Vol. 34 (2014)
No. 1

数 学 杂 志
J. of Math. (PRC)

反五对角与拟反五对角方程组的追赶法

倪有义, 蔡 静
(湖州师范学院理学院, 浙江 湖州 313000)

摘 要: 本文研究了反五对角和拟五对角线性方程组的求解问题. 利用矩阵分解方法以及将系数矩阵 A 分解成三个简单矩阵的乘积 $A = LUD$, 获得了反五对角线性方程组以及拟反五对角线性方程组的追赶法, 从而推广了对角型线性方程组追赶法.

关键词: 反对角线性方程组; 追赶法; LU 分解
MR(2010) **主题分类号**: 65F05; 65F50 **中图分类号**: O241.6
文献标识码: A **文章编号**: 0255-7797(2014)01-0137-08

1 引言

随着现代工业和科学的发展，线性方程组的应用出现在经济管理、工程计算等各个领域，许多的应用会导出一些具有特殊结构的稀疏线性方程组的计算问题 [1,2]. 伴随着这些方程组的出现，寻找简便而且准确的求解方法就显得十分重要而且具有现实意义.

在对角线性方程组的解法中，追赶法是一类比较常用的方法，因其计算公式简单，运算量和存储量小，在科学领域中被广泛运用，倍受广大科研技术人员关注. 近年来，关于对角型线性方程组的追赶法，已经有了比较细致的研究，主要有求解三对角线性方程组、循环三对角线性方程组、五对角线性方程组、拟五对角线性方程组、对称循环五对角线性方程组和七对角线性方程组的各类追赶法，参见文献 [3–9]. 但目前尚未见文献探讨反五对角与拟反五对角方程组的追赶法，而反五对角与拟反五对角方程组是反对角线性方程组中比较常见的一类，在力学、流体力学、工程学等领域有很重要的应用 (例如神舟飞船运行轨道的某些计算问题可归结为此类线性方程组的求解问题). 因此，本文将借鉴文献 [6, 7] 的思想，建立求解反五对角线性方程组和拟反五对角线性方程组的追赶法.

2 预备知识

定义 1 若方阵 $A = (a_{ij})$ 的元素当 $1 \le i \le n-3$, $1 \le j \le n-i-2$ 且 $4 \le i \le n$, $n-i+4 \le j \le n$ 时, 均有 $a_{ij} = 0$, 则称此矩阵为反五对角矩阵.

*收稿日期: 2011-12-29 **接收日期**: 2012-04-11
基金项目: 国家自然科学基金 (11071079); 浙江省自然科学基金 (Y6110043); 校教研重点项目 (GJB11007) 及国家特色专业建设点“数学与应用数学”.
作者简介: 倪有义 (1989–), 男, 浙江金华, 本科生, 研究方向: 计算数学.
通讯作者: 蔡静

引理 1[9] 对于任意阶数不小于 3 的反五对角矩阵 A, 一般记:

$$A=\begin{pmatrix} & & & & e_1 & d_1 & c_1 \\ & & & e_2 & d_2 & c_2 & b_2 \\ & & e_3 & d_3 & c_3 & b_3 & a_3 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \\ e_{n-2} & d_{n-2} & c_{n-2} & b_{n-2} & a_{n-2} & & \\ d_{n-1} & c_{n-1} & b_{n-1} & a_{n-1} & & & \\ c_n & b_n & a_n & & & & \end{pmatrix}.$$

若有 $d_i,e_i\neq 0$, 且 $|c_1|\geq|d_1|+|e_1|$, $|c_2|\geq|b_2|+|d_2|+|e_2|$, $|c_{n-1}|\geq|a_{n-1}|+|b_{n-1}|+|d_{n-1}|$, $|c_n|\geq|b_n|+|a_n|$, $|c_i|\geq|a_i|+|b_i|+|d_i|+|e_i|$ $(i=3,\cdots,n-2)$, 其中至少有一个不等号严格成立，则此矩阵的各阶顺序主子式不等于零，存在唯一的分解 $A=LU$. 若上述条件不变，其中 $|c_n|>|b_n|+|a_n|$, 则称 A 为具非零元素链对角占优矩阵.

