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ASYMPTOTIC LOWER BOUNDS OF LARGE DEVIATION FOR SUMS OF UPPER TAIL ASYMPTOTIC INDEPENDENT RANDOM VARIABLES

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Abstract: This paper investigates the asymptotic lower bounds of large deviation for random variable sums of the upper tail asymptotic independent random variables with long tailed in a multi-risk model. By using the classic method of large deviation, we obtain some expressions of random and nonrandom sums, which extend the corresponding independent and identically distributed results.

Keywords: large deviation; upper tail asymptotic independence; multi-risk model

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1 Introduction

Motivated by the recent work of [1], we investigate the tail probability of the non-random sums in a multi-risk model

$$\sum_{i=1}^{k}\sum_{j=1}^{n_i} C_{ij}X_{ij}. \tag{1.1}$$

Here $\{X_{ij}, j\geq 1\}_{i=1}^{k}$ are the sequences of upper tail asymptotic independent random variables with long tailed, and $\{C_{ij}, j\geq 1\}_{i=1}^{k}$ be another nonnegative real sequences. The corresponding random sums of (1.1) is

$$\sum_{i=1}^{k}\sum_{j=1}^{N_i(t)} C_{ij}X_{ij}, \tag{1.2}$$

where $\{N_i(t)\}$ be non-negative integer-value process with $\lambda_i(t) = N_i(t)$, while $\{N_i(t), i = 1, 2, \cdots, k\}$ and $\{X_{ij}, j\geq 1\}_{i=1}^{k}$ are mutually independent.

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Since asymptotic behavior of precise large deviations for non-random sums and random sums of random variables has important theoretical significance and extensive applications, it attracts much attention and there appears to be a lot of research literature. For recent works of this aspect, we refer the reader to [1–13]. Among these papers, [11] studied the asymptotic lower bounds of precise large deviations for sums of nonnegative and independent and identically distributed random variable sequence $\{X_j, j \geq 1\}$. [9] extended the results of [11] to nonnegative and negatively dependent random variables. Also, [1] extended those of [11] to a multi-risk model. We will extend and improve their results to the upper tail asymptotic independent structure.

At the end of this section, we introduce some corresponding notations and concepts of this paper required. Denoted by $F(x) = P(X \leq x)$, $\overline{F}(x) = 1 - F(x)$, and $\lfloor x \rfloor$ is the integer part of x. We use the following notations for two positive functions $a_1(x)$ and $a_2(x)$

$$a_1(x) \gtrsim a_2(x) \text{ if } \liminf_{x\to\infty} \frac{a_1(x)}{a_2(x)} \geq 1; \ a_1(x) \sim a_2(x) \text{ if } \lim_{x\to\infty} \frac{a_1(x)}{a_2(x)} = 1.$$

Definition 1.1 [11] we say that a distribution F on $(-\infty, +\infty)$ belongs to the long-tailed distribution class, denoted by$F \in \mathcal{L}$, if for any $y \in (-\infty, +\infty)$,

$$\overline{F}(x + y) \sim \overline{F}(x).$$

Remark It is known that the long-tailed distribution class is one of the most important heavy-tailed distribution classes, where we say X (or its distribution F) is heavy tailed if it has no exponential moments.Also, one can see that, a distribution $F \in \mathcal{L}$ if and only if there exists a function $h(\cdot) : [0, \infty) \mapsto [0, \infty)$ such that $h(x) \to \infty$, $\lim\limits_{x\to\infty} \frac{h(x)}{x} = 0$ and

$$\overline{F}(x + y) \sim \overline{F}(x) \tag{1.3}$$

holds uniformly for all $|y| \leq h(x)$.

Definition 1.2 [12] we say that random variable sequence $\{X_j, j \geq 1\}$ is upper tail asymptotic independent (UTAI), if all natural numbers $i \neq j$,

$$\lim_{\min\{x_i, x_j\}\to\infty} P\left(X_i > x_i | X_j > x_j\right) = 0.$$

Remark For research on this structure, we refer the reader to [12], which presented some examples to illustrate that this structure is wider than the other dependent structures.

