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QUASI-HADAMARD PRODUCT OF MEROMORPHIC UNIVALENT FUNCTIONS AT INFINITY

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Abstract: In this paper, we study quasi-Hadamard product problem for certain new subclasses of meromorphic starlike and convex functions in the punctured disk U^*. By using the method of convolution, we derive some results associated with the quasi-Hadamard product of functions belonging to these subclasses, which generalizes some known results.

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1 Introduction

Let Σ denote the class of functions f of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the punctured disk $U^* = \{z : 0 < |z| < 1\}$.

Also let Σ_α denote the class of functions of the form

$$F(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^{n-\frac{n}{\alpha}} \quad (\alpha \in N \setminus \{1\}), \tag{1.2}$$

which are analytic in the punctured disk U^* (cf. [1, 2]). When α goes to infinity then $(n - \frac{n}{\alpha})$ approaches n; hence $\Sigma_\alpha = \Sigma$.

Throughout this paper, let the functions of the form

$$f(z) = \frac{a_0}{z} + \sum_{n=1}^{\infty} a_n z^{n-\frac{n}{\alpha}} \ \ (a_0 > 0, a_n \geq 0, \alpha \in N \setminus \{1\}), \tag{1.3}$$

$$f_i(z) = \frac{a_{0,i}}{z} + \sum_{n=1}^{\infty} a_{n,i} z^{n-\frac{n}{\alpha}} \ \ (a_{0,i} > 0, a_{n,i} \geq 0, \alpha \in N \setminus \{1\}), \tag{1.4}$$

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$$g(z)=\frac{b_0}{z}+\sum_{n=1}^{\infty}b_nz^{n-\frac{n}{\alpha}}\ \ (b_0>0,b_n\ge 0,\alpha\in N\setminus\{1\}),\tag{1.5}$$

and

$$g_j(z)=\frac{b_{0,j}}{z}+\sum_{n=1}^{\infty}b_{n,j}z^{n-\frac{n}{\alpha}}\ \ (b_{0,j}>0,b_{n,j}\ge 0,\alpha\in N\setminus\{1\})\tag{1.6}$$

be regular and univalent in the punctured disk U^*.

For the function $F\in\Sigma_\alpha$, we define

$$\begin{aligned}I_\alpha^0F(z)&=F(z),\\ I_\alpha^1F(z)&=zF'(z)+\frac{2}{z},\\ I_\alpha^2F(z)&=z(I_\alpha^1F(z))'+\frac{2}{z},\end{aligned}$$

and for $k=1,2,\cdots$, we can write

$$I_\alpha^kF(z)=z(I_\alpha^{k-1}F(z))'+\frac{2}{z}=\frac{1}{z}+\sum_{n=1}^{\infty}\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^k a_nz^{n-\frac{n}{\alpha}},$$

where $\alpha\in N\setminus\{1\}$, $k\ge 0$ and $z\in U^*$. We note that when α goes to ∞ then $n(\frac{\alpha-1}{\alpha})$ approaches n; in this way we have $I_\alpha^k\to I^k$, which was introduced by Frasin and Darus [3] (see also [4]).

With the help of the differential operator I_α^k, we define the following subclasses of Σ_α.

Let $\Sigma_\alpha S^*(k,\beta,\gamma)$ be the class of functions F defined by (1.2) and satisfying the condition

$$\left|\frac{z(I_\alpha^kF(z))'}{I_\alpha^kF(z)}+1\right|<\beta\left|\frac{z(I_\alpha^kF(z))'}{I_\alpha^kF(z)}+2\gamma-1\right|\tag{1.7}$$

$$(z\in U^*,\ 0\le\gamma<1,\ 0<\beta\le 1,\ k\in N_0=N\cup\{0\},\ \alpha\in N\setminus\{1\}).$$

Also let $\Sigma_\alpha C(k,\beta,\gamma)$ be the class of functions F for which $-zF'(z)\in\Sigma_\alpha S^*(k,\beta,\gamma)$.

Using similar methods as given in [4], we can easily obtain the characterization properties for the classes $\Sigma_\alpha S^*(k,\beta,\gamma)$ and $\Sigma_\alpha C(k,\beta,\gamma)$ as follows.

