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THE NUMBER OF SMALL COVERS OVER PRODUCTS OF A SIMPLEX WITH 3-CUBE UP TO EQUIVARIANT COBORDISM

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Abstract: In this paper, we study equivariant cobordism classification of small covers. By using characteristic and Stong homomorphism, we determine the number of equivariant cobordism classes of small covers over products of a simplex with 3-cube, which extends the existing related result in literature.

Keywords: cobordism; small cover; tangential representation

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1 Introduction

The notion of small covers was first introduced by Davis and Januszkiewicz [1], where a small cover is a smooth closed manifold M^n with a locally standard $(\mathbb{Z}_2)^n$-action such that its orbit space is a simple convex polytope. For instance, the n-dimensional real projective space $\mathbb{R}P^n$ with a natural $(\mathbb{Z}_2)^n$-action is a small cover over an n-simplex. In recent years, several studies attempted to enumerate the number of Davis-Januszkiewicz equivalence classes and equivariant homeomorphism classes of small covers over a specific polytope, see [2–6].

By $\mathcal{M}_n$ we denote the set of equivariant unoriented cobordism classes of all n-dimensional small covers. Let $\mathcal{M}_* = \sum\limits_{n\geq 1} \mathcal{M}_n$. From [7, Theorems 1.4, 1.5, Corollary 5.8], $\mathcal{M}_*$ is generated by the classes of small covers over the product of simplices. When the dimension of each simplex is 1 or when the number of simplices is at most 3, we determine the number of small covers over the product of simplices up to equivariant cobordism [8]. In 2008, Wu determined equivariant cobordism classificaton of small covers over 3-dimensional prisms [9].

Let Δ^m, I^3 be m-simplex and 3-cube, respectively. The main results of this paper are stated as follows:

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Theorem 1 When $m \geq 2$, the number of equivariant cobordism classes of small covers over $\Delta^m \times I^3$ is

$$\frac{\prod\limits_{t=1}^{m+3}(2^{m+3}-2^{t-1})}{48(m+1)!}(25\cdot 2^{3m}-9\cdot 2^{2m+2}+6\cdot 2^{m+1}-1)+1.$$

All small covers over $\Delta^1 \times I^3$ equivariantly bound.

The paper is organized as follows. In Section 2, we review some basic facts about small covers and the tangential representation. In Section 3, using characteristic functions and Stong homomorphism, we prove Theorem 1.

2 Preliminaries

An n-dimensional convex polytope P^n is said to be simple, if exactly n faces of codimension one meet at each of its vertices. An n-dimensional smooth closed manifold M^n is said to be a small cover if it admits a smooth $(\mathbb{Z}_2)^n$-action such that the action is locally isomorphic to a standard action of $(\mathbb{Z}_2)^n$ on $\mathbb{R}^n$ and the orbit space $M^n/(\mathbb{Z}_2)^n$ is a simple convex polytope of dimension n.

Suppose that $\pi : M^n \to P^n$ is a small cover over a simple convex polytope P^n. Let $\mathcal{F}(P^n) = \{F_1, \cdots, F_\ell\}$ be the set of codimension-one faces (facets) of P^n. Then there are ℓ connected submanifolds $\pi^{-1}(F_1), \cdots, \pi^{-1}(F_\ell)$. Each submanifold $\pi^{-1}(F_i)$ is fixed pointwise by a $\mathbb{Z}_2$-subgroup $\mathbb{Z}_2(F_i)$ of $(\mathbb{Z}_2)^n$, so that each facet F_i corresponds to the $\mathbb{Z}_2$-subgroup $\mathbb{Z}_2(F_i)$. Obviously, the $\mathbb{Z}_2$-subgroup $\mathbb{Z}_2(F_i)$ actually agrees with an element ν_i in $(\mathbb{Z}_2)^n$ as a vector space. For each face F of codimension u, since P^n is simple, there are u facets $F_{i_1}, \cdots, F_{i_u}$ such that $F = F_{i_1} \cap \cdots \cap F_{i_u}$. Then, the corresponding submanifolds $\pi^{-1}(F_{i_1}), \cdots, \pi^{-1}(F_{i_u})$ intersect transversally in the $(n-u)$-dimensional submanifold $\pi^{-1}(F)$, and the isotropy subgroup $\mathbb{Z}_2(F)$ of $\pi^{-1}(F)$ is a subtorus of rank u and is generated by $\mathbb{Z}_2(F_{i_1}), \cdots, \mathbb{Z}_2(F_{i_u})$ (or is determined by $\nu_{i_1}, \cdots, \nu_{i_u}$ in $(\mathbb{Z}_2)^n$). Thus, this actually gives a characteristic function [1]