3 反五对角线性方程组的追赶法

在实际问题中, 经常遇到如下形式的线性方程组

$$\begin{pmatrix} & & & & e_1 & d_1 & c_1 \\ & & & e_2 & d_2 & c_2 & b_2 \\ & & e_3 & d_3 & c_3 & b_3 & a_3 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \\ e_{n-2} & d_{n-2} & c_{n-2} & b_{n-2} & a_{n-2} & & \\ d_{n-1} & c_{n-1} & b_{n-1} & a_{n-1} & & & \\ c_n & b_n & a_n & & & & \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ f_n \end{pmatrix}. \tag{2.1}$$

这种方程组称为反五对角线性方程组, 简记为 $Ax=f$.

假设

$$\begin{cases} |c_1|\geq|d_1|+|e_1|,\ d_1,e_1\neq 0; \\ |c_2|\geq|b_2|+|d_2|+|e_2|,\ b_2,d_2,e_2\neq 0; \\ |c_i|\geq|a_i|+|b_i|+|d_i|+|e_i|,\ a_i,b_i,d_i,e_i\neq 0\ \ (i=3,4,\cdots,n-2); \\ |c_{n-1}|\geq|a_{n-1}|+|b_{n-1}|+|d_{n-1}|,\ a_{n-1},b_{n-1},d_{n-1}\neq 0; \\ |c_n|\geq|b_n|+|a_n|,\ a_n,b_n\neq 0. \end{cases}$$

此时, A 为具非零元素链对角占优矩阵，可实现矩阵 LU 分解. 分解形式为

$$A=\begin{pmatrix} & & & & & l_1 \\ & & & & l_2 & m_2 \\ & & & l_3 & m_3 & s_3 \\ & & \vdots & \vdots & \vdots & \\ & \vdots & \vdots & \vdots & & \\ l_n & m_n & s_n & & & \end{pmatrix}\begin{pmatrix} 1 & & & & & \\ p_2 & 1 & & & & \\ q_3 & p_3 & 1 & & & \\ & \vdots & \vdots & \vdots & & \\ & & \vdots & \vdots & \vdots & \\ & & & q_n & p_n & 1 \end{pmatrix}=LU. \tag{2.2}$$

利用矩阵乘法，可得

$$\begin{cases} p_n = d_1/c_1, l_1 = c_1, m_2 = b_2, l_2 = c_2 - b_2 d_1/c_1, \\ s_i = a_i, m_i = b_i - a_i p_{n-i+3}, l_i = c_i - a_i p_{n-i+3} - m_i p_{n-i+2}, i = 3, 4, \cdots, n, \\ p_{n-i+1} = (d_i - m_i q_{n-i+2})/l_i, i = 2, 3, \cdots, n-1, \\ q_{n-i+1} = e_i/l_i, i = 1, 2, \cdots, n-2. \end{cases} \tag{2.3}$$

可将求解方程组 $Ax = f$ 化为依次求解 $\begin{cases} Ly = f, \\ Ux = y. \end{cases}$

算法 1

第一步：解方程 $Ly = f$, 即“追”过程, 算法如下:

$$\begin{cases} y_n = f_1/c_1, \\ y_{n-1} = (f_2 c_1 - b_2 y_n c_1)/(c_1 c_2 - b_2 d_1), \\ y_{n-i+1} = (f_i - m_i y_{n-i+2} - a_i y_{n-i+3})/l_i, i = 3, 4, \cdots, n. \end{cases}$$

第二步：解方程 $Ux = y$, 即“赶”过程，算法如下：

$$\begin{cases} x_1 = y_1, \\ x_2 = y_2 - x_1 p_2, \\ x_i = y_i - x_{i-2} q_i - x_{i-1} p_i, i = 3, 4, \cdots, n, \end{cases}$$

其中 l_i, m_i, p_i, q_i 的计算见 (2.3) 式.