Lemma 1.1 A function f defined by (1.3) belongs to the class $\Sigma_\alpha S^*(k,\beta,\gamma)$, if and only if

$$\sum_{n=1}^{\infty}\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^k\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]a_n\le 2\beta(1-\gamma)a_0.\tag{1.8}$$

Lemma 1.2 A function f defined by (1.3) belongs to the class $\Sigma_\alpha C(k,\beta,\gamma)$ if and only if

$$\sum_{n=1}^{\infty}\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{k+1}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]a_n\le 2\beta(1-\gamma)a_0.\tag{1.9}$$

We also note that when α goes to ∞ then we have $\Sigma_\alpha S^*(k,\beta,\gamma) \to \Sigma S^*(k,\beta,\gamma)$ and $\Sigma_\alpha C(k,\beta,\gamma) \to \Sigma C(k,\beta,\gamma)$, which are special classes that were introduced by El-Ashwah and Aouf [4].

Now, we introduce the following class of meromorphic univalent functions in U^*.

Definition 1.1 A function f of form (1.3), which is analytic in U^*, belongs to the class $\Sigma_\alpha^h(\beta,\gamma)$ if and only if

$$\sum_{n=1}^{\infty}\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^h\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]a_n \le 2\beta(1-\gamma)a_0, \tag{1.10}$$

where $0 \le \gamma < 1,\ 0 < \beta \le 1,\ \alpha \in N \setminus \{1\}$ and h is any fixed nonnegative real number. The class $\Sigma_\alpha^h(\beta,\gamma)$ is nonempty for any nonnegative real number h as the functions have the form

$$f(z) = \frac{a_0}{z} + \sum_{n=1}^{\infty}\frac{2\beta(1-\gamma)a_0}{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^h\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]}\lambda_n z^{n-\frac{n}{\alpha}}, \tag{1.11}$$

where $a_0 > 0$, $\lambda_n \ge 0$ and $\sum\limits_{n=1}^{\infty}\lambda_n \le 1$, satisfying inequality (1.10).

Clearly, we have the following relationships:

(i) $\Sigma_\alpha^k(\beta,\gamma) \equiv \Sigma_\alpha S^*(k,\beta,\gamma)$ and $\Sigma_\alpha^{k+1}(\beta,\gamma) \equiv \Sigma_\alpha C(k,\beta,\gamma)$;

(ii) $\Sigma_\alpha^{h_1}(\beta,\gamma) \subset \Sigma_\alpha^{h_2}(\beta,\gamma)$ $(h_1 > h_2 \ge 0)$;

(iii) $\Sigma_\alpha^h(\beta,\gamma) \subset \Sigma_\alpha^{h-1}(\beta,\gamma) \subset \cdots \subset \Sigma_\alpha C(k,\beta,\gamma) \subset \Sigma_\alpha S^*(k,\beta,\gamma)$ $(h > k+1)$.

Following the earlier works of Mogra [5, 6] and Aouf and Darwish [7] (see also [4, 8]), we define the quasi-Hadamard product of the functions $f(z)$ and $g(z)$ by

$$f * g(z) = \frac{a_0 b_0}{z} + \sum_{n=1}^{\infty} a_n b_n z^{n-\frac{n}{\alpha}}. \tag{1.12}$$

Similarly, we can define the quasi-Hadamard product of more than two functions, e.g.,

$$f_1 * f_2 * \cdots * f_p(z) = \left(\prod_{i=1}^{p} a_{0,i}\right)z^{-1} + \sum_{n=1}^{\infty}\left(\prod_{i=1}^{p} a_{n,i}\right)z^{n-\frac{n}{\alpha}}, \tag{1.13}$$

where the functions f_i $(i = 1, 2, \cdots, p)$ are given by (1.4).

The object of this paper is to derive certain results related to the quasi-Hadamard product of functions belonging to the classes $\Sigma_\alpha^h(\beta,\gamma)$, $\Sigma_\alpha S^*(k,\beta,\gamma)$ and $\Sigma_\alpha C(k,\beta,\gamma)$.