2 Asymptotic Lower Bounds for Nonrandom Sums

Theorem 2.1 For $i = 1, 2, \cdots, k$, let $\{X_{ij}, j \geq 1\}$ be a sequence of UTAI and nonnegative random variables with common distribution $F_i \in \mathcal{L}$, and let $\{n_i\}$ be a positive integer sequence. We assume that $\{X_{ij}, j \geq 1\}_{i=1}^k$ are mutually independent. Then, for any fixed $0 < C_{ij} < \infty$, $i = 1, 2, \cdots, k$, $j \geq 1$,

$$P\Big(\sum_{i=1}^{k} \sum_{j=1}^{n_i} C_{ij} X_{ij} > x\Big) \gtrsim \sum_{i=1}^{k} \sum_{j=1}^{n_i} P(C_{ij} X_{ij} > x) \tag{2.1}$$

holds as $x \to \infty$.

Proof We use induction to prove (2.1).

(I) When $k=1$, for an $h(x)$ satisfying (1.3), we have

$$\begin{aligned}
& P(\sum_{j=1}^{n_1} C_{1j}X_{1j} > x) \geq P(\sum_{j=1}^{n_1} C_{1j}X_{1j} > x, \bigcup_{j=1}^{n_1}(C_{1l}X_{1l} > x+h(x))) \\
\geq \ & \sum_{l=1}^{n_1} P(C_{1l}X_{1l} > x+h(x)) - \sum_{1\leq l<j\leq n_1} P(C_{1j}X_{1j} > x+h(x), C_{1l}X_{1l} > x+h(x)) \\
= \ & \sum_{l=1}^{n_1} P(C_{1l}X_{1l} > x+h(x)) \\
& - \sum_{1\leq l<j\leq n_1} P(C_{1j}X_{1j} > x+h(x) | C_{1l}X_{1l} > x+h(x)) P(C_{1l}X_{1l} > x+h(x)) \\
= \ & K_1 - K_2. \qquad (2.2)
\end{aligned}$$

By $F_1 \in \mathcal{L}$, we find

$$\liminf_{x\to\infty} \frac{K_1}{P(\sum\limits_{j=1}^{n_1} C_{1j}X_{1j} > x)} = \liminf_{x\to\infty} \frac{P(\sum\limits_{j=1}^{n_1} X_{1j} > \frac{x}{C_{1j}} + \frac{h(x)}{C_{1j}})}{P(\sum\limits_{j=1}^{n_1} C_{1j}X_{1j} > x)} \geq 1. \qquad (2.3)$$

For K_2, along with UTAI property, we have that

$$\begin{aligned}
& \limsup_{x\to\infty} \frac{K_2}{P\left(\sum\limits_{j=1}^{n_1} C_{1j}X_{1j} > x\right)} \\
& = \limsup_{x\to\infty} \left[\sum_{1\leq l<j\leq n_1} P(C_{1j}X_{1j} > x+h(x) | C_{1l}X_{1l} > x+h(x)) \frac{P(C_{1l}X_{1l} > x+h(x))}{P(\sum\limits_{j=1}^{n_1} C_{1j}X_{1j} > x)} \right] \\
& = 0. \qquad (2.4)
\end{aligned}$$

Hence, combining (2.2)–(2.4) leads to (2.1) when $k=1$, that is

$$P(\sum_{j=1}^{n_1} C_{1j}X_{1j} > x) \gtrsim \sum_{j=1}^{n_1} P(C_{1j}X_{1j} > x). \qquad (2.5)$$

(II) For the case in which $k=2$, there is

$$\begin{aligned}
& P(\sum_{j=1}^{n_1} C_{1j}X_{1j} + \sum_{j=1}^{n_2} C_{2j}X_{2j} > x) \\
\geq \ & P(\sum_{j=1}^{n_1} C_{1j}X_{1j} > x+h(x), \sum_{j=1}^{n_2} C_{2j}X_{2j} > -h(x)) \\
& + P(\sum_{j=1}^{n_1} C_{1j}X_{1j} > -h(x), \sum_{j=1}^{n_2} C_{2j}X_{2j} > x+h(x))
\end{aligned}$$

$$
\begin{aligned}
&= P(\sum_{j=1}^{n_1} C_{1j}X_{1j} > x + h(x))P(\sum_{j=1}^{n_2} C_{2j}X_{2j} > -h(x)) \\
&\quad + P(\sum_{j=1}^{n_1} C_{1j}X_{1j} > -h(x))P(\sum_{j=1}^{n_2} C_{2j}X_{2j} > x + h(x)).
\end{aligned}
$$