上述算法 1 就是求解反五对角线性方程组的追赶法.

算例演示：用追赶法求解反五对角线性方程组:

$$\begin{cases} -2x_5 - 2x_6 + 4x_7 = 6, \\ -x_4 - 2x_5 + 5x_6 - 2x_7 = 2, \\ -2x_3 - x_4 + 6x_5 - x_6 - 2x_7 = 0, \\ -2x_2 - x_3 + 6x_4 - x_5 - 2x_6 = 0, \\ -x_1 - x_2 + 5x_3 - 2x_4 - x_5 = -1, \\ -3x_1 + 6x_2 - x_3 - 2x_4 = -2, \\ 4x_1 - 2x_2 - x_3 = -3, \end{cases}$$

有

$$A = \begin{pmatrix} & & & & -2 & -2 & 4 \\ & & & -1 & -2 & 5 & -2 \\ & & -2 & -1 & 6 & -1 & -2 \\ & -2 & -1 & 6 & -1 & -2 & \\ -1 & -1 & 5 & -2 & -1 & & \\ -3 & 6 & -1 & -2 & & & \\ 4 & -2 & -1 & & & & \end{pmatrix}, \quad f = \begin{pmatrix} 6 \\ 2 \\ 0 \\ 0 \\ -1 \\ -2 \\ -3 \end{pmatrix},$$

$$\begin{aligned} & y_7 = f_1/c_1 = 6/4 = 3/2, \\ & y_6 = (f_2 c_1 - b_2 y_7 c_1)/(c_1 c_2 - b_2 d_1) \\ = \quad & [2 \times 4 - (-2) \times (3/2) \times 4]/[4 \times 5 - (-2) \times (-2)] = 5/4, \end{aligned}$$

$q_7 = e_1/l_1 = -1/2, l_2 = c_2 - b_2 d_1/c_1 = 5 - (-2) \times (-2)/4 = 4,$
$p_6 = (d_2 - m_2 q_7)/l_2 = [-2 - (-2) \times (-1/2)]/4 = -3/4,$
$m_3 = b_3 - a_3 d_1/c_1 = (-1) - (-2) \times (-2/4) = -2,$
$l_3 = c_3 - a_3 q_7 - m_3 p_6 = 6 - (-2) \times (-1/2) - (-2) \times (-3/4) = 7/2,$
$y_5 = (f_3 - m_3 y_6 - a_3 y_7)/l_3 = [0 - (-2)) \times (5/4) - (-2) \times (3/2)]/(7/2) = 11/7,$
$q_6 = e_2/l_2 = -1/4, p_5 = (d_3 - m_3 q_6)/l_3 = [-1 - (-2) \times (-1/4)]/(7/2) = -3/7,$
$m_4 = b_4 - a_4 p_6 = (-1) - (-2) \times (-3/4) = -5/2,$
$l_4 = c_4 - a_4 q_6 - m_4 p_5 = 6 - (-2) \times (-1/4) - (-5/2) \times (-3/7) = 31/7,$
$y_4 = (f_4 - m_4 y_5 - a_4 y_6)/l_4 = [0 - (-5/2) \times (11/7) - (-2) \times (5/4)]/(31/7) = 45/31,$
$q_5 = e_3/l_3 = -2/(7/2) = -4/7,$
$p_4 = (d_4 - m_4 q_5)/l_4 = [-1 - (-5/2) \times (-4/7)]/(31/7) = -17/31,$
$m_5 = b_5 - a_5 p_5 = -2 - (-1) \times (-3/7) = -17/7,$
$l_5 = c_5 - a_5 q_5 - m_5 p_4 = 5 - (-1) \times (-4/7) - (-17/7) \times (-17/31) = 96/31,$
$y_3 = (f_5 - m_5 y_4 - a_5 y_5)/l_5 = [-1 - (-17/7) \times 45/31 - (-1) \times 11/7]/(96/31) = 127/96,$
$q_4 = e_4/l_4 = -2/(31/7) = -14/31,$
$p_3 = (d_5 - m_5 q_4)/l_5 = [-1 - (-17/7) \times (-14/31)]/(96/31) = -65/96,$
$m_6 = b_6 - a_6 p_4 = -1 - (-2) \times (-17/31) = -65/31,$
$l_6 = c_6 - a_6 q_4 - m_6 p_3 = 6 - (-2) \times (-14/31) - (-65/31) \times (-65/96) = 353/96,$
$y_2 = (f_6 - m_6 y_3 - a_6 y_4)/l_6 = [-2 - (-65/31) \times 127/96 - (-2) \times 45/31]/(353/96) = 1,$
$q_3 = e_5/l_5 = -1/(96/31) = -31/96,$
$p_2 = (d_6 - m_6 q_3)/l_6 = [-3 - (-65/31) \times (-31/96)]/(353/96) = -1,$
$m_7 = b_7 - a_7 p_3 = -2 - (-1) \times (-65/96) = -257/96,$
$l_7 = c_7 - a_7 q_3 - m_7 p_2 = 4 - (-1) \times (-31/96) - (-257/96) \times (-1) = 1,$
$y_1 = (f_7 - m_7 y_2 - a_7 y_3)/l_7 = [-3 - (-257/96) \times 1 - (-1) \times 127/96]/1 = 1.$