2 Main Results

Unless otherwise mentioned, we shall assume throughout the following results that $z \in U^*$, $0 \le \gamma < 1$, $0 < \beta \le 1$, $k \in N_0$, $\alpha \in N \setminus \{1\}$ and h is any fixed nonnegative real number.

Theorem 2.1 Let the functions $f_i(z)$ defined by (1.4) be in the class $\Sigma_\alpha C(k,\beta,\gamma)$ for every $i = 1, 2, \cdots, p$; and let the functions $g_j(z)$ defined by (1.6) be in the class $\Sigma_\alpha^h(\beta,\gamma)$ for every $j = 1, 2, \cdots, q$. Then the quasi-Hadamard product $f_1 * f_2 * \cdots * f_p * g_1 * g_2 * \cdots * g_q(z)$ belongs to the class $\Sigma_\alpha^{p(k+2)+q(h+1)-1}(\beta,\gamma)$.

Proof Let $G(z)=f_1*f_2*\cdots*f_p*g_1*g_2*\cdots*g_q(z)$, then

$$G(z)=\left(\prod_{i=1}^{p}a_{0,i}\prod_{j=1}^{q}b_{0,j}\right)z^{-1}+\sum_{n=1}^{\infty}\left(\prod_{i=1}^{p}a_{n,i}\prod_{j=1}^{q}b_{n,j}\right)z^{n-\frac{n}{\alpha}}. \tag{2.1}$$

It is sufficient to show that

$$\sum_{n=1}^{\infty}\left\{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{p(k+2)+q(h+1)-1}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]\right.$$
$$\left.\left(\prod_{i=1}^{p}a_{n,i}\prod_{j=1}^{q}b_{n,j}\right)\right\}\leq 2\beta(1-\gamma)\left(\prod_{i=1}^{p}a_{0,i}\prod_{j=1}^{q}b_{0,j}\right). \tag{2.2}$$

Since $f_i\in\Sigma_\alpha C(k,\beta,\gamma)$, by Lemma 1.2 we have

$$\sum_{n=1}^{\infty}\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{k+1}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]a_{n,i}\leq 2\beta(1-\gamma)a_{0,i} \tag{2.3}$$

for every $i=1,2,\cdots,p$. Thus,

$$\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{k+1}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]a_{n,i}\leq 2\beta(1-\gamma)a_{0,i}$$

or

$$a_{n,i}\leq\frac{2\beta(1-\gamma)}{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{k+1}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]}a_{0,i}$$

for every $i=1,2,\cdots,p$. The right-hand expression of the last inequality is not greater than $\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-(k+2)}a_{0,i}$. Therefore,

$$a_{n,i}\leq\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-(k+2)}a_{0,i} \tag{2.4}$$

for every $i=1,2,\cdots,p$. Also, since $g_j\in\Sigma_\alpha^h(\beta,\gamma)$, we find from (1.10) that

$$\sum_{n=1}^{\infty}\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{h}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]b_{n,j}\leq 2\beta(1-\gamma)b_{0,j}, \tag{2.5}$$

which implies that

$$b_{n,j}\leq\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-(h+1)}b_{0,j} \tag{2.6}$$

for every $j=1,2,\cdots,q$.

Using (2.4)–(2.6) for $i=1,2,\cdots,p; j=q$; and $j=1,2,\cdots,q-1$ respectively, we have

$$\sum_{n=1}^{\infty}\left\{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{p(k+2)+q(h+1)-1}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]\left(\prod_{i=1}^{p}a_{n,i}\prod_{j=1}^{q}b_{n,j}\right)\right\}$$
$$\leq\sum_{n=1}^{\infty}\left\{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{p(k+2)+q(h+1)-1}\cdot\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-p(k+2)}\cdot\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-(q-1)(h+1)}\right.$$
$$\left.\cdot\left(\prod_{i=1}^{p}a_{0,i}\prod_{j=1}^{q-1}b_{0,j}\right)\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]b_{n,q}\right\}$$

$$\begin{aligned} &\le \left(\prod_{i=1}^{p} a_{0,i} \prod_{j=1}^{q-1} b_{0,j}\right) \left\{\sum_{n=1}^{\infty} \left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{h} \left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right] b_{n,q}\right\} \\ &\le 2\beta(1-\gamma)\left(\prod_{i=1}^{p} a_{0,i} \prod_{j=1}^{q} b_{0,j}\right). \end{aligned}$$

Thus, we have $G(z) \in \Sigma_\alpha^{p(k+2)+q(h+1)-1}(\beta,\gamma)$. This completes the proof of Theorem 2.1.