For any $0 < \delta < 1$, by (2.5) and $F_i \in \mathcal{L}$, $i = 1, 2$, for sufficiently large x, we get

$$
P(\sum_{j=1}^{n_i} C_{ij}X_{ij} > x + h(x)) \geq (1-\delta)[\sum_{j=1}^{n_i} C_{ij}X_{ij} > x], \;\; i = 1, 2. \tag{2.6}
$$

Since for $i = 1, 2$, $\{X_{ij}, j \geq 1\}$ are nonnegative, for any fixed $0 < C_{ij} < \infty$, $j = 1, 2, \cdots, n_i$, we have

$$
P(\sum_{j=1}^{n_i} C_{ij}X_{ij} > -h(x)) > (1-\delta), \;\; i = 1, 2. \tag{2.7}
$$

Then, using (2.6)–(2.7), we obtain

$$
P(\sum_{j=1}^{n_1} C_{1j}X_{1j} + \sum_{j=1}^{n_2} C_{2j}X_{2j} > x) \geq (1-\delta)^2 P[\sum_{j=1}^{n_1} P(C_{1j}X_{1j} > x) + P(\sum_{j=1}^{n_2} C_{2j}X_{2j} > x)].
$$

Therefore, letting $\delta \downarrow 0$, we obtain (2.1) when the case of $k = 2$.

(III) Now suppose that (2.1) holds for $k-1$. As for k, using a similar argument to that in (II), for any $0 < \delta < 1$ and any fixed C_{ij}, $i = 1, 2, \cdots, k$, $j \geq 1$, and when x is sufficiently large , we have

$$
\begin{aligned}
& P(\sum_{i=1}^{k}\sum_{j=1}^{n_i} C_{ij}X_{ij} > x) \\
\geq\; & P(\sum_{i=1}^{k-1}\sum_{j=1}^{n_i} C_{ij}X_{ij} > x + h(x)) + P(\sum_{j=1}^{n_k} C_{kj}X_{kj} > -h(x)) \\
\geq\; & (1-\delta)^2[\sum_{i=1}^{k-1}\sum_{j=1}^{n_i} P(C_{ij}X_{ij} > x + h(x))] + (1-\delta)^2 \sum_{j=1}^{n_k} P(C_{kj}X_{kj} > -h(x)) \\
=\; & (1-\delta)^2 \sum_{i=1}^{k}\sum_{j=1}^{n_i} P(C_{ij}X_{ij} > x + h(x)).
\end{aligned}
$$

Letting $\delta \downarrow 0$, we obtain the desired result, and the proof Theorem 2.1 is now complete.

3 Asymptotic Lower Bounds for Random Sums

Theorem 3.1 For $i = 1, 2, \cdots, k$, let$\{X_{ij}, j \geq 1\}$ be a sequence of UTAI and nonnegative random variables with common distribution $F_i \in \mathcal{L}$, and let $\{N_i(t)\}$ be nonnegative integer-value process with $\lambda_i(t) = N_i(t)$. We assume that $\{X_{ij}, j \geq 1\}_{i=1}^{k}$ and

$\{N_i(t), i=1,2,\cdots,k\}$ are mutually independent and that $\{N_i(t), i=1,2,\cdots,k\}$ satisfies

$$\text{Assumption I}: \ \frac{N_i(t)}{\lambda_i(t)} \to_p 1 \ \text{ as } \ t\to\infty.$$

Then, for any fixed $0<C_{ij}<\infty$, $i=1,2,\cdots,k$, $j\geq 1$,

$$P(\sum_{i=1}^{k}\sum_{j=1}^{N_i(t)} C_{ij}X_{ij}>x) \gtrsim \sum_{i=1}^{k}\sum_{j=1}^{N_i(t)} P(C_{ij}X_{ij}>x) \tag{3.1}$$

holds as $x\to\infty$.