利用以上结果, 我们可以得到关于 $x_i (i = 1, 2, 3, 4, 5, 6, 7)$ 的解:

$$
\begin{aligned}
& x_1 = y_1 = 1, x_2 = y_2 - x_1 p_2 = 1 - 1 \times (-1) = 2, \\
& x_3 = y_3 - x_1 q_3 - x_2 p_3 = 127/96 - 1 \times (-31/96) - 2 \times (-65/96) = 3, \\
& x_4 = y_4 - x_2 q_4 - x_3 p_4 = 45/31 - 2 \times (-14/31) - 3 \times (-17/31) = 4, \\
& x_5 = y_5 - x_3 q_5 - x_4 p_5 = 11/7 - 3 \times (-4/7) - 4 \times (-3/7) = 5, \\
& x_6 = y_6 - x_4 q_6 - x_5 p_6 = 5/4 - 4 \times (-1/4) - 5 \times (-3/4) = 6, \\
& x_7 = y_7 - x_5 q_7 - x_6 p_7 = 3/2 - 5 \times (-1/2) - 6 \times (-1/2) = 7.
\end{aligned}
$$

4 拟反五对角线性方程组的追赶法

在实际问题中，由于误差的原因，经常遇到如下形式的线性方程组

$$\begin{pmatrix} b_1 & a_1 & & & e_1 & d_1 & c_1 \\ a_2 & & & e_2 & d_2 & c_2 & b_2 \\ & & e_3 & d_3 & c_3 & b_3 & a_3 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \\ e_{n-2} & d_{n-2} & c_{n-2} & b_{n-2} & a_{n-2} & & \\ d_{n-1} & c_{n-1} & b_{n-1} & a_{n-1} & & & e_{n-1} \\ c_n & b_n & a_n & & & e_n & d_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ f_n \end{pmatrix}. \tag{3.1}$$

这种方程组称为拟反五对角线性方程组，简记 $Ax = f$. 可将系数矩阵 A 分解为 3 个矩阵的乘积 $A = LUD$, 其中

$$L\begin{pmatrix} & & & & p_1 \\ & & & p_1 & g_2 \\ & & p_3 & g_3 & a_3 \\ & \vdots & \vdots & \vdots & \\ p_n & g_n & a_n & & \end{pmatrix}, \quad U = \begin{pmatrix} 1 & & & & & & \\ 0 & 1 & & & & & \\ 0 & 0 & 1 & & & & \\ & 0 & q_4 & 1 & & & \\ & & h_5 & q_5 & 1 & & \\ & & & \vdots & \vdots & \vdots & \\ & & & & h_n & q_n & 1 \end{pmatrix}, D = \begin{pmatrix} 1 & & & s_1 & t_1 \\ t_2 & 1 & & & s_2 \\ t_3 & s_3 & 1 & & \\ \vdots & \vdots & \vdots & & \\ \vdots & \vdots & & \vdots & \\ t_n & s_n & & & 1 \end{pmatrix}.$$