Upon setting $h = k+1$ in Theorem 2.1, we obtain the following result.

Corollary 2.1 Let the functions $f_i(z)$ defined by (1.4) and the functions $g_j(z)$ defined by (1.6) belong to the class $\Sigma_\alpha C(k,\beta,\gamma)$ for every $i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots, q$. Then the quasi-Hadamard product $f_1 * f_2 * \cdots * f_p * g_1 * g_2 * \cdots * g_q(z)$ belongs to the class $\Sigma_\alpha^{p(k+2)+q(k+2)-1}(\beta,\gamma)$.

Theorem 2.2 Let the functions $f_i(z)$ defined by (1.4) be in the class $\Sigma_\alpha^h(\beta,\gamma)$ for every $i = 1, 2, \cdots, p$; and let the functions $g_j(z)$ defined by (1.6) be in the class $\Sigma_\alpha S^*(k,\beta,\gamma)$ for every $j = 1, 2, \cdots, q$. Then the quasi-Hadamard product $f_1 * f_2 * \cdots * f_p * g_1 * g_2 * \cdots * g_q(z)$ belongs to the class $\Sigma_\alpha^{p(h+1)+q(k+1)-1}(\beta,\gamma)$.

Proof Suppose that $G(z)$ be defined as (2.1). To prove the theorem, we need to show that

$$\begin{aligned} &\sum_{n=1}^{\infty} \left\{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{p(h+1)+q(k+1)-1} \left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right] \left(\prod_{i=1}^{p} a_{n,i} \prod_{j=1}^{q} b_{n,j}\right)\right\} \\ &\qquad \le 2\beta(1-\gamma)\left(\prod_{i=1}^{p} a_{0,i} \prod_{j=1}^{q} b_{0,j}\right). \end{aligned} \tag{2.7}$$

Since $f_i \in \Sigma_\alpha^h(\beta,\gamma)$, from (1.10) we have

$$\sum_{n=1}^{\infty} \left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{h} \left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right] a_{n,i} \le 2\beta(1-\gamma)a_{0,i}, \tag{2.8}$$

which implies that

$$a_{n,i} \le \left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-(h+1)} a_{0,i} \tag{2.9}$$

for every $i = 1, 2, \cdots, p$. Further, since $g_j \in \Sigma_\alpha S^*(k,\beta,\gamma)$, by Lemma 1.1 we have

$$\sum_{n=1}^{\infty} \left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{k} \left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right] b_{n,j} \le 2\beta(1-\gamma)b_{0,j}$$

for every $j = 1, 2, \cdots, q$. Whence we obtain

$$b_{n,j} \le \left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-(k+1)} b_{0,j} \tag{2.10}$$

for every $j = 1, 2, \cdots, q$.

Using (2.8)–(2.10) for $i = p; i = 1, 2, \cdots, p-1$; and $j = 1, 2, \cdots, q$ respectively, we get

$$\begin{aligned}
&\sum_{n=1}^{\infty}\left\{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{p(h+1)+q(k+1)-1}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]\left(\prod_{i=1}^{p}a_{n,i}\prod_{j=1}^{q}b_{n,j}\right)\right\}\\
\le\ &\sum_{n=1}^{\infty}\left\{\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{p(h+1)+q(k+1)-1}\cdot\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-(p-1)(h+1)}\cdot\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{-q(k+1)}\right.\\
&\left.\cdot\left(\prod_{i=1}^{p-1}a_{0,i}\prod_{j=1}^{q}b_{0,j}\right)\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]a_{n,p}\right\}\\
\le\ &\left(\prod_{i=1}^{p-1}a_{0,i}\prod_{j=1}^{q}b_{0,j}\right)\left\{\sum_{n=1}^{\infty}\left[n\left(\frac{\alpha-1}{\alpha}\right)\right]^{h}\left[n\left(\frac{\alpha-1}{\alpha}\right)(1+\beta)+(2\gamma-1)\beta+1\right]a_{n,p}\right\}\\
\le\ &2\beta(1-\gamma)\left(\prod_{i=1}^{p}a_{0,i}\prod_{j=1}^{q}b_{0,j}\right).
\end{aligned}$$

Therefore, we have $G(z) \in \Sigma_{\alpha}^{p(h+1)+q(k+1)-1}(\beta, \gamma)$. We complete the proof.

