorff measure.

Lemma 2.2 (see [12]) If $K \in S_0^n$, then $\frac{\omega_{n-1} r_K^{n+1}}{nc_{\phi^\star}|K|} \leq h_{\Gamma_\phi K}(u) \leq \frac{R_K}{c_{\phi^\star}}$ for all $u \in S^{n-1}$, where the real numbers R_K and r_K are defined by

$$R_K = \max_{u \in S^{n-1}} \rho_K(u) \text{ and } r_K = \min_{u \in S^{n-1}} \rho_K(u).$$

It will be helpful to also use the alternate definition of Orlicz projection body (see [11]):

$$h_{\Pi_\phi K}(x) = \inf\{\lambda > 0 : \int_{\mathcal{S}^{n-1}} \phi\left(\frac{1}{\lambda}(x \cdot u)\rho_{K^*}(u)\right) dV_k(u) \leq 1\},$$

if $K \in \mathcal{K}_0^n$, then the polar body K^* is defined by

$$K^* = \{x \in \mathbb{R}^n : x \cdot y \leq 1 \text{ for all } y \in K\},$$

it will be convenient to use the volume-normalized conical measure V_K defined by

$$| K | \, dV_K = \frac{1}{n} h_K dS_K.$$

Lemma 2.3 (see[11]) Suppose $\phi \in \mathcal{C}$ and $K \in \mathcal{K}_0^n$. If $x_0 \in \mathbb{R}^n \setminus \{0\}$, then

$$\int_{S^{n-1}} \phi\left(\frac{x_0 \cdot u}{\lambda_0 h_K(u_0)}\right) dV_K(v) = 1$$

if and only if $h_{\Pi_\phi K}(x_0) = \lambda_0$.

Lemma 2.4 (see[11]) If $\phi \in \mathcal{C}$ and $K \in \mathcal{K}_0^n$, then

$$\frac{1}{2nc_\phi R_K} \leq h_{\Pi_\phi K}(u) \leq \frac{1}{c_\phi r_K}$$

for all $u \in S^{n-1}$, where the real numbers R_K and r_K are defined by

$$R_K = \max_{u \in S^{n-1}} h_K(u) \text{ and } r_K = \min_{u \in S^{n-1}} h_K(u).$$

3 Proof of Main Theorems

Theorem 3.1 Suppose $\phi_i \in \mathcal{C}$ and $K_j \in {S_0}^n$. If $\phi_i \to \phi \in \mathcal{C}$ and $K_j \to K \in {S_0}^n$, then $\Gamma_{\phi_i} K_j \to \Gamma_\phi K$.

Proof (1) First, for fixed $j \in N^+$ (the set of all the positive integer), suppose $K_j \in S_0^n$ and $u_0 \in S^{n-1}$. We will show that $h_{\Gamma_{\phi_i} K_j}(u_0) \to h_{\Gamma_\phi K_j}(u_0)$. Let $h_{\Gamma_{\phi_i} K_j}(u_0) = \lambda_i$ and note that Lemma 2.2 gives

$$\frac{\omega_{n-1} r_{K_j}^{n+1}}{nc_{\phi_i^\star} \mid K_j \mid} \leq \lambda_i \leq \frac{R_{K_j}}{c_{\phi_i^\star}}.$$

Since $\phi_i^\star \to \phi^\star \in \mathcal{C}$, we have $c_{\phi_i^\star} \to c_{\phi^\star} \in (0, \infty)$ and thus there exist a, b such that $0 < a \leq \lambda_i \leq b < \infty$ for all i.

To show that the bounded sequence $\{\lambda_i\}$ converges to $h_{\Gamma_\phi K_j}(u_0)$, we show that every convergent subsequence of $\{\lambda_i\}$ converges to $h_{\Gamma_\phi K_j}(u_0)$. Denote an arbitrary convergent subsequence of $\{\lambda_i\}$ by $\{\lambda_i\}$ as well, and suppose that for this subsequence we have $\lambda_i \to \lambda_*$. Obviously, $0 < a \le \lambda_* \le b$. Since $h_{\Gamma_{\phi_i} K_j}(u_0) = \lambda_i$, Lemma 2.1 gives

$$1 = \int_{S^{n-1}} \phi_i^\star \left(\frac{u_0 \cdot u}{\lambda_i} \rho_{K_j}(u) \right) dV_{K_j}^*(u).$$

This, together with $\phi_i^\star \to \phi^\star \in \mathcal{C}$ and $\lambda_i \to \lambda_*$, gives

$$1 = \int_{S^{n-1}} \phi^\star \left(\frac{u_0 \cdot u}{\lambda_*} \rho_{K_j}(u) \right) dV_{K_j}^*(u).$$

By Lemma 2.1 this gives $h_{\Gamma_\phi K_j}(u_0) = \lambda_*$. This shows that $h_{\Gamma_{\phi_i} K_j}(u_0) \to h_{\Gamma_\phi K_j}(u_0)$.