$$\lambda : \mathcal{F}(P^n) \longrightarrow (\mathbb{Z}_2)^n$$

defined by $\lambda(F_i) = \nu_i$ such that whenever the intersection $F_{i_1} \cap \cdots \cap F_{i_u}$ is non-empty, $\lambda(F_{i_1}), \cdots, \lambda(F_{i_u})$ are linearly independent in $(\mathbb{Z}_2)^n$.

In fact, Davis and Januszkiewicz gave a reconstruction process of a small cover by using a characteristic function $\lambda : \mathcal{F}(P^n) \longrightarrow (\mathbb{Z}_2)^n$. Let $\mathbb{Z}_2(F_i)$ be the subgroup of $(\mathbb{Z}_2)^n$ generated by $\lambda(F_i)$. Given a point $p \in P^n$, by $F(p)$ we denote the minimal face containing p in its relative interior. Assume $F(p) = F_{i_1} \cap \cdots \cap F_{i_u}$ and $\mathbb{Z}_2(F(p)) = \bigoplus_{j=1}^{u} \mathbb{Z}_2(F_{i_j})$. Note that $\mathbb{Z}_2(F(p))$ is a u-dimensional subgroup of $(\mathbb{Z}_2)^n$. Let $M(\lambda)$ denote $P^n \times (\mathbb{Z}_2)^n/\sim$, where $(p, g) \sim (q, h)$ if $p = q$ and $g^{-1}h \in \mathbb{Z}_2(F(p))$. The free action of $(\mathbb{Z}_2)^n$ on $P^n \times (\mathbb{Z}_2)^n$ descends to an action on $M(\lambda)$ with quotient P^n. Thus $M(\lambda)$ is a small cover over P^n [1].

By $\Lambda(P^n)$ we denote the set of all characteristic functions on P^n. Then we have

Theorem 2.1 Let $\pi : M^n \to P^n$ be a small cover over a simple convex polytope P^n. Then all small covers over P^n are given by $\{M(\lambda)|\lambda \in \Lambda(P^n)\}$ from the viewpoint of cobordism.

Remark 1 Generally speaking, we can't make sure that there always exist small covers over a simple convex polytope P^n when $n \geq 4$. For example, see [1, Nonexample 1.22]. From [1], $\mathbb{R}P^m$ is a small cover over Δ^m, and the 3-dimensional torus T^3 is a small cover over I^3. Thus, $\mathbb{R}P^m \times T^3$ is a small cover over $\Delta^m \times I^3$.

Next we recall some results in [10]. Let $G = (\mathbb{Z}_2)^n$ and ρ_0 be the trivial element in $\mathrm{Hom}(G, \mathbb{Z}_2)$ (the set of all homomorphisms from G to $\mathbb{Z}_2$). The irreducible real G-representations are all one-dimensional and correspond to all elements in $\mathrm{Hom}(G, \mathbb{Z}_2)$. Given an element β of $\mathcal{M}_n$, let (M^n, ϕ) be a representative of β such that M^n is a small cover. Take an isolated point p in the fixed point set $(M^n)^G$, then the G-representation at p can be written as $\tau_p(M^n) = \bigoplus_{\rho\neq\rho_0} \lambda_\rho^{q_\rho}$, where $\lambda_\rho : G \times \mathbb{R} \longrightarrow \mathbb{R}$, $(g, x) \mapsto \rho(g) \cdot x$ with $\rho \in \mathrm{Hom}(G, \mathbb{Z}_2)$ is the irreducible real G-representation and $\Sigma_{\rho\neq\rho_0} q_\rho = n$ and if $q_\rho \neq 0$, then $q_\rho = 1$. $\mathcal{N}_{M^n} = \{[\tau_p(M^n)]|p \in (M^n)^G\}$ is called the tangential representation set of (M^n, ϕ), where by $[\tau_p(M^n)]$ we denote the isomorphism class of $\tau_p(M^n)$.