矩阵元素的计算公式如下:

$$\begin{cases} p_1 = c_1, q_n = d_1/c_1, h_n = e_1/c_1, \omega_n = a_1/c_1, \sigma_n = b_1/c_1; \\ g_2 = b_2, p_2 = c_2 - b_2 d_1/c_1, q_{n-1} = (c_1 d_2 - b_2 e_1)/(c_1 c_2 - b_2 d_1), h_{n-1} = c_1 e_2/(c_1 c_2 - b_2 d_1); \\ \omega_{n-1} = -a_1 b_2/(c_1 c_2 - b_2 d_1), \sigma_{n-1} = (a_2 c_1 - b_1 b_2)/(c_1 c_2 - b_2 d_1); \\ \alpha_i = a_i, g_i = b_i - a_i q_{n-i+3}, p_i = c_i - a_i h_{n-i+3} - g_i q_{n-i+2}; \\ q_{n-i+1} = (d_i - g_i h_{n-i+2})/p_i, h_{n-i+1} = e_i/p_i; \\ \omega_{n-i+1} = (-a_i \omega_{n-i+3} - g_i \omega_{n-i+2})/p_i, \sigma_{n-i+1} = (-a_i \sigma_{n-i+3} - g_i \sigma_{n-i+2})/p_i, i = 3, 4, \ldots, n-4; \\ g_{n-3} = b_{n-3} - \alpha_{n-3} q_6, p_{n-3} = c_{n-3} - \alpha_{n-3} h_6 - g_{n-3} q_5; \\ q_4 = (d_{n-3} - g_{n-3} h_5)/p_{n-3}; \\ \omega_4 = (e_{n-3} - \alpha_{n-3}\omega_6 - g_{n-3}\omega_5)/p_{n-3}; \\ \sigma_4 = (-\alpha_{n-3}\sigma_6 - g_{n-3}\sigma_5)/p_{n-3}; \\ g_{n-2} = b_{n-2} - \alpha_{n-2} q_5, p_{n-2} = c_{n-2} - \alpha_{n-2} h_5 - g_{n-2} q_4; \\ \omega_3 = (d_{n-2} - \alpha_{n-2}\omega_5 - g_{n-2}\omega_4)/p_{n-2}; \\ \sigma_3 = (e_{n-2} - \alpha_{n-2}\sigma_5 - g_{n-2}\sigma_4)/p_{n-2}; \\ g_{n-1} = b_{n-1} - \alpha_{n-1} q_4, p_{n-1} = c_{n-1} - \alpha_{n-1}\omega_4 - g_{n-1}\omega_3; \\ \omega_2 = e_{n-1}/p_{n-1}, \sigma_2 = (d_{n-1} - \alpha_{n-1}\sigma_4 - g_{n-1}\sigma_3)/p_{n-1}; \\ g_n = b_n - \alpha_n \omega_3, p_n = c_n - \alpha_n \sigma_3 - g_n \sigma_2; \\ \sigma_1 = (d_n - g_n \omega_2)/p_n, \omega_1 = e_n/p_n; \\ s_i = \omega_i, t_i = \sigma_i, i = 1, 2, 3; \\ s_4 = \omega_4 - q_4 s_3, t_4 = \sigma_4 - q_4 t_3; \\ s_i = \omega_i - h_i s_{i-2} - q_i s_{i-1}, t_i = \sigma_i - h_i t_{i-2} - q_i t_{i-1}, i = 5, 6, \ldots, n. \end{cases}$$