By taking $h = k$ in Theorem 2.2, we get the following result.

Corollary 2.2 Let the functions $f_i(z)$ defined by (1.4) and the functions $g_j(z)$ defined by (1.6) belong to the class $\Sigma_\alpha S^*(k, \beta, \gamma)$ for every $i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots, q$. Then the quasi-Hadamard product $f_1 * f_2 * \cdots * f_p * g_1 * g_2 * \cdots * g_q(z)$ belongs to the class $\Sigma_{\alpha}^{p(k+1)+q(k+1)-1}(\beta, \gamma)$.

By putting $h = k$ in Theorem 2.1 or $h = k+1$ in Theorem 2.2, we obtain the following result.

Corollary 2.3 Let the functions $f_i(z)$ defined by (1.4) be in the class $\Sigma_\alpha C(k, \beta, \gamma)$ for every $i = 1, 2, \cdots, p$; and let the functions $g_j(z)$ defined by (1.6) be in the class $\Sigma_\alpha S^*(k, \beta, \gamma)$ for every $j = 1, 2, \cdots, q$. Then the quasi-Hadamard product $f_1 * f_2 * \cdots * f_p * g_1 * g_2 * \cdots * g_q(z)$ belongs to the class $\Sigma_{\alpha}^{p(k+2)+q(k+1)-1}(\beta, \gamma)$.

Next, we discuss some applications of Theorems 2.1 and 2.2.

Taking into account the quasi-Hadamard product of functions $f_1(z), f_2(z), \cdots, f_p(z)$ only, in the proof of Theorem 2.1, and using (2.3) and (2.4) for $i = p$ and $i = 1, 2, \cdots, p-1$, respectively, we are led to

Corollary 2.4 Let the functions $f_i(z)$ defined by (1.4) belong to the class $\Sigma_\alpha C(k, \beta, \gamma)$ for every $i = 1, 2, \cdots, p$. Then the quasi-Hadamard product $f_1 * f_2 * \cdots * f_p(z)$ belongs to the class $\Sigma_{\alpha}^{p(k+2)-1}(\beta, \gamma)$.

Also, taking into account the quasi-Hadamard product of functions $g_1(z), g_2(z), \cdots, g_q(z)$ only, in the proof of Theorem 2.2, and using (2.10) and (2.11) for $j = q$ and $j = 1, 2, \cdots, q-1$, respectively, we are led to

Corollary 2.5 Let the functions $g_j(z)$ defined by (1.6) belong to the class $\Sigma_\alpha S^*(k, \beta, \gamma)$ for every $j = 1, 2, \cdots, q$. Then the quasi-Hadamard product $g_1 * g_2 * \cdots * g_q(z)$ belongs to the class $\Sigma_{\alpha}^{q(k+1)-1}(\beta, \gamma)$.

Remark 2.1 By letting $\alpha \to \infty$ in the proofs of Corollaries 3–5, we obtain the results obtained by El-Ashwah and Aouf [4, Theorems 3, 1 and 2, respectively].

Remark 2.2 By letting $\alpha \to \infty$ and $k = 0$ in the proofs of Corollaries 3–5, we obtain the results obtained by Mogra [6, Theorems 3, 1 and 2, respectively].

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无穷远点处亚纯单叶函数的拟Hadamard卷积

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摘 要: 本文研究了圆环域U^*内亚纯星象和凸象函数的某些新子类的拟Hadamard卷积. 利用卷积方法, 获得了该类函数的与拟Hadamard卷积有关的某些性质, 推广了一些已知结果.

关 键 词: 解析函数; 亚纯; 星象; 凸象; 拟Hadamard卷积

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