The homomorphisms $\rho_i : (g_1, \cdots, g_n) \longmapsto g_i$ form a standard basis of $\mathrm{Hom}(G, \mathbb{Z}_2)$. Let $R_n(G)$ denote the vector space over $\mathbb{Z}_2$ generated by the representation classes of dimension n. Then $R_*(G) = \sum_{n\geq 0} R_n(G)$ is isomorphic to the graded polynomial algebra $\mathbb{Z}_2[\rho_1, \cdots, \rho_n]$. Each $[\tau_p(M^n)]$ of $\mathcal{N}_{M^n}$ uniquely corresponds to a monomial of degree n in $\mathbb{Z}_2[\rho_1, \cdots, \rho_n]$ such that all n factors of the monomial form a basis of $\mathrm{Hom}(G, \mathbb{Z}_2)$. In [11], Stong showed that the natural homomorphism (Stong homomorphism) $\delta_n : \mathcal{M}_n \longrightarrow R_n(G)$ defined by

$$\delta_n([M^n, \phi]) = \sum_{p\in(M^n)^G} [\tau_p(M^n)]$$

is a monomorphism. This implies that for each β in $\mathcal{M}_n$, there exists a representative (M^n, ϕ) of β such that $\mathcal{N}_{M^n}$ is prime (i.e., either all elements of $\mathcal{N}_{M^n}$ are distinct or $\mathcal{N}_{M^n}$ is empty) and $\mathcal{N}_{M^n}$ is independent of the choice of representatives of β. Thus we can define $\mathcal{N}_\beta := \mathcal{N}_{M^n}$. Obviously we have $\beta_1 = \beta_2 \Longleftrightarrow \mathcal{N}_{\beta_1} = \mathcal{N}_{\beta_2}$, for $\beta_1, \beta_2 \in \mathcal{M}_n$.

Let $\pi : M^n \to P^n$ be a small cover over a simple convex polytope P^n. The set of the vertices of P^n is just the image of $(M^n)^G$ under the map π. Let E denote an edge (1-dimensional face) of P^n, then $\pi^{-1}(E)$ is a connected 1-dimensional G-submanifold of M^n by [1, Lemma 1.3]. For $p \in (M^n)^G$ and $\pi(p) \in E$, p is also a fixed point of this submanifold. We have a 1-dimensional real tangential representation $\tau_p(\pi^{-1}(E))$ of G at p. Suppose that $E_{i_1}, \cdots, E_{i_n}$ are the n edges that meet at $\pi(p)$. Then $\bigoplus_{k=1}^n \tau_p(\pi^{-1}(E_{i_k}))$ just gives $\tau_p(M^n)$. The isotropy group of $\pi^{-1}(E)$ is of rank n-1. Thus the tangential representation $\tau_p(\pi^{-1}(E))$ is determined by the vector orthogonal to the isotropy group (regarded as a subspace of $(\mathbb{Z}_2)^n$). Each edge is the intersection of n-1 facets. Suppose $E = \bigcap_{k=1}^{n-1} F_{j_k}$, where F_{j_k} denotes a facet. The vectors $\lambda(F_{j_k}), k = 1, \cdots, n-1$, span the isotropy group of $\pi^{-1}(E)$. So the characteristic function uniquely determines the tangential representation $\tau_p(M^n)$.

3 The Number of Small Covers

Let $e_1, e_2, \cdots, e_{m+3}$ be the standard basis of $(\mathbb{Z}_2)^{m+3}$. Using characteristic functions and Stong homomorphism, we give the proof of Theorem 1.

The Proof of Theorem 1 When $m = 1$, $\Delta^m \times I^3 = I^4$. From [8], all small covers over I^4 equivariantly bound. We shall be particularly concerned with the case $m \geq 2$.

In fact, $\Delta^m \times I^3 = \Delta^m \times I \times I \times I$. To be convenient, we introduce the following marks. By $F_1', \cdots, F_{m+1}'$ we denote all facets of m-simplex Δ^m. Let b_{11}, b_{12} be two vertices of the second factor I, b_{21}, b_{22} be two vertices of the third factor I and b_{31}, b_{32} be two vertices of the last factor I. Let

$$\begin{aligned}&F_i = F_i' \times I^3, 1 \leq i \leq m+1,\\&F_{m+2} = \Delta^m \times b_{11} \times I^2, F_{m+3} = \Delta^m \times b_{12} \times I^2, F_{m+4} = \Delta^m \times I \times b_{21} \times I,\\&F_{m+5} = \Delta^m \times I \times b_{22} \times I, F_{m+6} = \Delta^m \times I^2 \times b_{31}, F_{m+7} = \Delta^m \times I^2 \times b_{32}.\end{aligned}$$

Then $\mathcal{F}(\Delta^m \times I^3) = \{F_1, \cdots, F_{m+7}\}$.