可将求解方程组 $Ax=f$ 化为依次求解 $\begin{cases} Lz=f, \\ Uy=z, \\ Dx=y. \end{cases}$

算法 2

第一步: 解方程 $Lz=f$, 即"追" 过程, 算法如下:

$$\begin{cases} z_n=f_1/c_1, \\ z_{n-1}=(c_1f_2-c_1b_2z_n)/(c_1c_2-b_2d_1), \\ z_{n-i+1}=(f_i-g_iz_{n-i+2}-a_iz_{n-i+3})/p_i, i=3,4,\ldots,n. \end{cases}$$

第二步: 依次求解 $Uy=z$, $Dx=y$, 即"赶" 的过程, 算法如下:

$$\begin{cases} y_1=z_1, y_2=z_2, y_3=z_3, \\ y_4=z_4-q_4y_3, \\ y_i=z_i-h_iy_{i-2}-q_iy_{i-1}, i=5,6,\ldots,n, \end{cases}$$

$$\begin{cases} y_n=\frac{(y_{n-1}-t_{n-1}y_1+s_{n-1}t_2y_1-s_{n-1}y_2)(s_nt_2s_1-t_ns_1)-(y_n-t_ny_1+s_nt_2y_1-s_ny_2)(1-t_{n-1}s_1-s_{n-1}t_2s_1)}{(s_{n-1}t_2t_1-t_{n-1}t_1-s_{n-1}s_2)(s_nt_2s_1-t_ns_1)-(1-t_nt_1+s_nt_2t_1-s_ns_2)(1-t_{n-1}s_1+s_{n-1}t_2s_1)}; \\ x_{n-1}=\frac{(y_{n-1}-t_{n-1}y_1+s_{n-1}t_2y_1-s_{n-1}y_2)(1-t_nt_1+s_nt_2t_1-s_ns_2)}{(1-t_{n-1}s_1+s_{n-1}t_2s_1)(1-t_nt_1+s_nt_2t_1-s_ns_2)-(s_nt_2s_1-t_ns_1)(s_{n-1}t_2t_1-t_{n-1}t_1-s_{n-1}s_2)} \\ \qquad -\frac{(y_n-t_ny_1+s_nt_2y_1-s_ny_2)(s_{n-1}t_2t_1-t_{n-1}t_1-s_{n-1}s_2)}{(1-t_{n-1}s_1+s_{n-1}t_2s_1)(1-t_nt_1+s_nt_2t_1-s_ns_2)-(s_nt_2s_1-t_ns_1)(s_{n-1}t_2t_1-t_{n-1}t_1-s_{n-1}s_2)}; \\ x_i=y_i-t_ix_1-s_ix_2, i=3,4,\ldots,n-2; \\ x_2=y_2-t_2y_1+t_2s_1x_{n-1}+t_2t_1x_n-s_2x_n; \\ x_1=y_1-s_1x_{n-1}-t_1x_n, \end{cases}$$

其中 p_i,g_i,h_i,q_i,t_i,s_i 的值的计算见 (3.3) 式.

上述算法 2 就是求解拟反五对角线性方程组的追赶法.

算例演示: 用追赶法求解拟反五对角线性方程组:

$$\begin{cases} x_1+x_2-x_4-x_5+4x_6=18, \\ x_1-x_3-x_4+4x_5-x_6=8, \\ -x_2-x_3+4x_4-x_5-x_6=0, \\ -x_1-x_2+4x_3-x_4-x_5=0, \\ -x_1+4x_2-x_3-x_4+x_6=6, \\ 4x_1-x_2-x_3+x_5+x_6=10, \end{cases}$$

有

$$A=\begin{pmatrix} 1 & 1 & & -1 & -1 & 4 \\ 1 & & -1 & -1 & 4 & -1 \\ & -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 & \\ -1 & 4 & -1 & -1 & & 1 \\ 4 & -1 & -1 & & 1 & 1 \end{pmatrix}, \quad f=\begin{pmatrix} 18 \\ 8 \\ 0 \\ 0 \\ 6 \\ 10 \end{pmatrix};$$