Therefore, for any $\varepsilon > 0$, there exists $N_1 \in N^+$, for all $i > N_1$, we have

$$| \, h_{\Gamma_{\phi_i} K_j}(u_0) - h_{\Gamma_\phi K_j}(u_0) \, | < \frac{\varepsilon}{2}.$$

(2) Next, suppose $u_0 \in S^{n-1}$, we will show that

$$h_{\Gamma_\phi K_j}(u_0) \to h_{\Gamma_\phi K}(u_0).$$

Let $h_{\Gamma_\phi K_j}(u_0) = \lambda_j$, and note Lemma 2.2 gives

$$\frac{\omega_{n-1} r_{K_j}^{n+1}}{n c_{\phi^\star} \mid K_j \mid} \le \lambda_j \le \frac{R_{K_j}}{c_{\phi^\star}}.$$

Since $K_j \to K \in S_0^n$, we have $r_{K_j} \to r_K > 0$ and $R_{K_j} \to R_K < \infty$ and thus there exist c, d such that $0 < c \le \lambda_j \le d < \infty$, for all j. To show that the bounded sequence $\{\lambda_j\}$ converges to $h_{\Gamma_\phi K}(u_0)$, we show that every convergent subsequence of $\{\lambda_j\}$ converges to $h_{\Gamma_\phi K}(u_0)$. Denote an arbitrary convergent subsequence of $\{\lambda_j\}$ by $\{\lambda_j\}$ as well, and suppose that for this subsequence we have $\lambda_j \to \lambda_\diamond$. Obviously, $c \le \lambda_\diamond \le b$. Let $\bar{K}_j = \lambda_j^{-1} K_j$. Since $\lambda_j^{-1} \to \lambda_\diamond^{-1}$ and $K_j \to K$, we have

$$\bar{K}_j \to \lambda_\diamond^{-1} K.$$

Now (2.5), and the fact that $h_{\Gamma_\phi K_j}(u_0) = \lambda_j$, shows that $h_{\Gamma_\phi \bar{K}_j}(u_0) = 1$, i.e,

$$\int_{S^{n-1}} \phi^\star \left((u_0 \cdot u) \rho_{\bar{K}_j}(u) \right) dV_{\bar{K}_j}^*(u) = 1$$

for all j. But $\bar{K}_j \to \lambda_\diamond^{-1} K$ and the continuity of $\phi^\star$ now give

$$\int_{S^{n-1}} \phi^\star \left((u_0 \cdot u) \rho_{\lambda_\diamond^{-1} K}(u) \right) dV_{\lambda_\diamond^{-1} K}^*(u) = 1,$$

which by Lemma 2.1 give $h_{\Gamma_\phi \lambda_\diamond^{-1} K}(u_0) = 1$. This (2.5) and (2.2) now give $h_{\Gamma_\phi K}(u_0) = \lambda_\diamond$. This shows that $h_{\Gamma_\phi K_j}(u_0) \to h_{\Gamma_\phi K}(u_0)$.

Therefore, for any $\varepsilon > 0$, there exists $N_2 \in N^+$ for all $j > N_2$, we have

$$| \, h_{\Gamma_\phi K_j}(u_0) - h_{\Gamma_\phi K}(u_0) \, | < \frac{\varepsilon}{2}.$$

(3) To sum up, for all $\varepsilon > 0$, there exists $N = \max\{N_1, N_2\} \in N^+$ for all $i, j > N$, we have

$$\begin{aligned} & \mid h_{\Gamma_{\phi_i} K_j}(u_0) - h_{\Gamma_\phi K}(u_0) \mid \\ \leq \; & \mid h_{\Gamma_{\phi_i} K_j}(u_0) - h_{\Gamma_\phi K_j}(u_0) \mid + \mid h_{\Gamma_\phi K_j}(u_0) - h_{\Gamma_\phi K}(u_0) \mid < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon, \\ & h_{\Gamma_{\phi_i} K_j}(u_0) \to h_{\Gamma_\phi K}(u_0). \end{aligned}$$

Hence $\Gamma_{\phi_i} K_j \to \Gamma_\phi K$.