Proof Again by induction, as in the proof of Theorem 3.1, it is sufficient to show that (3.1) holds for $k=1,2$.

(I) Taking $k=1$, for any $0<\delta<1$ and any fixed C_{1j}, $j\geq 1$, and for sufficiently large t,

$$\begin{aligned}
& P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j}>x) = \sum_{n=1}^{\infty} P(\sum_{j=1}^{n} C_{1j}X_{1j}>x)P(N_1(t)=n) \\
\geq\ & \sum_{\lfloor(1-\delta)\lambda_1(t)\rfloor\leq k\leq\lfloor(1+\delta)\lambda_1(t)\rfloor} P(\sum_{j=1}^{n} C_{1j}X_{1j}>x)P(N_1(t)=n) \\
\geq\ & P(\sum_{j=1}^{\lfloor(1-\delta)\lambda_1(t)\rfloor} C_{1j}X_{1j}>x)P(|\frac{N_1(t)}{\lambda_1(t)}-1|<\delta) \\
\geq\ & (1-\delta)^2 \sum_{j=1}^{\lfloor(1-\delta)\lambda_1(t)\rfloor} P(C_{1j}X_{1j}>x),
\end{aligned}$$

where the last inequality holds due to Assumption I and (2.5). Letting $\delta\downarrow 0$, Theorem 3.1 holds for $k=1$, that is

$$P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j}>x) \gtrsim \sum_{j=1}^{N_1(t)} P(C_{1j}X_{1j}>x). \tag{3.2}$$

(II) When $k=2$, we get

$$\begin{aligned}
& P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j} + \sum_{j=1}^{N_2(t)} C_{2j}X_{2j}>x) \\
\geq\ & P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j}>x+h(x), \sum_{j=1}^{N_2(t)} C_{2j}X_{2j}>-h(x)) \\
& +P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j}>-h(x), \sum_{j=1}^{N_2(t)} C_{2j}X_{2j}>x+h(x))
\end{aligned}$$

$$
\begin{aligned}
&= P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j} > x+h(x))P(\sum_{j=1}^{N_2(t)} C_{2j}X_{2j} > -h(x)) \\
&\quad +P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j} > -h(x))P(\sum_{j=1}^{N_2(t)} C_{2j}X_{2j} > x+h(x)).
\end{aligned}
$$

By (3.2) and $F_i \in \mathcal{L}(i=1,2)$, for any $0<\delta<1$, for sufficiently large t, we arrive

$$
P(\sum_{j=1}^{N_i(t)} C_{ij}X_{ij} > x+h(x)) \geq (1-\delta)[P(\sum_{j=1}^{N_i(t)} C_{ij}X_{ij} > x)], i=1,2. \tag{3.3}
$$

Since for $i=1,2$, $\{X_{ij}, j\geq 1\}$ are nonnegative, and for any fixed $0<C_{ij}<\infty$, $j\geq 1$, we have

$$
P(\sum_{j=1}^{N_i(t)} C_{ij}X_{ij} > -h(x)) > (1-\delta),\ \ i=1,2. \tag{3.4}
$$

Then, using (3.3)–(3.4), we obtain

$$
\begin{aligned}
&P(\sum_{j=1}^{N_1(t)} C_{1j}X_{1j} + \sum_{j=1}^{N_2(t)} C_{1j}X_{1j} > x) \\
\geq\ & (1-\delta)^2 P[\sum_{j=1}^{N_1(t)} P(C_{1j}X_{1j} > x) + P(\sum_{j=1}^{N_2(t)} C_{1j}X_{1j} > x)].
\end{aligned}
$$

Therefore, letting $\delta \downarrow 0$, we obtain (3.1) for $k=2$. The proof Theorem 3.1 is completed.

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上尾渐近独立随机变量和的大偏差的渐近下界

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摘 要: 本文研究了多元风险模型中服从长尾分布的带上尾渐近独立的随机变量和的大偏差渐近下界. 利用大偏差的经典求法, 得到了随机变量的非随机和和随机和的大偏差表达式, 推广了独立同分布情形下的相关结论.

关键词: 大偏差; 上尾渐近独立性; 多元风险模型

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