$z_6=f_1/c_1=9/2; z_5=(c_1f_2-c_1b_2z_6)/(c_1c_2-b_2d_1)=50/15=10/3;$

$q_6=d_1/c_1=-1/4; q_5=(c_1d_2-b_2e_1)/(c_1c_2-b_2d_1)=-5/15=-1/3;$

$h_6 = e_1/c_1 = -1/4; p_3 = c_3 - a_3h_6 - g_3q_5 = 10/3, g_3 = b_3 - a_3q_6 = -5/4;$

$z_4 = (f_3 - g_3z_5 - a_3z_6)/p_3 = (26/3)/(10/3) = 13/5;$

$q_4 = (d_3 - g_3h_5)/p_3 = (-4/3)/(10/3) = -2/5, h_5 = c_1e_1/(c_1c_2 - b_2d_1) = -4/15;$

$p_4 = c_4 - a_4h_5 - g_4q_4 = 16/5, g_4 = b_4 - a_4q_5 = -4/3,$

$z_3 = (f_4 - g_4z_4 - a_4z_5)/p_4 = (34/5)/(16/5) = 17/8;$

$\omega_6 = a_1/c_1 = 1/4, \omega_5 = -a_1b_2/(c_1c_2 - b_2d_1) = 1/15,$

$\omega_4 = (e_3 - a_3\omega_6 - g_3\omega_5)/p_3 = (-2/3)/(10/3) = -1/5,$

$\omega_3 = (d_4 - a_4\omega_5 - g_5\omega_4)/p_4 = (-6/5)/(16/5) = -3/8;$

$g_5 = b_5 - a_5q_4 = -7/5, p_5 = c_5 - a_5\omega_4 - g_5\omega_3 = 131/40;$

$z_2 = (f_5 - g_5z_3 - a_5z_4)/p_5 = 463/131;$

$\sigma_6 = b_1/c_1 = 1/4, \sigma_5 = (a_2c_1 - b_1b_2)/(c_1c_2 - b_2d_1) = 1/3, \sigma_4 = (-a_3\sigma_6 - g_3\sigma_5)/p_3 = 1/5,$

$\sigma_3 = (e_4 - a_4\sigma_5 - g_4\sigma_4)/p_4 = (-2/5)/(16/5) = -1/8,$

$\sigma_2 = (d_5 - a_5\sigma_4 - g_5\sigma_3)/p_5 = (-39/40)/(131/40) = -39/131,$

$p_6 = c_6 - a_6\sigma_3 - g_6\sigma_2 = 454/131, g_6 = b_6 - a_6\omega_3 = -11/8,$

$z_1 = (f_6 - g_6z_2 - a_6z_3)/p_6 = (2225/131)/(454/131) = 2225/454;$

$y_1 = z_1 = 2225/454, y_2 = z_2 = 463/131, y_3 = z_3 = 17/8, y_4 = z_4 - q_4y_3 = 69/20,$

$y_5 = z_5 - h_5y_3 - q_5y_4 = 101/20, y_6 = z_6 - h_6y_4 - q_6y_5 = 53/8,$

$s_1 = \omega_1 = e_6/p_6 = 131/454, s_2 = \omega_2 = e_5/p_5 = 40/131, s_3 = \omega_3 = -3/8,$

$s_4 = \omega_4 - q_4s_3 = -7/20, s_5 = \omega_5 - h_5s_3 - q_5s_4 = -3/20, s_6 = \omega_6 - h_6s_4 - q_6s_5 = 1/8;$

$t_1 = \sigma_1 = (d_6 - g_6\omega_2)/p_6 = (186/131)/(454/131) = 186/454, t_2 = \sigma_2 = -39/131,$

$t_3 = \sigma_3 = -1/8, t_4 = \sigma_4 - q_4t_3 = 3/20,$

$t_5 = \sigma_5 - q_5t_3 - q_5t_4 = 7/20, t_6 = \sigma_6 - h_6t_4 - q_6t_5 = 3/8.$