From the proof of Theorem 3.1, we can obtain the following two results that proved by Lutwak, Yang and Zhang (see [12]).

Corollary 3.2 Suppose $\phi \in \mathcal{C}$ and $K_j \in {S_0}^n$. If $K_j \to K \in {S_0}^n$, then $\Gamma_\phi K_j \to \Gamma_\phi K$.

Corollary 3.3 Suppose $\phi_i \in \mathcal{C}$ and $K \in {S_0}^n$. If $\phi_i \to \phi \in \mathcal{C}$, then $\Gamma_{\phi_i} K \to \Gamma_\phi K$.

Now, we prove Theorem 2 that is illustrated in Section 1, it is just the following theorem.

Theorem 3.4 Suppose $\phi_i \in \mathcal{C}$ and $K_j \in {\mathcal{K}_0}^n$. If $\phi_i \to \phi \in \mathcal{C}$ and $K_j \to K \in {\mathcal{K}_0}^n$, then $\Pi_{\phi_i} K_j \to \Pi_\phi K$.

Proof (1) First, for fixed $j \in N^+$, suppose $K_j \in \mathcal{K}_0^n$ and $u_0 \in S^{n-1}$. We will show that $h_{\Pi_{\phi_i} K_j}(u_0) \to h_{\Pi_\phi K_j}(u_0)$. Let $h_{\Pi_{\phi_i} K_j}(u_0) = \lambda_i$, and note that Lemma 2.4 gives

$$\frac{1}{2nc_{\phi_i} R_{K_j}} \leq \lambda_i \leq \frac{1}{c_{\phi_i} r_{K_j}}.$$

Since $\phi_i \to \phi \in \mathcal{C}$, we have $c_{\phi_i} \to c_\phi \in (0, \infty)$ and thus there exist a, b such that $0 < a \leq \lambda_i \leq b < \infty$ for all i. To show that the bounded sequence $\{\lambda_i\}$ converges to $h_{\Pi_\phi K_j}(u_0)$, we show that every convergent subsequence of $\{\lambda_i\}$ converges to $h_{\Pi_\phi K_j}(u_0)$. Denote an arbitrary convergent subsequence of $\{\lambda_i\}$ by $\{\lambda_i\}$ as well, and suppose that for this subsequence we have $\lambda_i \to \lambda_*$. Obviously, $0 < a \leq \lambda_* \leq b$. Since $h_{\Pi_{\phi_i} K_j}(u_0) = \lambda_i$, Lemma 2.3 gives

$$1 = \int_{S^{n-1}} \phi_i \left(\frac{u_0 \cdot u}{\lambda_i h_{K_j}(u)} \right) dV_{K_j}(u).$$

This, together with the facts that $\phi_i \to \phi \in \mathcal{C}$ and $\lambda_i \to \lambda_* \in (0, \infty)$, gives

$$1 = \int_{S^{n-1}} \phi \left(\frac{u_0 \cdot u}{\lambda_* h_{K_j}(u)} \right) dV_{K_j}(u).$$

When combined with Lemma 2.3, this gives the desired $h_{\Pi_\phi K_j}(u_0) = \lambda_*$, and completes the argument showing that $h_{\Pi_{\phi_i} K_j}(u_0) \to h_{\Gamma_\phi K_j}(u_0)$.

Therefore, for all $\varepsilon > 0$, there exists $N_1 \in N^+$, when $i > N_1$, we have

$$\mid h_{\Pi_{\phi_i} K_j}(u_0) - h_{\Pi_\phi K_j}(u_0) \mid < \frac{\varepsilon}{2}.$$