We choose $F_1, F_2, \cdots, F_m, F_{m+2}, F_{m+4}, F_{m+6}$ such that they meet at one vertex of $\Delta^m \times I^3$. Without loss of generality, let $\lambda(F_i) = e_i, 1 \leq i \leq m; \lambda(F_{m+2}) = e_{m+1}, \lambda(F_{m+4}) = e_{m+2}, \lambda(F_{m+6}) = e_{m+3}$. By the linear independence condition of characteristic functions, we have $\lambda(F_{m+1}) = e_1 + \cdots + e_m + e_{k_1} + \cdots + e_{k_i}$, where $m+1 \leq k_1 < \cdots < k_i \leq m+3, 0 \leq i \leq 3$. Then our argument is divided into two cases:

(I) $\lambda(F_{m+1}) = e_1 + \cdots + e_m$.

(II) $\lambda(F_{m+1}) = e_1 + \cdots + e_m + e_{k_1} + \cdots + e_{k_i}$, where $m+1 \leq k_1 < \cdots < k_i \leq m+3, 1 \leq i \leq 3$.

(I) $\lambda(F_{m+1}) = e_1 + \cdots + e_m$.

In this case, by the linear independence condition of characteristic functions, we have

$$\begin{aligned}\lambda(F_{m+3}) \quad = \quad & e_{m+1} + e_{t_1} + \cdots + e_{t_j}, e_{m+1} + e_{m+2} + e_{t_1} + \cdots + e_{t_j},\\& e_{m+1} + e_{m+3} + e_{t_1} + \cdots + e_{t_j}\end{aligned}$$

or $e_{m+1} + e_{m+2} + e_{m+3} + e_{t_1} + \cdots + e_{t_j}$, where $1 \leq t_1 < \cdots < t_j \leq m, 0 \leq j \leq m$. When $\lambda(F_{m+3}) = e_{m+1}$, by Stong homomorphism, the small cover constructed from such λ equivariantly bounds. Here we only consider non-bounding small covers. Thus, $\lambda(F_{m+3}) \neq e_{m+1}$. Our argument is divided into four cases:

(I$_1$) $\lambda(F_{m+3}) = e_{m+1} + e_{t_1} + \cdots + e_{t_j}$, where $1 \leq t_1 < \cdots < t_j \leq m, 1 \leq j \leq m$,

(I$_2$) $\lambda(F_{m+3}) = e_{m+1} + e_{m+2} + e_{t_1} + \cdots + e_{t_j}$, where $1 \leq t_1 < \cdots < t_j \leq m, 0 \leq j \leq m$,

(I$_3$) $\lambda(F_{m+3}) = e_{m+1} + e_{m+3} + e_{t_1} + \cdots + e_{t_j}$, where $1 \leq t_1 < \cdots < t_j \leq m, 0 \leq j \leq m$,

(I$_4$) $\lambda(F_{m+3}) = e_{m+1} + e_{m+2} + e_{m+3} + e_{t_1} + \cdots + e_{t_j}$, where $1 \leq t_1 < \cdots < t_j \leq m, 0 \leq j \leq m$.

(I$_1$) $\lambda(F_{m+3}) = e_{m+1} + e_{t_1} + \cdots + e_{t_j}$, where $1 \leq t_1 < \cdots < t_j \leq m, 1 \leq j \leq m$. In this case, by the linear independence condition of characteristic functions, we have

$$\lambda(F_{m+5}) = e_{m+2} + e_{l_1} + \cdots + e_{l_k}$$

or $e_{m+2} + e_{m+3} + e_{l_1} + \cdots + e_{l_k}$, where $1 \le l_1 < \cdots < l_k \le m+1, 0 \le k \le m+1$. When $\lambda(F_{m+5}) = e_{m+2}$, by Stong homomorphism, the small cover constructed from λ equivariantly bounds. Thus, $\lambda(F_{m+5}) \ne e_{m+2}$. When