利用以上结果, 我们最终可以得到关于 $x_i(i = 1, 2, 3, 4, 5, 6)$ 的解:

$$\begin{aligned}
& x_6 = 6; x_5 = 5; x_1 = y_1 - s_1x_5 - t_1x_6 = 2225/454 - 131/454 \times 5 - 186/454 \times 6 = 1; \\
& x_2 = y_2 - t_2y_1 + t_2s_1x_5 + t_2t_1x_6 - s_2x_6 \\
= \ & 463/131 - (-39/131) \times 2225/454 + (-39/131) \times 131/454 \\
& \times 5 + (-39/131) \times 186/454 \times 6 - 40/131 \times 6 = 2; \\
& x_3 = y_3 - t_3x_1 - s_3x_2 = 17/8 - (-1/8) \times 1 - (-3/8) \times 2 = 3; \\
& x_4 = y_4 - t_4x_1 - s_4x_2 = 69/20 - 3/20 \times 1 - (-7/20) \times 2 = 4.
\end{aligned}$$

5 结论

本文通过 LU 分解推导出了反五对角线性方程组的追赶法. 针对拟反五对角线性方程组的特点，沿用 LU 分解和追赶法的基本思想，把拟反五对角线性方程组的系数矩阵 A 分解成

3 个矩阵的乘积 LUD，与 LU 算法相比，虽然增加了一个矩阵 D，可是简化了推导过程的复杂度. 追赶法求解 n 阶反五对角线性方程组只需要 $O(11n)$ 的运算量，追赶法求解 n 阶拟反五对角线性方程组的运算量仅为 $O(39n)$ ，运算量比较小.

参 考 文 献

[1] 张明望. 一类非单调线性互补问题的高阶 Dikin 型仿射尺度算法. 数学杂志 [J], 2004, 24(5): 585–590.

[2] 邱茂路. 矩阵约当标准化的一个新方法 [J]. 数学杂志, 2001, 21(2): 237–240.

[3] 李文强, 马明. 求解循环三对角方程组的追赶法 [J]. 科技导报, 2009, 27(14): 69–72.

[4] 李文强, 刘晓. 追赶法并行求解循环三对角方程组 [J]. 科技导报, 2009, 27(18): 90–93.

[5] 夏爱生, 李长国, 胡宝安, 王瑞, 刘艳娜. 求解五对角方程组追赶法 [J]. 军事交通学院学报, 2008, 10(4): 87–89.

[6] 王礼广, 蔡放, 熊岳山. 五对角线性方程组追赶法 [J]. 南华大学学报（自然科学版）, 2008, 22(1): 1–4.

[7] 李文强, 马民, 李卫霞. 追赶法求解拟五对角线性方程组 [J]. 科技导报, 2010, 22(18): 60–63.

[8] 洪全兴. 解对称循环五对角线性方程组的一种方法 [D]. 厦门: 厦门大学, 2009.

[9] 叶新涛, 张志. 七对角线性方程组的追赶法 [J]. 绍兴文理学院学报, 2010, 30(10): 1–5.

CHASE-AFTER METHODS OF ANTI-PENTADIAGONAL AND QUASI ANTI-PENTADIAGONAL LINEAR EQUATIONS

NI You-yi, CAI Jing

(*School of Science, Huzhou Teachers College, Huzhou 313000, China*)

Abstract: In this paper, the problem of solving the anti-pentadiagonal and quasi anti-pentadiagonal linear equation is discussed, respectively. By using matrix decomposition method and decomposing the coefficient matrix A into the product of three simple matrix $A = LUD$, we deduce the chase-after methods of anti-pentadiagonal linear equations and quasi anti-pentadiagonal linear equations, which generalize the ones of diagonal linear equation.

Keywords: anti-diagonal linear equations; chase-after method; LU decomposition

2010 MR Subject Classification: 65F05; 65F50