(2) Next, suppose $u_0 \in S^{n-1}$, we will show that $h_{\Pi_\phi K_j}(u_0) \to h_{\Pi_\phi K}(u_0)$. Let

$$h_{\Pi_\phi K_j}(u_0) = \lambda_j,$$

and note Lemma 2.4 gives $\frac{1}{2nc_\phi R_{K_j}} \leq \lambda_j \leq \frac{1}{c_\phi r_{K_j}}$. Since $K_j \to K \in \mathcal{K}_0^n$, we have $r_{K_j} \to r_K > 0$ and $R_{K_j} \to R_K < \infty$ and thus there exist c, d such that $0 < c \leq \lambda_j \leq d < \infty$ for all j. To show that the bounded sequence $\{\lambda_j\}$ converges to $h_{\Pi_\phi K}(u_0)$, we show that every convergent subsequence of $\{\lambda_j\}$ converges to $h_{\Pi_\phi K}(u_0)$. Denote an arbitrary convergent subsequence of $\{\lambda_j\}$ by $\{\lambda_j\}$ as well, and suppose that for this subsequence we have $\lambda_j \to \lambda_\diamond$. Obviously, $0 < c \leq \lambda_\diamond \leq b$. Let $\bar{K}_j = \lambda_j K_j$. Since $\lambda_j \to \lambda_\diamond$ and $K_j \to K$, we have

$$\bar{K}_j \to \lambda_\diamond K.$$

The fact that $h_{\Pi_\phi K_j}(u_0) = \lambda_j$, together with (2.2) and (1.3), shows that $h_{\Pi_\phi \bar{K}_j}(u_0) = 1$, i.e.,

$$\int_{S^{n-1}} \phi\left(\frac{u_0 \cdot u}{h_{\bar{K}_j}(u)}\right) dV_{\bar{K}_j}(u) = 1$$

for all j. But $\bar{K}_j \to \lambda_\diamond K$ implies that the functions $h_{\bar{K}_j} \to h_{\lambda_\diamond K}$, uniformly, and the measures $S_{\bar{K}_j} \to S_{\lambda_\diamond K}$, weakly. This in turn implies that the measures $V_{\bar{K}_j} \to V_{\lambda_\diamond K}$, weakly, and hence using the continuity of ϕ we have

$$\int_{S^{n-1}} \phi\left(\frac{u_0 \cdot u}{h_{\lambda_\diamond K}(u)}\right) dV_{\lambda_\diamond K}(u) = 1,$$

which by Lemma 2.3 give $h_{\Pi_\phi \lambda_\diamond K}(u_0) = 1$. This, together with (2.2) and (1.3), yields the desired $h_{\Pi_\phi K}(u_0) = \lambda_\diamond$, and shows that $h_{\Pi_\phi K_j}(u_0) \to h_{\Pi_\phi K}(u_0)$.

Therefore, for any $\varepsilon > 0$, there exists $N_2 \in N^+$, for all $j > N_2$, we have

$$\mid h_{\Pi_\phi K_j}(u_0) - h_{\Pi_\phi K}(u_0) \mid < \frac{\varepsilon}{2}.$$

(3) To sum up, for all $\varepsilon > 0$, there exists $N = \max\{N_1, N_2\} \in N^+$, for all $i, j > N$, we have

$$\begin{aligned} & \mid h_{\Pi_{\phi_i} K_j}(u_0) - h_{\Pi_\phi K}(u_0) \mid \\ \leq\ & \mid h_{\Pi_{\phi_i} K_j}(u_0) - h_{\Pi_\phi K_j}(u_0) \mid + \mid h_{\Pi_\phi K_j}(u_0) - h_{\Pi_\phi K}(u_0) \mid < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon, \\ & h_{\Pi_{\phi_i} K_j}(u_0) \to h_{\Pi_\phi K}(u_0), \end{aligned}$$

Hence $\Pi_{\phi_i} K_j \to \Pi_\phi K$.

From the proof of Theorem 3.4, we can obtain the following results that were proved by Lutwak, Yang and Zhang (see [11]).

Corollary 3.5 Suppose $\phi \in \mathcal{C}$ and $K_j \in {\mathcal{K}_0}^n$. If $K_j \to K \in {\mathcal{K}_0}^n$, then $\Pi_\phi K_j \to \Pi_\phi K$.

Corollary 3.6 Suppose $\phi_i \in \mathcal{C}$ and $K \in {\mathcal{K}_0}^n$. If $\phi_i \to \phi \in \mathcal{C}$, then $\Pi_{\phi_i} K \to \Pi_\phi K$.

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关于Orlicz 凸体的收敛性的注

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摘要: 本文研究了Orlicz 投影体和 Orlicz 质心体的性质. 利用几何分析的方法, 获得了 Orlicz 投影算子和 Orlicz 质心算子的连续性.

关键词: Orlicz 投影体; Orlicz 质心体; 收敛性

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