$$\lambda(F_{m+5}) = e_{m+2} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m+1$$

and $1 \le k \le m+1$, by the linear independence condition of characteristic functions and Stong homomorphism, $\lambda(F_{m+7}) = e_{m+3} + e_{f_1} + \cdots + e_{f_l}, 1 \le f_1 < \cdots < f_l \le m+2, 1 \le l \le m+2$. When

$$\lambda(F_{m+5}) = e_{m+2} + e_{m+3} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m+1$$

and $0 \le k \le m+1$, similarly we have

$$\lambda(F_{m+7}) = e_{m+3} + e_{g_1} + \cdots + e_{g_h}, 1 \le g_1 < \cdots < g_h \le m+1, 1 \le h \le m+1.$$

Thus, the values of λ have $3 \cdot 2^{3m+2} - 5 \cdot 2^{2m+2} + 9 \cdot 2^m - 1$ possible choices in case ($\mathbf{I}_1$).

($\mathbf{I}_2$) $\lambda(F_{m+3}) = e_{m+1} + e_{m+2} + e_{t_1} + \cdots + e_{t_j}$, where $1 \le t_1 < \cdots < t_j \le m, 0 \le j \le m$. By the linear independence condition of characteristic functions and Stong homomorphism, we have

$$\lambda(F_{m+5}) = e_{m+2} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m, 1 \le k \le m$$

or

$$\lambda(F_{m+5}) = e_{m+2} + e_{m+3} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m, 0 \le k \le m.$$

When

$$\lambda(F_{m+5}) = e_{m+2} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m$$

and

$$1 \le k \le m, \lambda(F_{m+7}) = e_{m+3} + e_{f_1} + \cdots + e_{f_l}, 1 \le f_1 < \cdots < f_l \le m+2, 1 \le l \le m+2.$$

When

$$\lambda(F_{m+5}) = e_{m+2} + e_{m+3} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m$$

and

$$0 \le k \le m, \lambda(F_{m+7}) = e_{m+3} + e_{g_1} + \cdots + e_{g_h}, 1 \le g_1 < \cdots < g_h \le m, 1 \le h \le m.$$

Thus, the values of λ have $5 \cdot 2^{3m} - 3 \cdot 2^{2m+1} + 2^m$ possible choices in case ($\mathbf{I}_2$).

($\mathbf{I}_3$) $\lambda(F_{m+3}) = e_{m+1} + e_{m+3} + e_{t_1} + \cdots + e_{t_j}$, where $1 \le t_1 < \cdots < t_j \le m, 0 \le j \le m$. If we first consider $\lambda(F_{m+7})$ and lastly consider $\lambda(F_{m+5})$ in this case, then the problem is reduced to case ($\mathbf{I}_2$), so the values of λ also have $5 \cdot 2^{3m} - 3 \cdot 2^{2m+1} + 2^m$ possible choices in case ($\mathbf{I}_3$).

($\mathbf{I_4}$) $\lambda(F_{m+3}) = e_{m+1} + e_{m+2} + e_{m+3} + e_{t_1} + \cdots + e_{t_j}$, where $1 \le t_1 < \cdots < t_j \le m, 0 \le j \le m$. By the linear independence condition of characteristic functions and Stong homomorphism, we have

$$\lambda(F_{m+5}) = e_{m+2} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m, 1 \le k \le m$$

or

$$\lambda(F_{m+5}) = e_{m+2} + e_{m+3} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m, 0 \le k \le m.$$

When

$$\lambda(F_{m+5}) = e_{m+2} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m$$

and $1 \le k \le m$, $\lambda(F_{m+7}) = e_{m+3} + e_{f_1} + \cdots + e_{f_l}$, $1 \le f_1 < \cdots < f_l \le m+2$, $f_1 \ne m+1, \cdots, f_l \ne m+1$, $1 \le l \le m+1$. When

$$\lambda(F_{m+5}) = e_{m+2} + e_{m+3} + e_{l_1} + \cdots + e_{l_k}, 1 \le l_1 < \cdots < l_k \le m$$

and

$$0 \le k \le m, \lambda(F_{m+7}) = e_{m+3} + e_{g_1} + \cdots + e_{g_h}, 1 \le g_1 < \cdots < g_h \le m, 1 \le h \le m.$$

Thus, the values of λ have $3 \cdot 2^{3m} - 2^{2m+2} + 2^m$ possible choices in case ($\mathbf{I}_4$).

So in case (I), the values of λ have $25 \cdot 2^{3m} - 9 \cdot 2^{2m+2} + 6 \cdot 2^{m+1} - 1$ possible choices.

(II) $\lambda(F_{m+1}) = e_1 + \cdots + e_m + e_{k_1} + \cdots + e_{k_i}$, where $m+1 \le k_1 < \cdots < k_i \le m+3$, $1 \le i \le 3$. In this case, no matter which value of $\lambda(F_{m+1})$ is chosen, the small cover constructed from λ equivariantly bounds. We only give the proof of the case $\lambda(F_{m+1}) = e_1 + \cdots + e_m + e_{m+1}$ because when other values of $\lambda(F_{m+1})$ are chosen, the proof is similar.

When $\lambda(F_{m+1}) = e_1 + \cdots + e_m + e_{m+1}$, by the linear independence condition of characteristic functions and Stong homomorphism, we have $\lambda(F_{m+3}) = e_{m+1} + e_{m+2}$. Similarly we have $\lambda(F_{m+5}) = e_{m+2} + e_{m+3}$ and $\lambda(F_{m+7}) = e_{m+3}$. By Stong homomorphism, the small cover constructed from such λ equivariantly bounds.

We may choose other basis of $(\mathbb{Z}_2)^{m+3}$. There are $\frac{\prod\limits_{t=1}^{m+3}(2^{m+3}-2^{t-1})}{48(m+1)!}$ choices for a basis of $(\mathbb{Z}_2)^{m+3}$ in this case if we consider equivariant cobordism classification by Stong homomorphism. Thus, there are

$$\frac{\prod\limits_{t=1}^{m+3}(2^{m+3} - 2^{t-1})}{48(m+1)!}(25 \cdot 2^{3m} - 9 \cdot 2^{2m+2} + 6 \cdot 2^{m+1} - 1)$$

non-bounding small covers over $\Delta^m \times I^3$ up to equivariant cobordism.

Adding the small cover that equivariantly bounds, we give the calculation formula of the number of small covers over $\Delta^m \times I^3$ up to equivariant cobordism.

References

[1] Davis M W, Januszkiewicz T. Convex polytopes, coxeter orbifolds and torus actions [J]. Duke Math. J., 1991, 62(2): 417–451.

[2] Cai M Z, Chen X, Lü Z. Small covers over prisms [J]. Topology Appl., 2007, 154(11): 2228–2234.

[3] Choi S. The number of small covers over cubes [J]. Algebr. Geom. Topol., 2008, 8(4): 2391–2399.

[4] Choi S. The number of orientable small covers over cubes [J]. Proc. Japan Acad. Ser. A Math. Sci., 2010, 86(6): 97–100.

[5] Chen Y C, Wang Y Y. Orientable small covers over products of a prism with a simplex [J]. An. St. Univ. Ovidius Constanta, 2011, 19(3): 71–84.

[6] Wang Y Y, Chen Y C. Small covers over products of a polygon with a simplex [J]. Turkish J. Math., 2012, 36(1): 161–172.

[7] Lü Z, Tan Q B. A differential operator and tom Dieck-Kosniowski-Stong localization theorem [J]. arXiv:1008.2166.

[8] Chen Y C, Wang Y Y. The number of small covers over cubes and the product of at most three simplices up to equivariant cobordism [J]. Proc. Japan Acad. Ser. A Math. Sci., 2011, 87(6): 95–98.

[9] Wu C Y. Classificaton of small covers over prisms up to equivariant cobordism [J]. Adv. Math. (China), 2008, 37(5): 584–590.

[10] Lü Z. 2-torus manifolds, cobordism and small covers [J]. Pacific J. Math., 2009, 241(2): 285–308.

[11] Stong R E. Equivariant bordism and $(\mathbb{Z}_2)^k$-actions [J]. Duke Math. J., 1970, 37(4): 779–785.

单形和3维立方体乘积上小覆盖的等变协边类的个数

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摘要: 本文研究了小覆盖的等变协边分类. 利用示性函数和Stong同态确定了单形和3维立方体乘积上小覆盖的等变协边类的个数, 推广了现有文献中的相关结果.

关键词: 协边; 小覆盖; 切表示

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